# Research of Spare Parts Joint Support Method Based on Support Degree

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## Abstract

**Through making a analysis of the spare parts joint support system that consists of two supply departments, this paper has established support degree models of spare parts joint support. The arithmetic of spare parts joint support storage based on support degree is presented. Then optimal method of spare parts joint support is determined. Applicability of the method is given by way of a numerical example.**

## Keywords

**Support Degree, Joint Support, Spare Parts Storage, Spare Parts Supply.**

## 1. Introduction

One spare-parts joint guarantee system consists of supplier 1, supplier 2 and several clients. When a client has parts shortage, either supplier 1 or supplier 2 can supply the parts with only 1 piece every time, but the client could only choose one of the suppler. The demand amount for spare-parts is random variable (Distribution function is known). The system structure is as below:

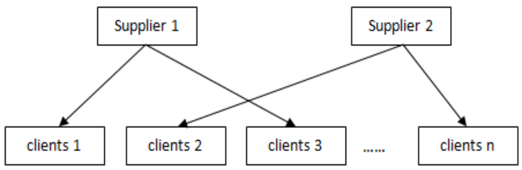


Figure 1. Spare-parts joint guarantee system

When determining the storage volume, either supplier 1 or supplier 2 should consider the following factors: the delivery distance between storage location and the location of client, the average time per km when delivering the parts from supplier to client, the annual demand amount of the client, the average time for urgently producing a single part when supplier has a stock-out, the guaranteed amount of stock, the density function of total annual demand amount by each supplier, the average downtime of equipment, etc. [1][2][3]

This essay solely considers getting the maximum degree of joint guarantee in the case of spare-part replacement after equipment failure, which requires the maximum guarantee degree of single part of the client. Under the limitation of budget for actual guarantee expenditure, the plan which realizes the maximum guarantee degree will be the optimal one, meanwhile, the optimal storage amount for supplier 1 and supplier 2 can be confirmed.

## 2. Code Description

$p$ -purchasing price of single part;

$D_{1k}$ -the distance between supplier 1 and the client $k$ ;

$D_{2k}$ -the distance between supplier 2 and the client $k$ ;

$k_0$ -the average time per km when delivering the parts from supplier to client;

$x_k$ -the annual demand amount of the client $k$ , which is random variable;

$N_k$-the equipment amount of client $k$;

$t_1$- the average time for supplier 1 to urgently produce a single part when supplier has a stock-out;

$t_2$- the average time for supplier 2 to urgently produce a single part when supplier has a stock-out;

$f_{01}(r_{01})$-the density function of total annual demand amount guaranteed by supplier1;

$f_{02}(r_{02})$-the density function of total annual demand amount guaranteed by supplier2;

$X_{01}$-storage amount for supplier 1;

$X_{02}$-storage amount for supplier 2;

$C(X_{01}, X_{02})$-actual guarantee expenditure;

$C_0$-limitation of budget;

$\lambda_k$-the parameter to determine which supplier to guarantee the client $k$; if client $k$ is guaranteed by supplier 1, $\lambda_k = 1$; if client $k$ is guaranteed by supplier 2, $\lambda_k = 0$.

## 3. Modeling

Determining the storage amount of joint guarantee from the perspective of guarantee degree, is to get the maximum value of guarantee degree with the actual guarantee expenditure under the budget limitation.

1. The calculation of average downtime of guarantee by supplier 1: The total annual average time for client to get the guarantee from supplier 1:

$$T_{01} = k_0 \sum_{k=1}^{n} \lambda_k D_{1k} E(X_k) + t_1 \int_{X_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r_{01} \tag{1}$$

It is known that the equipment amount guaranteed by supplier 1 is $\sum_{k=1}^{n} \lambda_k N_k$ >0,then within the given time $t_0$, the average downtime of single equipment caused by lack of the part:

$$\overline{T}_{01} = \frac{T_{01}}{\sum_{k=1}^{n} \lambda_k N_k} = \frac{k_0 \sum_{k=1}^{n} \lambda_k D_{1k} E(X_k) + t_1 \int_{X_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r_{01}}{\sum_{k=1}^{n} \lambda_k N_k} \tag{2}$$

2. The calculation of average downtime of guarantee by supplier 2: The total annual average time for client to get the guarantee from supplier 2:

$$T_{02} = k_0 \sum_{k=1}^{n} (1-\lambda_k) D_{2k} E(X_k) + t_2 \int_{X_{02}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r_{02} \tag{3}$$

It is known that the equipment amount guaranteed by supplier 1 is $\sum_{k=1}^{n} \lambda_k N_k$ >0,then within the given time $t_0$, the average downtime of single equipment caused by lack of the part:

$$\overline{T}_{02} = \frac{T_{02}}{\sum_{k=1}^{n} (1-\lambda_k) N_k} = \frac{k_0 \sum_{k=1}^{n} (1-\lambda_k) D_{2k} E(X_k) + t_2 \int_{X_{02}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r_{02}}{\sum_{k=1}^{n} (1-\lambda_k) N_k} \tag{4}$$

