Grey Relational Analysis Procedure for Constructional Engineering Software Quality Assessment with Triangular Fuzzy Information

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Abstract
In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the constructional engineering software quality with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating the constructional engineering software quality in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the constructional engineering software quality is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords
Multiple Attribute Decision-Making (MADM), Triangular Fuzzy Information, Grey Relational Analysis (GRA), Software Quality, Constructional Engineering

1. Introduction
With the development of science and technology, the software systems are more and more complex and the function is more and more powerful, so the demands of software quality are more and more high. The software quality directly impacts the use and maintenance of software, which is impacted by the uncertain facts in the process of software development, and this brings a lot of difficulties to the software quality evaluation [1-3]. If we can early get the necessary levels in the early software development process, this will be great significance for the quality controlling of the achieving the ultimate software, shortening the software development cycle, reducing the cost of the development and maintenance of software [5]. The technology of software quality prediction model is the key technology of software quality evaluation, but the models which current technology of software quality prediction are based on are relatively rough, and the methods used are mostly statistical algorithm So how to choose the suitable method which can make the software quality prediction model accurately and effectively establish the uncertainty and nonlinear relationship between internal properties and external properties is the concern research topic [6, 7].

Multiple attribute decision making (MADM) problems are to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In the recent years, MADM has received a great deal of attention from researchers in many disciplines [8-11]. Grey system theory is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. In grey system theory, according to the degree of information, if the system information is fully known, the system is called a white system; if the information is unknown, it is called a black system. A system with information known partially is called a grey system. The grey system theory includes five major parts: grey prediction, grey relational analysis (GRA), grey decision, grey programming and grey control. GRA is part of grey system theory, which is suitable for solving problems with complicated interrelationships between multiple factors and variables. So, GRA method has been widely used to solve the uncertainty...
problems under the discrete data and incomplete information [12-17]. In addition, GRA method is one of the very popular methods to analyze various relationships among the discrete data sets and make decisions in multiple attribute situations. The major advantages of the GRA method are that the results are based on the original data, the calculations are simple and straightforward, and, finally, it is one of the best methods to make decisions under business environment.

The problem of evaluating the software quality with triangular fuzzy information is the multiple attribute decision making (MADM) problems [1-17]. The aim of this paper is to investigate the MADM problems for the evaluating the software quality with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating marine service industry in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the constructional engineering software quality is given to verify the developed approach and to demonstrate its practicality and effectiveness. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to triangular fuzzy sets. In Section 3 we extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the constructional engineering software quality in triangular fuzzy setting. In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

In this section, we briefly describe some basic concepts and basic operational laws related to triangular fuzzy numbers.

Definition 1[18]. A triangular fuzzy number $\tilde{a}$ can be defined by a triplet $(a^L, a^M, a^U)$. The membership function $\mu_\tilde{a}(x)$ is defined as:

$$
\mu_\tilde{a}(x) = \begin{cases} 
0, & x < a^L, \\
\frac{x-a^L}{a^U-a^L}, & a^L \leq x \leq a^M, \\
\frac{x-a^U}{a^U-a^L}, & a^M \leq x \leq a^U, \\
0, & x \geq a^U.
\end{cases}
$$

(1)

where $0 < a^L \leq a^M \leq a^U$, $a^L$ and $a^U$ stand for the lower and upper values of the support of $\tilde{a}$, respectively, and $a^M$ for the modal value.

Definition 2[18]. Basic operational laws related to triangular fuzzy numbers:

- $\tilde{a} \oplus \tilde{b} = [a^L, a^M, a^U] \oplus [b^L, b^M, b^U] = [a^L+b^L, a^M+b^M, a^U+b^U]$ 
- $\tilde{a} \otimes \tilde{b} = [a^L, a^M, a^U] \otimes [b^L, b^M, b^U] = [a^L b^L, a^M b^M, a^U b^U]$ 
- $\lambda \otimes \tilde{a} = \lambda \otimes [a^L, a^M, a^U] = [\lambda a^L, \lambda a^M, \lambda a^U]$, $\lambda > 0$.

