Research on group decision making behavior based on multi-attribute hierarchical relation

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Abstract

Most large group decision-making methods don't take correlation between attributes into account and consider that attributes are all independent of each other, it is one-sided. Therefore, this paper put forward a large group decision-making method based on multi-attribute hierarchical relation. Using the preference vector aggregation method, obtained the weighted sorting scheme. In addition, this paper presents a new similarity index and applies the method to the past natural disasters throughout the country to evaluate disaster grade.

Keywords

Large group decision, hierarchical relation,vector aggregation, mixed similarity.

1. Introduction

Since ancient times, china is a country which has a frequent occurrence of natural disasters, every year there are many natural disasters, Tangshan earthquake, Wen Chuan earthquake, Guangdong Mian Shan debris flow, flood disaster, tens of thousands of people affected by disasters. According to statistics, In 2015, a total of 186 million 203 thousand people were affected by various natural disasters, and the direct economic loss was 270 billion 410 million Yuan. Emergency decision-making for major disasters is one of multiple attribute group decision. As the environment becomes more and more complex, more and more factors need to be considered in decision making, and gradually become the focus of research. At present, group decision includes social choice theory and the analytic hierarchy process (AHP) and Delphi method, but these methods are applicable to small group decision and have some defects for the multi-attribute large group decision. Therefore, it is of great academic and practical significance to study complex large group decision-making method.

Foreign studies on complex large groups mainly include, a clustering method based on clustering for large group preference proposed by Za ře [1]. Combining AHP and voting theory, Srdjevic proposed a large group decision-making method for hydropower industry [2]. Saaty proposes a four dimension decision-making method in literature [3], and it is used for large group decision based on the internet. In addition, in 2007, Hoffmann proposed a heterogeneous large group expert probabilistic aggregation method [4]. In the aspect of large group decision-making in China, Xu Xuanhua proposed a method of large group preference aggregation based on vector space clustering method in 2005 [5]. Hu Lihui improved and improved the method in 2007 [6].Chen Xiaohong and Liu Rong proposed an improved clustering algorithm in 2006 ,which is adapt to large group preference for aggregating over 50 persons [7][8]. Chen Xiaohong and Xu Xuanhua proposed a large group decision-making clustering method based on entropy weight method in 2007[9][10].

Similarity was first widely used in psychology, and later it became a common measuring tool in uncertain information. It was especially prominent in fuzzy information. In 1965, Zadeh proposed the fuzzy set theory for the first time [11]. In 1993, Pappis and Karacapilidi proposed similarity between two fuzzy sets. In 1997, Wang proposed two new similarity models. By using the ratio of the maximum function and the minimum function of two fuzzy sets, the similarity [12] of two fuzzy sets was constructed. Li and Cheng proposed a similarity of two intuitionistic fuzzy sets, and used this method to solve pattern recognition problems for the first time. In 2003, Liang and Shi proposed a new method of similarity computation [13]. In 2004, Huang and Yang based on the Hausdorff distance, uses Hausdorff distance between two intuitionistic fuzzy sets as the similarity between two intuitionistic fuzzy sets, which can be applied to solve the fuzzy linear programming problem [14]. In 2005, Nehi and Maleki proposed parameter type intuitionistic trapezoidal fuzzy numbers, which attracted extensive attention in academia [15]. since then, the use of similarity to solve multi-attribute decision-making problem is not very effective, but there is still a lot of work to be studied.

In this paper, the method of determining attribute weight and the establishment of similarity model between preferences are discussed on the basis of assuming the hierarchical relationship between attributes. For the evaluation system, the general evaluation index has 3 or more indicators, and the traditional group decision making method is difficult to achieve, which requires us to find new ways to solve. In addition, for the determination of attribute weights, the trigonometric function is used to construct the triangle function to determine the weights of attributes. Based on the previous similarity model, this paper constructs a mixed similarity model based on the distance similarity model in vector space and the cosine similarity.

2. Determination of attribute weights

In practical problems, there is a complex relationship between attributes, such as hierarchy relationship, binary relationship, the recursive relationship, this paper discusses and studies the hierarchical relationships between attributes, and hierarchical relationships between attributes based on attribute weights. In this paper, the method of trigonometric function is used to describe the objective information contained in attributes.

Suppose there is P schemes for a decision problem, which is expressed by $S = \{S_1, S_2, \cdots, S_P\}$, and n decision attributes, named by $A = \{A_1, A_2, \cdots, A_n\}$. There are m experts who make decisions about this decision problem, and evaluate the above P schemes from the N attributes, expressed by V_{ij}^k, represents the evaluation value of the first k expert on the first j attribute of the i scheme. And $V_{ij}^k \geq 0 (i = 1,2, \cdots, p; j = 1,2, \cdots, n; k = 1,2, \cdots, m)$, vector $V_i^k = \left(v_{i1}^k, v_{i2}^k, \cdots, v_{in}^k\right)$ represents a preference vector for the Kth expert about a programS_i.

