The Development Process and Future Trend of Financial Mathematics

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Abstract

Financial mathematics is a new discipline, which is the cross of finance and mathematics. Its most significant feature is the effective use of mathematical methods to identify and demonstrate some rules of financial and economic operation. The study of the history of the development of financial mathematics has important guiding significance for the study of financial mathematics theory and practice. At first, the paper introduces the concept and theory of financial mathematics, and then discusses the development process and the current situation of modern financial mathematics. Finally, the paper shows some views on the future development trend of financial mathematics and future challenges faced by financial mathematics.

Keywords

Financial Mathematics, Development History, Development Trend.

1. Introduction

The large number of applications of modern mathematical tools is the most important characteristic of modern financial mathematics theory. Along with the creative application of the achievements of control theory and stochastic process in the financial field, a new emerging edge discipline has emerged - financial mathematics, also known as Mathematical Finance. Financial mathematics originated from the study of financial problems. With the development of financial market, finance is more and more closely connected with mathematics, and the development of modern finance also promoted the development of some branches of mathematics. At the same time, the mathematical theories and methods provide a powerful tool for the development of finance.

Financial mathematics is a cross discipline between finance and mathematics. It establishes the mathematical model of the financial market and uses mathematical tools to study the pricing, hedging and optimal investment consumption of risky assets. The core problem is the portfolio theory and the asset pricing theory under the uncertain multi-period condition. Modern financial theory refers to the achievements of the application of financial mathematics in financial economics, such as the prevention and control of financial risks, the operation of capital markets, the structure and pricing of capital assets and so on. Financial mathematics is a mathematical theory based on probability statistics and functional analysis, with random analysis and martingale theory as the core. Modern financial theory is maturing with the development of financial market. Financial markets refer to financial securities markets such as bonds, funds, stocks, futures and options. Securities originated in the late medieval Italy Venice, Genoa and other cities, which issued military bonds. After the Second World War, due to the rapid development of the US economy, requiring the continuous improvement of financial markets in order to prevent, control and resolve financial risks, a variety of financial derivatives continue to appear. That need to solve the pricing problem of financial derivatives and the decision-making problem of securities investment. In recent decades, a very noteworthy advance in modern financial theory has been the emergence of a model of financial market equilibrium. Financial mathematics is the main cornerstone to support this progress, that is, every step of the modern financial theory is closely related to the application of financial mathematics. In recent years, the development of modern financial theory has been advancing by leaps and bounds [1, 2].

In summary, financial mathematics is the interdisciplinary of finance, mathematics, statistics, economics and computer science, belonging to the level of applied science. Financial mathematics is also a higher level of quantitative analysis subject after the qualitative description sector. In order to get a better understanding of the achievements of the application of financial mathematics and the prospect of financial theory, this paper makes a detailed description of the theory and development of financial mathematics. The study of the history of the development of financial mathematics has important guiding significance for the study of financial mathematical theory and financial practice.

2. Historical Development of Financial Mathematics

The history of financial mathematics dates back to 1900, when the French Mathematician L. Bachelier's doctoral thesis—Theory of Speculation, which proclaimed the birth of financial mathematics [3]. In this paper, he first uses Brownian motion to describe the changes of stock price. He believed that in the capital market the ones can buy and the other ones can sell, because the buyers are bullish and the sellers are bearish. Its price fluctuation is in line with Brownian Motion and its statistical distribution is a normal distribution, which was five years earlier than Einstein's 1905 study of Brownian movement.

However, Bachelet's work has not attracted the attention of the financial community for more than 50 years. In the early 1950s, Paul A. Samuelson rediscovered Baseler's work through statistician Savage, which marked the beginning of modern finance. Then modern finance went through two major revolutions, the first in 1952. That year, H. Markowitz (1952) published his doctoral thesis, and proposed the " mean-variance theory of portfolio selection " [4]. Its meaning is to lead the idea that people are looking for the "best" stocks to the understanding of the quantification and balance on risk and return. A given level of risk maximizes the expected return, or a given level of income minimizes the risk, which is the main idea of the "mean-variance theory". We can think of it as a constrained optimization problem. After that, W.F. Sharpe (1964) and J. Lintner (1965) further expanded the work of Markowitz and proposed the "capital asset pricing model" (CAPM) [5, 6]. The point is to determine the relevance of each stock to the entire market. Thus, for the above optimization problem, the holding of each stock can be determined by the average return and the correlation coefficient between the stock and the market.

It is worth mentioning that another influential work in the 1960s was the "efficient market hypothesis" of Samuelson (1965) and E. Fama (1965), which is essentially a description of market completeness [7, 8]. They proved that in a functioning market, the capital price process is a (lower) martingale. In other words, the future earning is unpredictable in fact. This work actually paves the way for the second revolution.

