An Denoising Method based on Improved Wavelet Threshold Function

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Abstract

The ECG signal is an important parameter for the diagnosis of heart disease. In the process of collection and transportation, ECG signals easily mixed with human body noise or the noise generated by the instrument. Therefore, the noise greatly affects the accuracy of the measurement. Wavelet threshold denoising method is widely used in denoising of ECG signal. Based on soft threshold and hard threshold denoising, an improved wavelet threshold denoising algorithm is proposed to remove ECG noise in this paper. The improved denoising algorithm can avoid the false Gibbs phenomenon and the drawbacks of excessive smoothing caused by the soft threshold and the hard threshold method. Finally, the ECG data in MIT-BIH database is used to simulate. The experimental result shows that the improved threshold denoising algorithm can effectively remove the noise in the signal, and the signal to noise ratio is higher than the soft threshold and hard threshold method.

Keywords

ECG Signal, Wavelet Denoising, Threshold, SNR.

1. Introduction

According to some reports, Cardiovascular disease has become one of the major threats to human health. Human ECG signal, as an important basis for the diagnosis of cardiovascular disease parameters, in the signal acquisition and transmission process, it is often subject to outside or human body noise interference, such noise includes: power frequency interference, EMG interference and baseline drift caused by some certainfactors [1]. At the same time, because the acquisition of ECG signal is non-intrusive acquisition, the signal-to-noise ratio of signal is low. In order to get more pure ECG signal for the diagnosis of the disease, it is very important to remove the noise and preserve the complete ECG signal [2].

There are many common denoising algorithms, such as low-pass filtering, high pass filtering, median filtering and so on. These filtering methods are difficult to preserve the detail signal, so it is not suitable for the denoising of ECG signals. Wavelet transform is widely used in signal denoising because of its good multi-scale and multi-resolution analysis. The most commonly used in wavelet denoising algorithm is modulus maxima denoising algorithm, correlation denoising algorithm and threshold denoising algorithm .Threshold denoising algorithm was proposed by Donoho and others [3] based on wavelet transform in 1994. The method of threshold denoising is clear, flexible and changeable, the effect of denoising is ideal. Therefore, it applied to ECG signal denoising by many scholars. Traditional threshold denoising algorithms are both hard threshold denoising and soft threshold denoising, but the two methods have some shortcomings. The hard threshold method will cause the vibration when the wavelet reconstruction, and the soft threshold method will make the processed wavelet coefficients and the real wavelet coefficients have a constant deviation [4]. Therefore, it is very important to propose a suitable threshold function for denoising. Based on the merits and demerits of soft and hard thresholds, a new threshold function is constructed to combine the advantages and disadvantages of soft and hard threshold functions, and overcome the shortcomings of the soft and hard threshold functions, and to achieve better denoising effect.

2. Wavelet Threshold Denoising

2.1 Denoising Principle

Because the wavelet basis is unconditional, so after wavelet transform, the useful component of the signal is spread over a small number of coefficients while the unwanted component will be on most expansion coefficients. According to the energy conservation of Parseval of discrete wavelet transform, the energy of the time-domain signal is equal to the sum of the components in the wavelet transform domain. Obviously, the wavelet transform domain coefficient which corresponds to a useful component of the signal must be larger, and which of a useless component must be smaller [5]. Based on the principle mentioned above, it can use the appropriate threshold function to deal with the coefficients of wavelet transform domain. Firstly, the threshold function generates a threshold value, then wavelet coefficients after transformed will compare with it, removing the larger wavelet coefficients while retaining the smaller wavelet coefficients. Finally, the processed wavelet threshold coefficients will be reconstructed. In this way, the useful signal can be preserved and the noise can be removed.

2.2 Wavelet Threshold Algorithm

Assuming the signal as:

$$y_i = x_i + n_i \tag{1}$$

x_i is an original signal, n_i is Standard Gaussian White Noise, y_i is the signal after denoising.

Wavelet threshold denoising algorithm has the following steps:

According to the characteristics of signal and the requirements of denoising, it can choose the appropriate wavelet basis function and determine the decomposition level.

Based on the previous step, Mallat decomposition algorithm is used to decompose the signal y_i , and obtaining the wavelet threshold coefficients d_i and l_i. The decomposition formula is as follows:

$$\begin{cases} d_i = \sum_{n=1}^{N} d_{i-1} h_{n-2} \\ l_i = \sum_{n=1}^{N} d_{i-1 g_{n-2}} \end{cases} \tag{2}$$

d_i is the low-frequency wavelet coefficient; l_i is high-frequency wavelet coefficient; i is the corresponding decomposition level; h and g are orthogonal filter banks.

