Dynamic weighted multi-criteria fuzzy decision- making based on vague sets

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Abstract

The paper discussed dynamic weighted multi-criteria fuzzy decision-making based on vague sets by applying dynamic weighting function to it. The new weighting method, whose advantages have been proved by numerous examples, was put forwarded by combining dynamic weighting with multi-criteria fuzzy decision-making after the advantages and deficiencies of the existing dynamic weighting function were analyzed. In the paper, new ideas will be provided to multi-criteria decision-making based on vague sets to make fuzzy comprehensive to be more wildly used.

Keywords

Vague set; Dynamic weighting; Multi-criteria decision- making.

1. Introduction

Gau and Buehere proposed vague set theory [1] in 1993. With further researches and development of intelligent systems, vague sets, fuzzy sets, rough sets, artificial neural networks, genetic algorithms are more and more widely used, becoming important soft computing methods. Unlike fuzzy set theory, vague theory, being able to express simultaneously the information of "support", "against", and "uncertain" has been widely used in many fields, such as controlling, decision making, fuzzy fault diagnosing and so on. Since Vague has the above characteristics, it is often applied to fuzzy decision deciding by scholars. As consequence, fuzzy multi-criteria decision-making based on Vague has been formed.

Recently, scholars have initially studied on the method of multi-objective decision making based on Vague Set Theory. In 1994, Chen and Tan [2] have studied for the first time the multi-criteria fuzzy decision making based on Vague Set Theory. In 2000, Hong and Choi [3] noted that Chen and Tan didn't fully considerate all the possible options, thus failed to provide three options of maxima, minimum and medium according to the attitude of policy-makers when they face risks. In 2001, Li Fan and other scholars [4] studied the fuzzy multi-objective decision-making based on vague set. In 2004, Liu Wenhua [5] proposed three methods: new scoring function, weighted scoring function and distance method for multi- objective decision making under fuzzy conditions by refining "abstaining section". In 2005, Lin Zhigui and his colleagues [6] have improved sorting function proposed in the literature [1, 2]. Wang Yu discussed the methods of fuzzy multi-objective decision making based on vague sets with fuzzy vague sets. Zhou Zhen and other scholars have studied fuzzy multi-criteria decision-making based on Vague set [8] and interval value of vague sets [9] in accordance with the improved sorting function.

In the method of multi-criteria decision making based on vague set, scoring functions $S\left(A_{ij}\right)$ and weighting $w_j, w_k, \cdots w_p$ have decisive effect on the results of the evaluation, so researchers focused on the scoring function and the weighting. According to researches, there are three kinds of Weighting Model currently, including linear weighting, which is like $y=\sum_{j=1}^{m} w_j x_j$, non-linear weighting model, similar to the application of nonlinear models $y=\prod_{j=1}^{m} x_j^{w_j}$,which highlights the consistency of the evaluated object and the less influence of different coefficients of weighting, the third category is a more ideal (TOPSIS) method, which sets an ideal target (samples) $x_1^*, x_2^*, \cdots, x_m^*$ for the evaluated objects, and then compares the ideal target and the evaluating index of the evaluated objects and finally determine the ranking[11]. In the methods of comprehensive weighting evaluation described above, the weighting coefficients are determined, namely steady weighting. There are three methods to determine the weighting $w_j, w_k, \cdots w_p$, including Subjective Weighting, Objective Weighting and Synthetic subjective and objective weighting. Though these methods are simple and feasible for simple practical problems, they appear to be less scientific because of relatively strong subjectivity. So they are not suitable for some more general problem of comprehensive evaluation, being not able to provide effective basis for decision-makers. Dynamic weighting method seems to be better, because it overcomes the drawbacks of the method described above. The weighting of this method is no longer a steady weighting, but a weighting function with property value as argument, which is consistent with common sense. After all, improving the results from zero to pass exams is relatively easier than from passing to full mark. The method is mainly used to solve the problems of more general comprehensive evaluation in practice.

