Model for China's Green Building Assessment Integrated Carbon Emissions with Interval-Valued Intuitionistic Fuzzy Information

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Abstract

In this paper, we investigate the multiple attribute decision making problems for evaluating the building engineering quality with interval-valued intuitionistic fuzzy information. We utilize the interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator to aggregate the interval-valued intuitionistic fuzzy information corresponding to each alternative and get the overall value of the alternatives, then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally an illustrative example for evaluating the building engineering quality with intuitionistic fuzzy information has been given to show the developed approach.

Keywords

Multiple Attribute Decision Making (MADM), interval-valued Intuitionistic Fuzzy Information, interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator, China's Green Building Assessment, Integrated Carbon Emissions

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [3] whose basic component is only a membership function. The intuitionistic fuzzy set has received more and more attention since its appearance [4-28]. Later, Atanassov and Gargov[29-30] further defined the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. Xu and Chen[31] and Wei & Wang[32] developed interval-valued intuitionistic fuzzy weighted geometric (IVIFWG) operator, the interval-valued intuitionistic fuzzy ordered weighted geometric (IVIFOWG) operator and the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator. Xu and Chen[33] proposed the interval-valued intuitionistic fuzzy hybrid aggregation (IVIFHA) operator. Xu and Yager [34] developed the dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator and uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator. Wei[35] proposed the dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator and uncertain dynamic intuitionistic fuzzy weighted geometric (UDIFWG) operator. Wei[36] proposed the induced intuitionistic fuzzy ordered weighted geometric (I-IFOWG) operator and induced interval-valued intuitionistic fuzzy ordered weighted geometric (I-IIFOWG) operator. Xu and Chen[37] proposed the interval-valued intuitionistic fuzzy Bonferroni mean. Xu[38] developed a series of operators for aggregating intuitionistic fuzzy information or interval-valued intuitionistic fuzzy information.

In this paper, we investigate the multiple attribute decision making problems for evaluating the building engineering quality with interval-valued intuitionistic fuzzy information. We utilize the interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator to aggregate the interval-valued intuitionistic fuzzy information corresponding to each alternative and get the overall value of the alternatives, then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally an illustrative example for evaluating the building engineering quality with intuitionistic fuzzy information has been given to show the developed approach.

2. Preliminaries

In the following, we shall introduce some basic concepts related to intuitionistic fuzzy sets.

Definition 1[1,2]. An IFS A in X is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\} \tag{1}$$

where $\mu_A:X\to[0,1]$ and $\nu_A:X\to[0,1]$, with the condition $0\le\mu_A(x)+\nu_A(x)\le1$, $\forall x\in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non- membership degree of the element x to the set A.

Definition 2[1,2]. For each IFS A in X, if

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x),\ \forall x\in X. \tag{2}$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A.

Atanassov and Gargov[29-39] further introduced the interval-valued intuitionistic fuzzy set (IVIFS).

Definition 3. [31-32] Let X be a universe of discourse, An IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A}=\{\langle x,\tilde{\mu}_A(x),\tilde{\nu}_A(x)\rangle|x\in X\} \tag{3}$$

where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and $0\le\sup(\tilde{\mu}_A(x))+\sup(\tilde{\nu}_A(x))\le1$, $\forall x\in X$.

Definition 4. Let $\tilde{a}=([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, a score function S of an interval-valued intuitionistic fuzzy value can be represented as follows [31-33]:

$$S(\tilde{a})=\frac{a-c+b-d}{2},\quad S(\tilde{a})\in[-1,1]. \tag{4}$$

Definition 5. Let $\tilde{a}=([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, a accuracy function H of an interval-valued intuitionistic fuzzy value can be represented as follows [31-33]:

$$H(\tilde{a})=\frac{a+b+c+d}{2},\quad H(\tilde{a})\in[0,1]. \tag{5}$$

to evaluate the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a}=([a,b],[c,d])$, where $H(\tilde{a})\in[0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a}$.

Based on the score function S and the accuracy function H, in the following, Xu[31] give an order relation between two interval-valued intuitionistic fuzzy values, which is defined as follows:

Definition 6. Let $\tilde{a}_1=([a_1,b_1],[c_1,d_1])$ and $\tilde{a}_2=([a_2,b_2],[c_2,d_2])$ be two interval-valued intuitionistic fuzzy values, $s(\tilde{a}_1)=\frac{a_1-c_1+b_1-d_1}{2}$ and $s(\tilde{a}_2)=\frac{a_2-c_2+b_2-d_2}{2}$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1)=\frac{a_1+c_1+b_1+d_1}{2}$ and $H(\tilde{a}_2)=\frac{a_2+c_2+b_2+d_2}{2}$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a})<S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$; if $S(\tilde{a})=S(\tilde{b})$, then if $H(\tilde{a})=H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a}=\tilde{b}$;

if $H(\tilde{a})<H(\tilde{b})$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$.

3. Interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator

Einstein operations includes the Einstein product and Einstein sum, which are examples of t-norms and tconorms, respectively. They are defined as follows[42]:

Einstein product $\otimes_{\varepsilon}$ is a t-norm and Einstein sum $\oplus_{\varepsilon}$ is a t-conorm, where

$$a\oplus_{\varepsilon} b=\frac{a+b}{1+a\cdot b},\ a\otimes_{\varepsilon} b=\frac{a\cdot b}{1+(1-a)\cdot(1-b)},\ \forall(a,b)\in[0,1]^2 \tag{6}$$

In the section, we shall investigate the interval-valued intuitionistic fuzzy aggregation operators with the help of the Einstein operations.

Definition 7. Let $\tilde{a}_j=\left([a_j,b_j],[c_j,d_j]\right)$ $(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic fuzzy values, and let IVIFEWA: $Q^n\to Q$, if

$$\text{IVIFEWA}_{\omega}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\mathop{\oplus_{\varepsilon}}_{j=1}^{n}(\omega_j\tilde{a}_j) \tag{7}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j\,(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then IVIFEWA is called the interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator.

Based on Einstein product operations of the interval-valued intuitionistic fuzzy numbers described, we can drive the Theorem 1.

Theorem 1. Let $\tilde{a}_j=\left([a_j,b_j],[c_j,d_j]\right)$ $(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic fuzzy numbers, then their aggregated value by using the IVIFEWA operator is also an IVIFVN, and

$$\begin{aligned}&\text{IVIFEWA}_{\omega}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)\\&=\mathop{\oplus_{\varepsilon}}_{j=1}^{n}(\omega_j\tilde{a}_j)\\&=\left(\left[\frac{\prod_{j=1}^{n}(1+a_j)^{\omega_j}-\prod_{j=1}^{n}(1-a_j)^{\omega_j}}{\prod_{j=1}^{n}(1+a_j)^{\omega_j}+\prod_{j=1}^{n}(1-a_j)^{\omega_j}},\frac{\prod_{j=1}^{n}(1+b_j)^{\omega_j}-\prod_{j=1}^{n}(1-b_j)^{\omega_j}}{\prod_{j=1}^{n}(1+b_j)^{\omega_j}+\prod_{j=1}^{n}(1-b_j)^{\omega_j}}\right],\right.\\&\qquad\left.\left[\frac{2\prod_{j=1}^{n}c_j^{\omega_j}}{\prod_{j=1}^{n}(2-c_j)^{\omega_j}+\prod_{j=1}^{n}c_j^{\omega_j}},\frac{2\prod_{j=1}^{n}d_j^{\omega_j}}{\prod_{j=1}^{n}(2-d_j)^{\omega_j}+\prod_{j=1}^{n}d_j^{\omega_j}}\right]\right)\end{aligned} \tag{8}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j\,(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$.

It can be easily proved that the IVIFEWA operator has the following properties.

Theorem 2. (Idempotency) If all $\tilde{a}_j\,(j=1,2,\cdots,n)$ are equal, i.e. $\tilde{a}_j=\tilde{a}$ for all j, then

$$\text{IVIFEWA}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\tilde{a} \tag{9}$$

Theorem 3. (Boundedness) Let $\tilde{a}_j\left(j=1,2,\cdots,n\right)$ be a collection of IVIFVN, and let

$$\tilde{a}^-=\min_j \tilde{a}_j,\ \tilde{a}^+=\max_j \tilde{a}_j$$

Then

$$\tilde{a}^-\leq \text{IVIFEWA}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\leq \tilde{a}^+ \tag{10}$$

Theorem 4. (Monotonicity) Let $\tilde{a}_j\left(j=1,2,\cdots,n\right)$ and $\tilde{a}'_j\left(j=1,2,\cdots,n\right)$ be two set of IVIFVNs, if $\tilde{a}_j\leq\tilde{a}'_j$, for all j, then

$$\text{IVIFEWA}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\leq \text{IVIFEWA}_{\omega}\left(\tilde{a}'_1,\tilde{a}'_2,\cdots,\tilde{a}'_n\right) \tag{11}$$

4. Model for China's Green Building Assessment Integrated Carbon Emissions with Interval-Valued Intuitionistic Fuzzy Information

In this section, consider a multiple attribute decision making problems to evaluate the China's green building integrated carbon emissions with interval-valued intuitionistic fuzzy information. Let $A=\left\{A_1,A_2,\cdots,A_m\right\}$ be a discrete set of alternatives, and $G=\left\{G_1,G_2,\cdots,G_n\right\}$ be the set of attributes, $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$ is the weighting vector of the attribute $G_j\left(j=1,2,\cdots,n\right)$, where $\omega_j\in\left[0,1\right]$, $\sum_{j=1}^{n}\omega_j=1$. Suppose that $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\left[a_{ij},b_{ij}\right],\left[c_{ij},d_{ij}\right]\right)_{m\times n}$ is the interval-valued intuitionistic fuzzy decision matrix, where $\left[a_{ij},b_{ij}\right]$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, $\left[c_{ij},d_{ij}\right]$ indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\left[a_{ij},b_{ij}\right]\subset\left[0,1\right]$, $\left[c_{ij},d_{ij}\right]\subset\left[0,1\right]$, $b_{ij}+d_{ij}\leq 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

In the following, we apply the interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator to MADM problems to evaluate the China's green building integrated carbon emissions with interval-valued intuitionistic fuzzy information.