Definition 3[18]. A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is convex if and only if for all $x_1, x_2$ in $X$,

$$
\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \text{Min}\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right), \lambda \in [0,1]
$$

(2)
Definition 4 [18]. Let \( \tilde{a} = [a_L, a_M, a_U] \) and \( \tilde{b} = [b_L, b_M, b_U] \) be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as
\[
\| \tilde{a} - \tilde{b} \| = \sqrt{\frac{1}{3} \left[ (a_L - b_L)^2 + (a_M - b_M)^2 + (a_U - b_U)^2 \right]}
\]  

(3)

3. An Approach to Constructional Engineering Software Quality Assessment with Triangular Fuzzy Information

The problem of evaluating the constructional engineering software quality with triangular fuzzy information is the multiple attribute decision making (MADM) problems. In this section, consider a multiple attribute decision making problems to evaluate the constructional engineering software quality with triangular fuzzy information: Let \( A = \{ A_1, A_2, \ldots, A_m \} \) be a discrete set of alternatives. Let \( G = \{ G_1, G_2, \ldots, G_n \} \) be a set of attributes. The information about attribute weights is completely known. Let \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) be the weight vector of attributes, where \( \omega_j \geq 0, \quad j = 1, 2, \ldots, n \).

Suppose that \( A = (\tilde{a}_{ij}) \) is the decision making matrix, where \( \tilde{a}_{ij} \) is a preference value, which take the form of triangular fuzzy numbers, given by the decision maker, for the alternative \( A_i \in A \) with respect to the attribute \( G_j \in G \).

In the following, we shall extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the constructional engineering software quality in triangular fuzzy setting. The method involves the following steps:

Step 1. Normalize each attribute value \( \tilde{a}_{ij}(k) \) in the matrix \( \tilde{A} = \left[ \tilde{a}_{ij} \right] \) into a corresponding element in the matrix \( \tilde{R} = \left[ \tilde{r}_{ij} \right] \) using the following formulas:
\[
\begin{align*}
\tilde{r}_{ij}^L & = a_{ij}^L / \sum_{i=1}^{m} a_{ij}^U, \\
\tilde{r}_{ij}^M & = a_{ij}^M / \sum_{i=1}^{m} a_{ij}^M, \quad \text{for benefit attribute } G_j, \\
\tilde{r}_{ij}^U & = a_{ij}^U / \sum_{i=1}^{m} a_{ij}^L 
\end{align*}
\]

\( i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, t. \)  

(4)

\[
\begin{align*}
\tilde{r}_{ij}^L & = (1/\tilde{r}_{ij}^U) / \left( \sum_{i=1}^{m} 1/\tilde{r}_{ij}^L \right), \\
\tilde{r}_{ij}^M & = (1/\tilde{r}_{ij}^M) / \left( \sum_{i=1}^{m} 1/\tilde{a}_{ij}^M \right), \quad \text{for benefit attribute } G_j, \\
\tilde{r}_{ij}^U & = (1/\tilde{r}_{ij}^U) / \left( \sum_{i=1}^{m} 1/\tilde{a}_{ij}^U \right) 
\end{align*}
\]

\( i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, t. \)  

(5)

Step 2. Defining the triangular fuzzy positive-ideal solution (TFPIS, \( Y^+ \)) and triangular fuzzy negative-ideal solution (TFNIS, \( Y^- \)) as
\[
\tilde{R}^+ = \left[ r_{1}^+, r_{2}^+, \ldots, r_{n}^+ \right], \quad \tilde{R}^- = \left[ r_{1}^-, r_{2}^-, \ldots, r_{n}^- \right]
\]

where
\( \tilde{r}_j^+ = (\max_{i} r_{ij}^L, \max_{i} r_{ij}^M, \max_{i} r_{ij}^U), \tilde{r}_j^- = (\min_{i} r_{ij}^L, \min_{i} r_{ij}^M, \min_{i} r_{ij}^U). \)

Step 3. Calculating the fuzzy grey relational coefficient of each alternative from TFPIS and TFNIS using the following equation, respectively:

\[
\xi_{ij}^+ = \frac{\min_{1 \leq i \leq n, 1 \leq j \leq m} \| \tilde{r}_j^+ - \tilde{r}_i^+ \| + \rho \max_{1 \leq i \leq n, 1 \leq j \leq m} \| \tilde{r}_j^+ - \tilde{r}_j^- \|}{\| \tilde{r}_j^+ - \tilde{r}_j^- \|} \tag{6}
\]

\[
\xi_{ij}^- = \frac{\min_{1 \leq i \leq n, 1 \leq j \leq m} \| \tilde{r}_j^- - \tilde{r}_i^- \| + \rho \max_{1 \leq i \leq n, 1 \leq j \leq m} \| \tilde{r}_j^- - \tilde{r}_j^+ \|}{\| \tilde{r}_j^- - \tilde{r}_j^+ \|} \tag{7}
\]

where the identification coefficient \( \rho = 0.5 \).