Quantizing the wavelet coefficients. Selecting the appropriate threshold function and determining the threshold to deal with the wavelet threshold coefficients which obtained from the previous step.

Reconstructing the wavelet, and restoring the signal. The residual coefficients and the wavelet threshold coefficients are reconstructed, and finally, the denoised signal is obtained.

3. Threhold Setting and Selection of Threhold Function

3.1 Threshold Improvement

Because the threshold of the wavelet threshold denoising algorithm is used as the boundary between the useful signal and the noise, so the selection of threshold determines whether the denoising algorithm is ideal directly. If the threshold is too large, it will filter out some useful signals, showing the phenomenon of "over the kill"; if the threshold is too small, some of the noise signal will be retained, showing the phenomenon of "over retention". Traditional threshold estimation is the estimation of noise variance which proposed by Donoho[6]. The definition is as follows:

$$\lambda = \sigma\sqrt{2\ln(N)} \tag{3}$$

N is the length of the signal, σ is the standard deviation of noise. It can be seen from the principle of wavelet threshold denoising algorithm that with the increase of the number of decomposition level, the noise coefficient is decreasing simultaneously. Therefore, it is prudent to use a relatively small threshold when the decomposition level is higher. So, a new threshold estimation is proposed in this paper, the definition is as follows:

$$T = \frac{\lambda}{(1+\ln(J)/\beta)^2} \tag{4}$$

λ is the Donoho threshold, T is the improved threshold, J is the decomposition level, β is the regulation factor , andβ>0.From the formula, when the decomposition level J increases, the threshold T will decay simultaneously. The attenuation of threshold T can be controlled by adjusting theβvalue, so the adaptability of threshold estimation is increased, and the flexibility of the algorithm is improved.

3.2 Threshold Function Improvment

Threshold function is the key to quantify the wavelet coefficients, traditional threshold function includes two kinds.

Hard threshold function:

$$\widehat{\omega}_{j,k} = \begin{cases} \omega_{j,k}\,, & |\widehat{\omega}_{j,k}| \geq \lambda \\ 0\,, & |\widehat{\omega}_{j,k}| < \lambda \end{cases} \tag{5}$$

Soft threshold function:

$$\widehat{\omega}_{j,k} = \begin{cases} \operatorname{sgn}(\omega_{j,k})(|\omega_{j,k}| - \lambda), & |\widehat{\omega}_{j,k}| \geq \lambda \\ 0\,, & |\widehat{\omega}_{j,k}| < \lambda \end{cases} \tag{6}$$

$\omega_{j,k}$ is a high frequency coefficient after wavelet decomposition, $\widehat{\omega}_{j,k}$ is the wavelet coefficient, λ is the estimated threshold.

It can be observed from the threshold function that the hard threshold function is discontinuous at -T and +T, which will cause the reconstructed signal to oscillate near the discontinuity point and reduce the smoothness of the reconstructed signal. Although the soft threshold function overcomes this drawback because of the function is continuous throughout the domain, but it can be observed from the function that the wavelet coefficients treated by soft threshold function have a constant deviation compare with the original wavelet coefficients[7,8], so it has some limitations.

The selection of threshold function should meet the following points:

The function should be continuous in order to avoid producing the shock at the break point which will reduce the signal smoothness when reconstructing the signal.

The function should have higher order of conductibility to facilitate a variety of mathematical operations.

The asymptote of the function is a straight line$Y = x$[9], so it is can avoid the constant deviation like the soft threshold function has.

Based on the points mentioned above, aiming at the shortcomings of traditional methods, a new threshold function is proposed.

$$\widehat{\omega}_{j,k} = \begin{cases} u * \omega_{j,k} + (1-u) * \operatorname{sgn}(\omega_{j,k})\left(|\omega_{j,k}| - \frac{\lambda}{\ln(|\omega_{j,k}|/\lambda + e - 1)^{\frac{1}{J}}}\right), & |\omega_{j,k}| \geq \lambda \\ 0 & , |\omega_{j,k}| < \lambda \end{cases} \tag{7}$$

And $u = 1 - e^{-\alpha\left(T - \frac{T}{\ln(|\omega_{j,k}| - \lambda + e)}\right)^2}$ is a regulatory factor in this function. The curve of the function can be adjusted by this regulatory factor, so that it can be applied to a variety of situations. α is a positive number in this facator.