Step 1. Utilize the decision information given in matrix $\tilde{R}$, and the IVIFEWA operator

$$\tilde{r}_i=\left(\left[a_i,b_i\right],\left[c_i,d_i\right]\right)=\text{IVIFEWA}_{\omega}\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right),\ i=1,2,\cdots,m. \tag{12}$$

to derive the overall preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i ,where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ is the weighting vector of the attributes.

Step 3. Calculate the scores $S\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$ of the collective overall interval-valued intuitionistic fuzzy preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s)

Step 4. Rank all the alternative $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

5. Numerical example

This section presents a numerical example to illustrate the method proposed in this paper. Suppose a school plans to evaluate the China's green building integrated carbon emissions. There is a panel with five possible green building projects $A_i\left(i=1,2,3,4,5\right)$ to select. The company selects four attribute to evaluate the five possible green building projects: ①G_1 is the energy saving and energy use; ②G_2 is the indoor environmental quality; ③G_3 is the saving material and material resources; ④G_4 is the water saving and water resource utilization. The five possible green building projects $A_i\left(i=1,2,3,4,5\right)$ are to be evaluated using the interval-valued intuitionistic fuzzy numbers by under the above four attributes (whose weighting vector $\omega=\left(0.2,0.1,0.3,0.4\right)^T$), and construct, respectively, the decision matrix as listed in the following matrices $\tilde{R}=\left(\tilde{r}_{ij}\right)_{5\times 4}$ as follows:, and

$$\tilde{R}=\begin{bmatrix} ([0.2,0.5],[0.3,0.4]) & ([0.4,0.5],[0.1,0.2]) & ([0.3,0.6],[0.2,0.3]) & ([0.3,0.7],[0.1,0.3]) \\ ([0.2,0.7],[0.2,0.3]) & ([0.3,0.6],[0.2,0.4]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.8],[0.1,0.2]) \\ ([0.1,0.6],[0.3,0.4]) & ([0.1,0.4],[0.3,0.5]) & ([0.2,0.6],[0.2,0.3]) & ([0.2,0.4],[0.1,0.5]) \\ ([0.3,0.6],[0.2,0.4]) & ([0.4,0.6],[0.2,0.3]) & ([0.1,0.4],[0.3,0.6]) & ([0.3,0.7],[0.1,0.2]) \\ ([0.4,0.7],[0.1,0.3]) & ([0.5,0.6],[0.3,0.4]) & ([0.2,0.5],[0.3,0.4]) & ([0.5,0.6],[0.2,0.4]) \end{bmatrix}$$

In the following, we apply the IVIFEWA operator to MADM problems to evaluate the China's green building integrated carbon emissions with interval-valued intuitionistic fuzzy information. To get the most desirable green building projects, the following steps are involved:

Step 1. Utilize the IVIFEWA operator, we obtain the overall preference values $\tilde{r}_i$ of the green building projects $A_i\left(i=1,2,\cdots,5\right)$.

$$\tilde{r}_1=\left([0.2679,0.5905],[0.0337,0.0770]\right)$$
$$\tilde{r}_2=\left([0.3701,0.7143],[0.0216,0.0475]\right)$$
$$\tilde{r}_3=\left([0.1722,0.5017],[0.0257,0.1452]\right)$$
$$\tilde{r}_4=\left([0.2212,0.5514],[0.0439,0.1238]\right)$$
$$\tilde{r}_5=\left([0.3540,0.5728],[0.0413,0.1093]\right)$$

Step 2. calculate the scores $S\left(\tilde{r}_i\right)\left(i=1,2,\cdots,5\right)$ of the overall interval-valued intuitionistic fuzzy preference values $\tilde{r}_i\left(i=1,2,\cdots,5\right)$

$$S\left(\tilde{r}_1\right)=0.3739, S\left(\tilde{r}_2\right)=0.5077, S\left(\tilde{r}_3\right)=0.2515$$
$$S\left(\tilde{r}_4\right)=0.3025, S\left(\tilde{r}_5\right)=0.3881$$

Step 3. Rank all the green building projects $A_i\left(i=1,2,3,4,5\right)$ in accordance with the scores $S\left(\tilde{r}_i\right)$ $\left(i=1,2,\cdots,5\right)$ of the overall interval-valued intuitionistic fuzzy preference values $\tilde{r}_i\left(i=1,2,\cdots,5\right)$: $A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$, and thus the most desirable building project is A_2.

6. Conclusion

In this paper, we investigate the multiple attribute decision making problems for evaluating the building engineering quality with interval-valued intuitionistic fuzzy information. We utilize the interval-valued intuitionistic fuzzy Einstein weighted averaging (IVIFEWA) operator to aggregate the interval-valued intuitionistic fuzzy information corresponding to each alternative and get the overall value of the alternatives, then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally an illustrative example for evaluating the building engineering quality with intuitionistic fuzzy information has been given to show the developed approach.

Acknowledgements

The work was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People’s Republic of China (No. 16YJCZH126).

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