Step 4. Calculating the degree of fuzzy grey relational coefficient of each alternative from TFPIS and TFNIS using the following equation, respectively:

\[
\xi_i^+ = \sum_{j=1}^{n} w_j \xi_{ij}^+, \xi_i^- = \sum_{j=1}^{n} w_j \xi_{ij}^-, i = 1, 2, \cdots, m. \tag{8}
\]

Step 5. Calculating the fuzzy relative relational degree of each alternative from TFPIS using the following equation,

\[
\xi_i = \frac{\xi_i^+}{(\xi_i^- + \xi_i^+), i = 1, 2, \cdots, m.} \tag{9}
\]

Step 6. According to the fuzzy relative relational degree, the ranking order of all alternatives can be determined. If any alternative has the highest \( \xi_i \) value, then, it is the most important alternative.

4. Numerical example

With the rapid development and the increasingly widespread application of information technology, the software becomes more and more important. Also, because of the increasing size and the complexity of software, the constructional engineering software quality has become difficult to control and manage. Improving the quality of software has become the focus of software industry. Constructional engineering software quality assurance becomes an important approach for improving constructional engineering software quality, which provides developers and managers with the information reflecting the product quality through monitoring the execution of software producing task by independent review. In this section, we present an empirical case study of evaluating the constructional engineering software quality. The project's aim is to evaluate the best constructional engineering software quality from the different software systems, which provide alternatives of software systems to university. The constructional engineering software quality of five possible software systems \( A_i (i = 1, 2, 3, 4, 5) \) is evaluated. A software selection problem can be calculated as a multiple attribute group decision making problem in which alternatives are the software packages to be selected and criteria are those attributes under consideration. A computer center in a university desires to select a new information system in order to improve work productivity. After preliminary screening, five constructional engineering software systems \( A_i (i = 1, 2, \cdots, 5) \) have remained in the candidate list. Three decision makers (experts) form a committee to act as decision makers. The computer center in the university must take a decision according to the following four attributes: \( G_1 \) is the costs of hardware/software investment; \( G_2 \) is the contribution to organization performance; \( G_3 \) is the effort to transform from current system; \( G_4 \) is the outsourcing software developer reliability. The five possible constructional engineering software system \( A_i (i = 1, 2, \cdots, 5) \) are to be evaluated by using triangular fuzzy numbers by the the decision makers under the above four.
attributes (whose weighting vector \( w = (0.20, 0.15, 0.25, 0.40)^T \)), and construct, respectively, the decision matrix as follows \( \tilde{R} = (\tilde{r}_{ij})_{5 \times 4} : \)

\[
\begin{array}{cccc}
G_1 & G_2 & G_3 & G_4 \\
A_1 & [0.65, 0.70, 0.75] & [0.57, 0.61, 0.65] & [0.76, 0.78, 0.81] & [0.47, 0.50, 0.53] \\
A_2 & [0.70, 0.75, 0.78] & [0.52, 0.62, 0.68] & [0.45, 0.52, 0.50] & [0.54, 0.57, 0.60] \\
A_3 & [0.80, 0.82, 0.83] & [0.75, 0.78, 0.80] & [0.62, 0.64, 0.67] & [0.78, 0.80, 0.81] \\
A_4 & [0.67, 0.70, 0.73] & [0.82, 0.83, 0.85] & [0.83, 0.84, 0.85] & [0.82, 0.84, 0.85] \\
A_5 & [0.63, 0.64, 0.66] & [0.63, 0.64, 0.66] & [0.68, 0.70, 0.76] & [0.79, 0.82, 0.80] \\
\end{array}
\]