This article established a new model of multi-criteria decision making based on vague set, which is verified with examples, by using a dynamic weighting function to determine the weighting of multiple criteria decision making based on vague set.

2. Concept of vague set

Supposing u is the domain, of which $u_i\left(i=1,2,\cdots n\right)$ represents an element, and the vague set A in the domain refers to a pair of membership functions t_A and f_A, that is $t_A: U \to [0,1]$, $f_A: U \to [0,1]$ with the requirement of $0 \le t_A\left(u_i\right) + f_A\left(u_i\right) \le 1$, in which $t_A\left(u_i\right)$ is called the true membership function of vague set A, showing supporting the fact that $u_i \in A$ are the next session of the degree of membership; $f_A\left(u_i\right)$ is called the false membership function of Vague set A, showing opposing the fact. $\pi_A\left(u_i\right) = 1 - t_A\left(u_i\right) - f_A\left(u_i\right)$ is called the degree of hesitation of $u_i\left(i=1,2,\cdots n\right)$ with respect to A, that is the missing information. Obviously, with the condition of $0 \le \pi_A\left(u_i\right) \le 1$, when $\pi_A\left(u_i\right)$ is greater, $u_i\left(i=1,2,\cdots n\right)$ indicates more missing information with respect to A, which is briefly recorded as $\langle t_A\left(u_i\right), 1 - f_A\left(u_i\right)\rangle$ or $\langle u_i, t_A\left(u_i\right), f_A\left(u_i\right)\rangle$. When U is discrete, $A = \sum_{i=1}^{n} [t_A\left(u_i\right), 1 - f_A\left(u_i\right)] / u_i$ and when U continuous, $A = \int [t_A\left(u\right), 1 - f_A\left(u\right)] / udu$

3. Multi-criteria decision making based on vague set

Supposing $U = \{u_1, u_2, \cdots u_m\}$ is the scheme set, $C = \{c_1, c_2, \cdots c_n\}$ the criteria set and under the evaluation criterion c_j the vague value of scheme u_i is $A_{ij} = [t_{ij}, 1 - f_{ij}]$, in which t_{ij} indicates the degree of scheme u_i meeting the condition of c_j, f_{ij} indicates the degree of scheme u_i discontenting with the condition of c_j, and $1 - f_{ij}$ indicates the degree of ignorance of c_j, So the vague value of the scheme set $U = \{u_1, u_2, \cdots u_m\}$ can be expressed as[12]

$$M = \begin{bmatrix} [t_{11}, 1 - f_{11}] & [t_{12}, 1 - f_{12}] & \cdots & [t_{1n}, 1 - f_{1n}] \\ [t_{21}, 1 - f_{21}] & [t_{22}, 1 - f_{22}] & \cdots & [t_{2n}, 1 - f_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [t_{m1}, 1 - f_{m1}] & [t_{m2}, 1 - f_{m2}] & \cdots & [t_{mn}, 1 - f_{mn}] \end{bmatrix} \quad (1)$$

with the criteria set $C=\{c_1,c_2,\cdots c_n\}$.

Scoring functions $S\left(A_{ij}\right)=S\left([t_{ij},1-f_{ij}]\right)\in[-1,1]$ are needed when decision makers want to choose a scheme which can better meet the criteria set. The comprehensive evaluation of each scheme is $W_C\left(u_i\right)=\sum_{j=1}^{n}w_jS\left(A_{ij}\right)$, in which $w_1,w_2,\cdots w_n$ is the weight of evaluation criteria.