3.The calculation of average guarantee degree: For every single client's demand for guarantee in one year, the guarantee will be conducted by either supplier 1 or supplier 2, without guaranteed by them together. Thus, the average downtime of single equipment caused by lack of the spare part will be

$$T = \lambda_k \overline{T}_{01} + (1-\lambda_k) \overline{T}_{02} \tag{5}$$

It's given that the total equipment amount of joint guarantee is $\sum_{k=1}^{n} N_k$, then within given time $t_0$,the average guarantee degree of single equipment will be:

$$A(X_{01}, X_{02}) = 1 - \frac{T}{t_0} = 1 - \frac{\lambda_k \overline{T}_{01} + (1-\lambda_k)\overline{T}_{02}}{t_0} \tag{6}$$

Substitute formula(2)and formula(4)into formula(6) to get:

$$A(X_{01}, X_{02}) = 1 - \lambda_k \frac{k_0 \sum_{k=1}^{n} \lambda_k D_{1k} E(X_k) + t_1 \int_{X_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r_{01}}{t_0 \sum_{k=1}^{n} \lambda_k N_k} - (1-\lambda_k) \frac{k_0 \sum_{k=1}^{n} (1-\lambda_k) D_{2k} E(X_k) + t_2 \int_{X_{02}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r_{02}}{t_0 \sum_{k=1}^{n} (1-\lambda_k) N_k} \tag{7}$$

4. Objective function: When guarantee expenditure $C(X_{01}, X_{02}) \le C_0$, the plan which realizes the maximum guarantee degree will be the optimal one, so the objective function to get the optimal plan will be:

$$\max \quad A(X_{01}, X_{02}) \tag{8}$$

## 4. Modeling solution

1.Objective function detailing: Under the constraint condition that $C(X_{01}, X_{02}) \le C_0$ ,given that $(X_{01} + X_{02})p = C_0$, $C_0^{'} = \frac{C_0}{p}$.Then

$$X_{02} = C_0^{'} - X_{01} \tag{9}$$

Substitute (9)into(7) to get one variable function of $X_{01}$:

$$A(X_{01}, X_{02}) = 1 - \frac{\lambda_k \left[ k_0 \sum_{k=1}^{n} \lambda_k D_{1k} E(X_k) + t_1 \int_{X_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r_{01} \right]}{t_0 \sum_{k=1}^{n} \lambda_k N_k} - \frac{(1-\lambda_k) \left[ k_0 \sum_{k=1}^{n} (1-\lambda_k) D_{2k} E(X_k) + t_2 \int_{C_0^{'} - X_{01}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r_{02} \right]}{t_0 \sum_{k=1}^{n} (1-\lambda_k) N_k} \tag{10}$$

2.All clients are guaranteed by supplier 1: given that all clients are guaranteed by supplier 1, $\lambda_k = 1$ ($k = 1,2,3 \cdots n$),then(10)can be transformed to be the one variable function of $X_{01}$:

$$A(X_{01}) = 1 - \frac{k_0 \sum_{k=1}^{n} D_{1k} E(X_k) + t_1 \int_{X_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r_{01}}{t_0 \sum_{k=1}^{n} N_k} \tag{11}$$

As $k_0 \sum_{k=1}^{n} D_{1k} E(X_k)$ is constant, and $\frac{\mathrm{d}\left( t_1 \int_{S_{01}}^{+\infty} r_{01} f(r_{01}) \mathrm{d}r \right)}{\mathrm{d}X_{01}} = -X_{01} f(X_{01}) t_1 < 0$, so $A_0(X_{01})$ is monotone increasing in section $[0,\ C_0^{'}]$. Round down on $C_0^{'}$,and given that $X_{01} = \lfloor C_0^{'} \rfloor$, then $A_0(X_{01})$ is the maximum value in this section.

3.All clients are guaranteed by supplier 2: given that all clients are guaranteed by supplier 2, $\lambda_k = 0$ ($k = 1,2,3 \cdots n$)then(10)can be transformed to be the one variable function of $X_{02}$:

$$A(X_{02}) = 1 - \frac{k_0 \sum_{k=1}^{n} D_{2k} E(X_k) + t_2 \int_{X_{02}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r_{02}}{t_0 \sum_{k=1}^{n} N_k} \tag{12}$$

As $k_0 \sum_{k=1}^{n} D_{2k} E(X_k)$ is constant, and $\frac{\mathrm{d}\left( t_2 \int_{X_{02}}^{+\infty} r_{02} f(r_{02}) \mathrm{d}r \right)}{\mathrm{d}X_{02}} = -X_{02} f(X_{02}) t_2 < 0$, so $A(X_{02})$ is monotone increasing in section $[0,\ C_0^{'}]$. Round down on $C_0^{'}$,and given that $X_{02} = \lfloor C_0^{'} \rfloor$, then $A(X_{02})$ is the maximum value in this section.