From the formula (7), it is shows that the new threshold function proposed in this paper contains the regulatory factor u,so the new threshold function has strong flexibility. When $\omega_{j,k} \to \infty$, $\widehat{\omega}_{j,k} \to \omega_{j,k}$, so line $Y = x$ can be the asymptote of the new threshold function. When $\omega_{j,k} \to \pm T$, $\widehat{\omega}_{j,k} \to 0$, it indicate that the function is continuous at $\pm T$. The analysis shows that the new threshold function is continuous and higher order derivable with the line Y = x as its asymptote. The condition of threshold function improvement which were mentioned before is satisfied. And when $u \to 0$, this function is close to the soft threshold function, when $u \to \infty$, this function is close to the hard threshold function. Obviously, this new function combines the advantages of both soft threshold function and hard threshold function, overcomes its shortcomings. This new threshold function also contains the decomposition level J, J>1, when the decomposition level J increases, the function value $\widehat{\omega}_{j,k}$ will decay simultaneously, it is in accordance with the threshold variation rule and improves the practicability of the new threshold function.

4. Simulation Experiment and Result Analysis

In order to evaluate the denoising effect of different methods objectively, in this paper, select two important denoising indexes[10]: signal to noise ratio(SNR) and root mean square error(RMSE). The definition is as follows, there y_i is the noisy signal, $\hat{y}_i$is the denoised signal, N is the signal length.

$$SNR = 10\lg\frac{\sum {y_i}^2}{\sum(y_i - \hat{y}_i)^2} \quad (8)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i-1}^{N}[y_i - \hat{y}_i]^2} \quad (9)$$

The experimental simulation of 107 ECG data which from MIT-BIH database is carried out on the Matlab2014b platform. Intercept 10s ECG data as original signal, then add 25dB Gauss white noise as noise signal, and noisy signal is formed from it. Figure1and figure2 is the original signal and noisy signal. Then, the denoising of the noisy signal is achieved by three threshold denoising algorithms: hard threshold, soft threshold and improved threshold. Finally, the two indexes mentioned above are used to evaluate the denoising effect, and got the final conclusion.

In this experient, select sym7 wavelet base, the decomposition level is 6. Selecting the hard threshold function, soft threshold function and the improved threshold function as threshold function, and the α=10, β=1.5. Comparison about the effect of different methods of denoising is shown in the following figures.See Fig. 1 and Fig. 2.