If $W_C\left(u^*\right)=\max\{W_C\left(u_i\right),i=1,2,\cdots;m\}$, the scheme of u^* is the best scheme.

$$S(A)=t_A-f_A$$

There are three kinds of existing scoring functions. A representative one is the scoring function $S(A)$ on the basis of the absolute gap between true and false proposed in the lecture 2], that is $S(A)=t_A-f_A$

$$J_\infty(A)=t_A+\frac{t_A}{t_A+f_A}\pi_A \tag{2}$$

The second one is the scoring function $J_\infty(A)$ on the basis of relative gap between true and false, that is

$$J_\infty(A)=t_A+\frac{t_A}{t_A+f_A}\pi_A \tag{3}$$

And the last one is the new scoring function $P(A)$, proposed by Zhang Enyu, Wang Yu and other scholars on the basis of the above two scoring functions.

$$P(A)=\begin{cases}0,t_A=f_A=0\\ \frac{1}{2}S(A)+J_\infty(A)-\frac{1}{2}=\frac{t_A-f_A}{2}\\ +\frac{t_A-f_A}{2(t_A+f_A)},or\end{cases} \tag{4}$$

The scoring function can better reflect the features of both the scoring function $S(A)$ and $J_\infty(A)$. By synthesizing their shared features, one can obtain the result which can be more in line with their intuitive judgment.

As stated in the introduction, the weighting is set as steady weighting. Though it is feasible, it can not handle relatively complex problem, being unable to better distinguish the quality and quantity of indexes.

3.1 dynamic weighting

Since be proposed, dynamic weighting was widely used. For instance, Guo Yajun [15] proposed a second weighting method of dynamic weighting. Zhong Qi [16] applied dynamic weighting to evaluate the efficiency of bank operating. Wang Xuerong[17] proposed a dynamic integrated evaluation method for performance based on dynamic weighting. Han Zhonggeng [18] applied dynamic weighted comprehensive evaluation to the comprehensive evaluation of the water quality in Yangtze River and so on.

Dynamic weighting not only take the difference of quality between indexes into consideration, but also that of quantity. As for different evaluation indexes, both the same and different weighting functions can be chose in accordance with practical problems. The commonly used weighting functions are as follows.

1) The power function

If the influence of a certain index to the evaluation result increased as a power function when the index increases, the dynamic weighting function for evaluation index can be set as a power function. That is

$$w_j\left(x\right)=x^{\frac{1}{k}},k>0$$.

in which $1\leq j\leq n$， n is as the index, and k must be set according to the practical situation. For example, when k equals to three the diagram of the dynamic weighting function is as image3-1

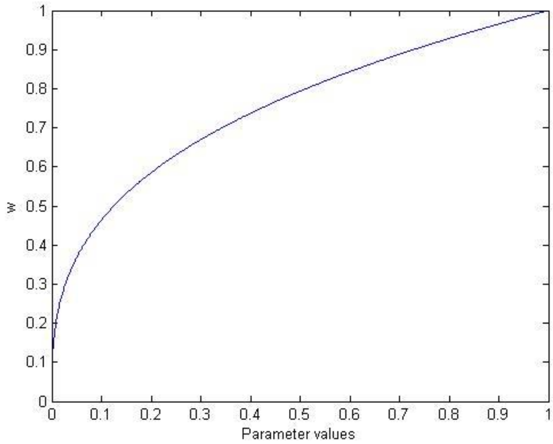

Figure 3-1

2) Partial large normal distribution function

If the effect of an index on the results of comprehensive evaluation increases slowly firstly, then fast and finally verge to the maximum slowly and steadily, the accordingly image will be normal distribution curve graphic (left side) shape. Therefore, the dynamic weighting function for evaluation index can be set as the partial large normal distribution function. That is,

$$w_j\left(x\right)=\begin{cases}0, & x\leq\alpha\\ 1-e^{-\left(\frac{x-\alpha}{\sigma}\right)^2}, & x>\alpha\end{cases}$$

α, σ are parameters of a function, in which α generally equals to 0.05, σ generally depends on $w\left(a_k\right)=0.9$ and a_k normally equals to 0.8, indicating that it is considered as zero when the index is less than a certain amount and close to 1 when more than a certain amount. The diagram of its weighting function is as follows:

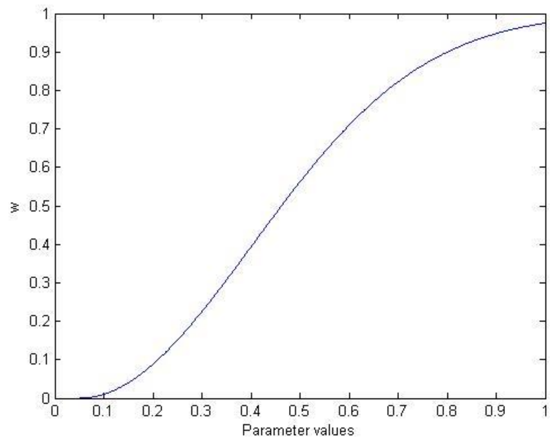

Figure 3-2

3) The distribution function of S type

If the effect of an indicator for the results of comprehensive evaluation increases as a curve of S shaped with the increase of the index value, then the dynamic weighting function can be set as the distribution function of S type. That is

$$w(x)=\begin{cases}2\left(\dfrac{x-a}{b-a}\right)^2, 当a\le x\le c时\\ 1-2\left(\dfrac{x-b}{b-a}\right)^2, \quad 当c\le x\le b时\end{cases}$$

Wherein the parameters is $c=\frac{1}{2}(a+b)$, and $w(c)=0.5$. Generally a is equal to zero, and b 0.95, representing the endpoints of the effective range. And then the diagram of the weighting function shows as the image 3-3:

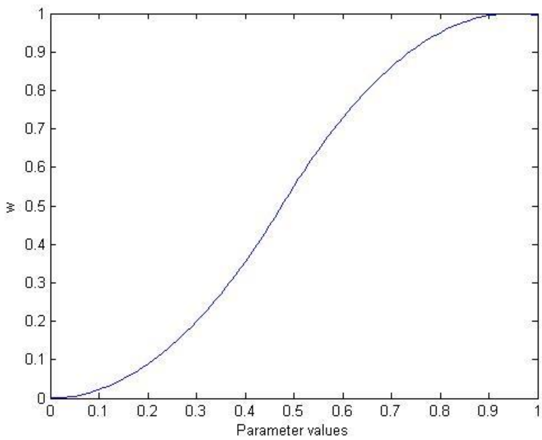

Figure 3-3

3.2 dynamic weighted multi-criteria decision making based on vague set

Based on the above discussion, a new comprehensive evaluating model has been built with newly proposed scoring function in the literature [14] and dynamic weighting function. That is the model of dynamic weighted multi-criteria decision making based on vague set. Specific steps are as follows:

Step 1 Collect the evaluation of every scheme set $U=\{u_1,u_2,\cdots u_m\}$ under every norm set $C=\{c_1,c_2,\cdots c_n\}$ in questionnaires;

Step 2 Convert the evaluation data of every norm set in every scheme set into vague estimation A_{ij};

Step3. Obtain scores $P(A_{ij})$ of every norm sets under every scheme sets by using the scoring function $P(A)$ represented in formula (4).

Step4. Determine dynamic weighting function $w(x)$ in accordance with actual indicators, and then obtain total points $W_c(u_i)$ of scheme sets with dynamic weighting function;

$W_c(u^*)=\max\{W_c(u_i), i=1,2,\cdots m\}$.

Step5. After rank the total points $W_c(u_i)$, choose the best scheme u^*, meeting the function $W_c(u^*)=\max\{W_c(u_i), i=1,2,\cdots m\}$.

4. Conclusion

This paper analyzes the scoring function and weighting of multi-criteria decision making based on vague set, the disadvantages of steady weights, such as high subjectivity and poor scientific, and types and advantages of dynamic weighting, basing on which, the model of multi-criteria decision making based on vague set has been established by applying dynamic weighting to it and using scoring function $P(A)$. Compared with steady weighting, dynamic weighting with the advantage of higher distinction has better evaluating efforts. It provides Multi-criteria comprehensive evaluation based on vague sets with new weighting methods, making fuzzy comprehensive evaluation be more widely used.

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