4.Joint guarantee: when clients are guaranteed by both supplier1 and supplier 2, have derivation on (10)about $X_{01}$, given that derivative is 0, and then:

$$\frac{\lambda_k X_{01} f(X_{01}) t_1}{t_0 \sum_{k=1}^{n} \lambda_k N_k} - \frac{(1-\lambda_k)(C_0^{'} - X_{01}) f(C_0^{'} - X_{01}) t_2}{t_0 \sum_{k=1}^{n} (1-\lambda_k) N_k} = 0 \tag{12}$$

$$\Rightarrow \frac{X_{01} f(X_{01})}{(C_0^{'} - X_{01}) f(C_0^{'} - X_{01})} = \frac{(1-\lambda_k) t_2 \sum_{k=1}^{n} \lambda_k N_k}{\lambda_k t_1 \sum_{k=1}^{n} (1-\lambda_k) N_k} \quad (13)$$

Get $X_{01} = \widehat{X}_{01}$ from (13), then substitute it into(9)to get $X_{02}$. Round up on $X_{01}$ and round down on $X_{02}$ separately to get the optimal plan.

## 5. Example analyzing

The guarantee system consists of supplier 1, supplier 2 and client 1, client 2, client 3. Each client can be guaranteed by either supplier 1 or supplier 2. The equipment amount of client 1, client 2, client 3 is 1, 2, 3 respectively. One equipment needs 2 parts, price is 1500 CNY, the annual amount of demands for the parts is $N(2,1)$. It is known that budget limitation is 21000 CNY, the distance between supplier 1 and client 1, 2, 3 is 190km,240km,210km respectively, the distance between supplier 2 and client 1, 2, 3 is 200km,220km,240km respectively, the average time per km when delivering the parts from suppliers to clients is 0.01h, the average time for supplier 1 to urgently produce a single part when supplier has a stock-out is 160h, the average time for supplier 2 to urgently produce a single part when supplier has a stock-out is 170h. With the above, please get the storage amount for supplier 1 and supplier 2 respectively, and which clients they will guarantee separately, in this way to maximize the annual expectation of total guarantee degree.

There are 8 plans for as shown in figure 1:

Table 1.plans for supplier 1 and supplier 2 to guarantee the clients

| plans | Client 1 | Client 2 | Client 3 |
|---|---|---|---|
| 1 | supplier 1 | supplier 1 | supplier 1 |
| 2 | supplier 1 | supplier 2 | supplier 1 |
| 3 | supplier 1 | supplier 1 | supplier 2 |
| 4 | supplier 1 | supplier 2 | supplier 2 |
| 5 | supplier 2 | supplier 2 | supplier 2 |
| 6 | supplier 2 | supplier 2 | supplier 1 |
| 7 | supplier 2 | supplier 1 | supplier 1 |
| 8 | supplier 2 | supplier 1 | supplier 2 |

For on the 8 plans, use formula (7) get the result of joint guarantee amount based on guarantee degree, as shown in figure 2:

Table 2. Result of joint guarantee amount based on guarantee degree

| plans | guarantee degree | guarantee cost（yuan） | storage amount | |
|---|---|---|---|---|
| | | | supplier 1 | supplier 2 |
| 1 | 0.9561 | 18000 | 12 | 0 |
| 2 | 0.9636 | 21000 | 6 | 8 |
| 3 | 0.9104 | 19500 | 1 | 12 |
| 4 | 0.9552 | 21000 | 2 | 12 |
| 5 | 0.9071 | 18000 | 0 | 12 |
| 6 | 0.9101 | 19500 | 2 | 11 |
| 7 | 0.9128 | 18000 | 7 | 5 |
| 8 | 0.9315 | 18000 | 8 | 4 |

The optimal plan is plan 2: the supplier 1 guarantees client 1 and client 3, the supplier 2 guarantees client 2, the storage amount of supplier 1 is 6, the storage amount of supplier 2 is 8, guarantee cost is 21000 CNY, and the total guarantee degree is 0.9636.

## 6. Conclusion

Adopting probabilistic method, this essay establishes the guarantee model of spare parts joint guarantee, gives the solution calculation method of storage amount based on guarantee degree, and determines the optimal plan for spare parts joint guarantee, through analysis and problem simplification on the joint guarantee system with 2 suppliers. This given method provides a theoretical basis for determining the optimal spare parts storage amount for guarantee system with several suppliers.

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