In the following, we shall extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the constructional engineering software quality in triangular fuzzy setting. To get the most desirable constructional engineering software system (s), the following steps are involved:

Step 1. Calculate the normalized decision matrix \( \tilde{R} : \)

\[
\tilde{R} = \begin{bmatrix}
0.1733, 0.1939, 0.2174 & 0.1592, 0.1753, 0.1976 & 0.2117, 0.2241, 0.2425 & 0.1309, 0.1416, 0.1559 \\
0.1867, 0.2078, 0.2261 & 0.1453, 0.1782, 0.2067 & 0.1253, 0.1494, 0.1497 & 0.1504, 0.1615, 0.1765 \\
0.2133, 0.2271, 0.2406 & 0.2095, 0.2241, 0.2432 & 0.1727, 0.1839, 0.2066 & 0.2173, 0.2266, 0.2382 \\
0.1877, 0.1939, 0.2116 & 0.2291, 0.2385, 0.2584 & 0.2312, 0.2414, 0.2545 & 0.2284, 0.2380, 0.2500 \\
0.1680, 0.1773, 0.1913 & 0.1760, 0.1839, 0.1824 & 0.1894, 0.2011, 0.2275 & 0.2201, 0.2323, 0.2353 \\
\end{bmatrix}
\]

Step 2. Determining TFPIs and TFNIS as:

\[
\tilde{R}^+ = \begin{bmatrix}
0.2133, 0.2271, 0.2406 & 0.2291, 0.2385, 0.2584 & 0.2312, 0.2414, 0.2545 & 0.2284, 0.2380, 0.2500 \\
0.1680, 0.1773, 0.1913 & 0.1453, 0.1753, 0.1824 & 0.1253, 0.1494, 0.1497 & 0.1309, 0.1416, 0.1559 \\
\end{bmatrix}
\]

Step 3. Calculating the fuzzy grey relational coefficient of each alternative from TFPIs and TFNIS

\[
\tilde{\xi}^+ = \left( \tilde{\xi}^+_i \right)_{5 \times 4} = \begin{bmatrix}
0.6058 & 0.4384 & 0.7534 & 0.3449 \\
0.7085 & 0.4312 & 0.3333 & 0.3993 \\
1.0000 & 0.7535 & 0.4714 & 0.8157 \\
0.6094 & 1.0000 & 1.0000 & 1.0000 \\
0.5118 & 0.4486 & 0.5778 & 0.8307 \\
\end{bmatrix}
\]

\[
\tilde{\xi}^- = \left( \tilde{\xi}^-_i \right)_{5 \times 4} = \begin{bmatrix}
0.7360 & 0.8092 & 0.3730 & 1.0000 \\
0.6370 & 0.7814 & 1.0000 & 0.7167 \\
0.5118 & 0.4642 & 0.5300 & 0.3740 \\
0.7556 & 0.4031 & 0.3333 & 0.3449 \\
1.0000 & 0.7328 & 0.4358 & 0.3686 \\
\end{bmatrix}
\]

Step 4. Calculating the degree of fuzzy grey relational coefficient of each alternative from TFPIs and TFNIS

\[
\tilde{\xi}_1^+ = 0.5132, \tilde{\xi}_2^+ = 0.4494, \tilde{\xi}_3^+ = 0.7572, \tilde{\xi}_4^+ = 0.9219, \tilde{\xi}_5^+ = 0.6464 \\
\tilde{\xi}_1^- = 0.7618, \tilde{\xi}_2^- = 0.7813, \tilde{\xi}_3^- = 0.4541, \tilde{\xi}_4^- = 0.4329, \tilde{\xi}_5^- = 0.5663 \\
\]

Step 5. Calculating the fuzzy relative relational degree of each alternative from TFPIs

\[
\xi_1 = 0.4025, \xi_2 = 0.3652, \xi_3 = 0.6251, \xi_4 = 0.6805, \xi_5 = 0.5330 \\
\]
Step 6. According to the fuzzy relative relational degree, the ranking order of the five constructional engineering software systems is: $A_4 > A_3 > A_5 > A_1 > A_2$. Thus, the most desirable constructional engineering software system is $A_4$.

5. Conclusion

In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the constructional engineering software quality with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating marine service industry in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the constructional engineering software quality is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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