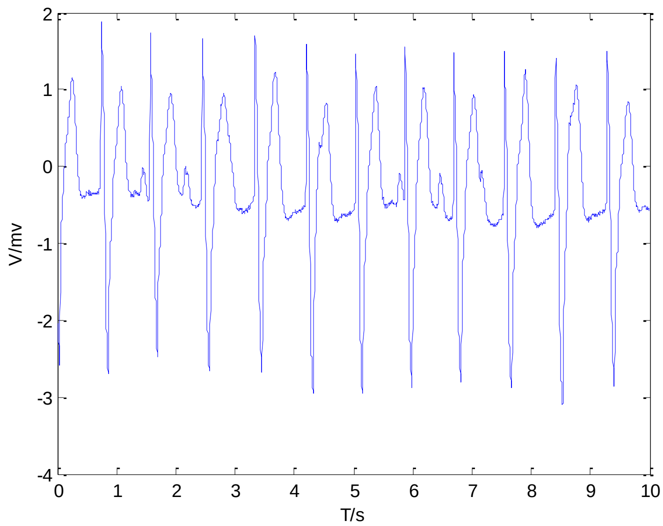

Fig. 1 Original signal

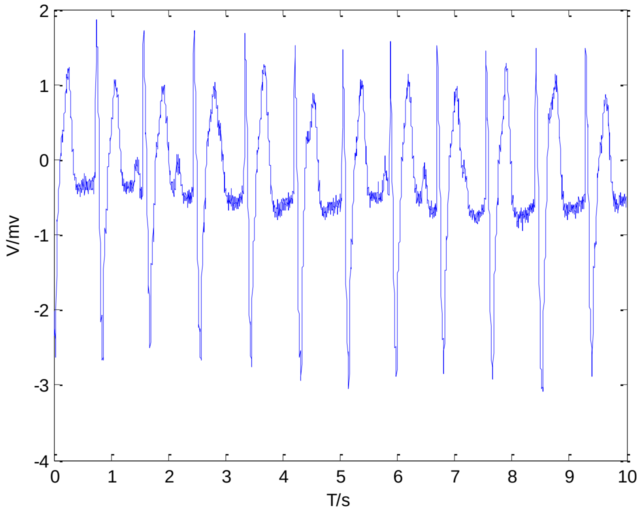

Fig. 2 Noisy signal

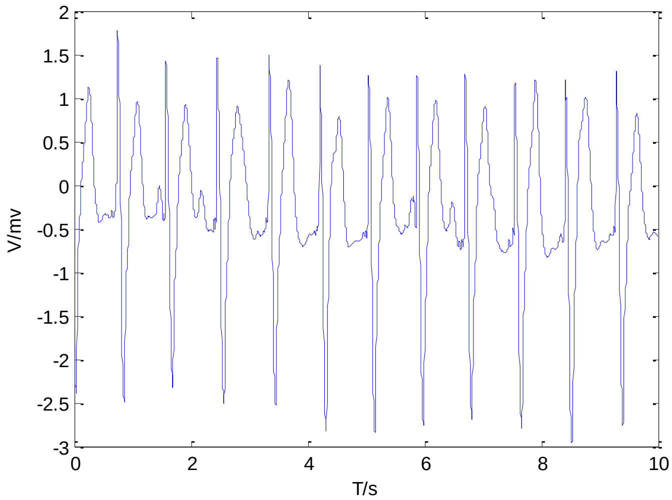

Fig. 3 Soft threshold method

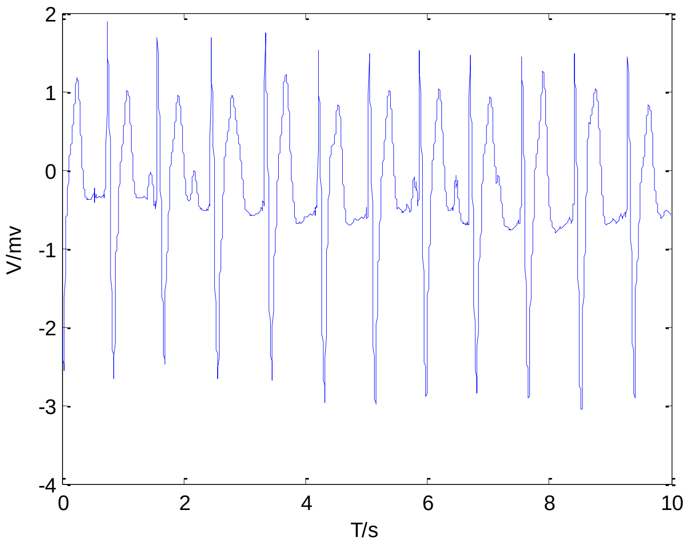

Fig. 4 Hard threshold method

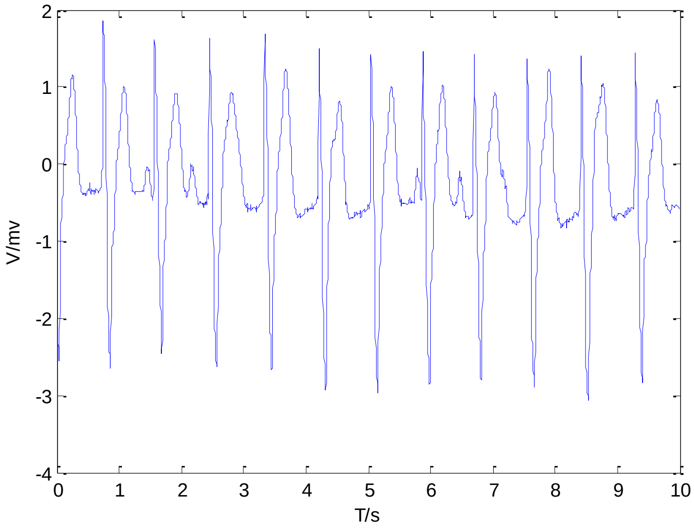

Fig. 5 Improved threshold method

Figures 3 ~ 5 are graphs showing the effect of denoising by wavelet threshold denoising which based on hard threshold function, soft threshold function or improved threshold function. The results show the reconstructed signal is more smooth by the improved threshold function method, so that the useful information of the original signal can be retained better.

In order to evaluate the denoising effect of the three methods more objectively, SNR and RMSE were calculated for each method. The calculation results are shown in Table 1.

Table 1. Calculation results of the three methods

Index	Hard threshold method	Soft threshold method	Improved threshold method
SNR	28.791	25.533	30.143
RMSE	0.0052	0.0093	0.0046

From the experimental results, it is obviously that the threshold function presented in this paper is better than the hard threshold function and soft threshold function, SNR and RMSE have improved significantly. It further shows that the improved method has higher reliability and better denoising effect.

5. Conlusion

In this paper, the principle and steps of wavelet threshold denoising are introduced at first, the merits and demerits of hard threshold function and soft threshold function are analyzed. Then, the basic requirements of threshold function are summed up, and a new threshold function is constructed based on the classical algorithm. This new function has the merits of both the hard threshold function and the soft threshold function, it can overcome the discontinuity caused by the shock and fixed deviation when we using hard threshold function and the soft threshold function. At last, the new function is compared with the hard threshold function and the soft threshold function by simulation experiment, the simulation results indicate that the new function can get higher SNR and smaller RMSE in denoising. Obviously, it better than the hard threshold function and the soft threshold function.

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