The second revolution in financial mathematics occurred in 1973. That year, F. Black and M. Scholes (1973) published the famous Black-scholes formula, and showed the display expression of European option pricing [9]. Merton and Scholes in an article in honor of Blake (Merton and Scholes, 1995) described the difficulty Black and Scholes's article was accepted once. The reason was that their work was ahead of that era [10]. Soon, Merton obtained another method of derivation, and promoted. In 1979, Cox, Ross, and Rubinstein (Cox, Ross, and Rubinstein, 1979) published a binary tree model. At the same time Harrison and Kreps proposed a multi-period martingale theory and arbitrage [11,12]. In 1981, Harrison and Pliska (1981) proposed the equivalent martingale measure [13]. This work served for the purpose of risk management essentially.

3. The Present Situation and Development Trend of Modern Financial Mathematics

In the late 1980s, with the further development of the financial markets, it was found that all the financial models studied above assumed that investors had complete information about the market. In fact, investors could only observe the price process itself that depicted the state of the system. While the Brownian motion and the drift coefficient of dynamic assets was not observable, that is, investors could only get part of information in the market. Therefore, many scholars have launched the systematic study based on incomplete information on the investment and consumption by a variety of mathematical methods, and have made some progress. In this paper, the main theory used in the study are listed below.

3.1 Martingale Theory

The introduction of martingale theory is the latest research of modern financial theory. In 1979, Harrison J.M. and Kreps S.R. proposed the martingale approach of option pricing theory [14]. They used the martingale-measure concept of martingale theory to characterize the no-arbitrage market and the incomplete market and used the equivalent martingale measure to price and hedge options. Another function of using martingale theory to study financial theory is that it can solve the pricing problem of derivative securities when financial market is incomplete so that the modern financial theory has made a breakthrough.

3.2 Differential Game Theory

The study of option pricing and investment decision-making by differential game is another important direction of modern financial theory, which has made certain achievements at present. When the financial market does not meet the steady-state assumption or abnormal fluctuation, the prices of securities are often not subject to geometric Brownian motion. In this case, there is a large deviation with the method of stochastic dynamic model for the security investment decision in theory or in practice. It is possible to broaden the hypothesis by studying the financial decision problem with the differential game method. Assuming that the uncertainty of the disturbance is the hostile one, the worst case can be optimized to obtain a strong "robust" investment strategy. In addition, the Bellman equation for solving the differential game is a first-order partial differential equation, which is much simpler than the second-order partial differential equation for solving the stochastic control problem. Therefore, the use of differential games to study the financial problem has broad application prospects.

3.3 Optimal Stopping Time Theory

The optimal stopping time theory is one of the practical fields in the probability system. In recent years, many economists and financial mathematicians have combined this theory with modern portfolio theory and have achieved good results [15]. However, the literature in this field is still limited, which is still in its infancy.

3.4 Intelligent Optimization

The combination of intelligent optimization methods (genetic algorithm, simulated annealing algorithm, artificial neural network, wavelet analysis, etc.) and traditional methods applied in the problems of risk control and investment decision-making is a broader research field, which provides us with a broader research subject. Extensive research topics. As the development of this area is relatively late, many problems have not yet been resolved. But we still believe that financial scientists, mathematicians and artificial intelligence experts work together in this emerging research field will be able to achieve breakthroughs.

4. Conclusion

In addition to the continued development and improvement of the above basic theory, a lot of work can be done in the field of financial mathematics. Such as American options, Asian options, the term structure of interest rates, market price volatility and unexpected events. For example, unexpected events in the financial field have a significant impact. For example, the 2009 global financial crisis caused huge losses to some countries. The prediction theory based on the traditional stationary stochastic process is completely inapplicable. The traditional theory may explain the market in 95% of the time. However, if people acknowledged that the incident occurred in the remaining 5% of the time, then the picture that the traditional theory described would not reflect the actual situation. Nowadays, chaos theory and fractal theory are often used to explain how the price of stock jumps and falls. Because of multiple factors, quantification and alarm sensitivity, financial distress warning is often difficult, which is an important research field of financial mathematics.

Financial mathematics is a kind of analysis research of financial theory and practice by using mathematical theory and methods. It has injected great impetus into the development of financial economy, and promoted financial theory, financial practice management and financial innovation. With the development of financial globalization, financial mathematics will become a wonderful work in the field of international finance, which is highly valued by the international financial and applied mathematics circles. We believe that financial mathematics will be more in-depth development and wider application in the 21st century.

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