Suppose there is n layer evaluation index, n_i indices in layer i, rank the index according to its importance, mark it from high to low: A_i^1,A_i^2,…… , $A_i^{n_i}$. The formula is defined as follows,

$$W_i^j = \cos\left(\frac{j}{n_i+1}\pi\right)+1 \qquad (j=1, 2, \ldots, n_i) \tag{1}$$

In this paper, the attribute weight is determined by trigonometric function. The method of weight determination is simple and practical, and the time complexity is low. The good properties of this method can be seen by an example.

Theorem 2.1: if the formula (2-1) satisfies the definition of weight, it is defined j and $W_i^j$is one-to-one correspondence. The less important attribute is, the less the value of function is, the converse is also true.

Proof: obviously, $W_i^j \geq 0$, and W_1 is a decreasing function.

Because $0 \leq \frac{j}{n_i+1}\pi < \pi$

So cosine function in (0, π)is monotonically decreasing, and $-1 \leq \cos\left(\frac{j}{n_i+1}\pi\right) < 1$.

So $0 \leq W_i^j < 2$ and function value decreases gradually as the weight importance decreases.

Proof is over.

Normalization processing get the weight of the i evaluation index in the level j:

$$W = \overline{W} = \frac{w_i^j}{\sum_{i=1}^{n_i} w_i^j} \tag{2}$$

3. Construction of similarity model

3.1 Definitions

Similarity is an important measure of similarity between two things, there are many kinds of definition method for similarity between vectors, and calculation method of similarity model based on vector space model is the most commonly used method, in this model, each preference is mapped into a feature vector. There are two basic models, one is based on distance, and the other is based on cosine. As follows:

Here, this article treats the preference of the expert as a vector of the attributes of the program, assuming that there are two vector preferences:$X = (X_1, X_2, \cdots, X_n)$,$Y = (Y_1, Y_2, \cdots, Y_n)$, are two n-dimensional vectors on the real number set R and all the components are positive real numbers.

In 1901, Jaccard defined the J- similarity between two vectors [16], as follows:

Definition 3: The J- similarity between two vectors X and Y is defined as follows:

$$J(X, Y) = \frac{XY}{\|X\|_2^2 + \|Y\|_2^2 - XY} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2 + \sum_{i=1}^{n} Y_i^2 - \sum_{i=1}^{n} X_i Y_i} \quad (3)$$

Here $XY = \sum_{i=1}^{n} X_i Y_i$ is vector inner product of X and Y, $\|X\|_2 = \sqrt{\sum_{i=1}^{n} X_i^2}$,$\|Y\|_2 = \sqrt{\sum_{i=1}^{n} Y_i^2}$ is Euclidean norm of X and Y.

Similarity model based on distance is defined as follows:

Definition 4: the E-similarity between the two vectors X and Y is as follows:

$$E(X, Y) = 1 - \frac{\sqrt{\sum_{i=1}^{n} (X_i - Y_i)^2}}{n} \quad (4)$$

Theorem 3.1:Suppose that $X = (x_1, x_2, \cdots, x_n)$,$Y = (y_1, y_2, \cdots, y_n)$, is n-dimensional vector on the real number set R, and all the components are positive real numbers. The similarity between the two vectors satisfies the following properties:

(1) Reflexivity: if $X = Y$, that is, $x_i = y_i$, for any i=1,2,…….,n, then $S(X, Y) = 1$;

(2) Symmetry: $S(X, Y) = S(Y, X)$;

(3) Boundedness: $0 \leq S(X, Y) \leq 1$;

If the new similarity satisfies the above theorem, it shows that the similarity is valid. The following proves that the similarity model satisfies the above properties.

Proof:

(1)When $X = Y$, that is $x_i = y_i$, for any $i = 1,2, \cdots, n$,here is the following equation:

$$E(X, Y) = 1 - \frac{\sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}}{n} = 1 - \frac{\sqrt{\sum_{i=1}^{n} (y_i - y_i)^2}}{n} = 1 - 0 = 1$$

It satisfies the first property.

(2) $E(Y, X) = 1 - \frac{\sqrt{\sum_{i=1}^{n} (y_i - x_i)^2}}{n} = 1 - \frac{\sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}}{n} = E(X, Y)$,$E(Y, X) = E(X, Y)$satisfies symmetry.

(3)Because $\frac{\sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}}{n} \in (0,1)$, so $E(X, Y) \in (0,1)$.

Proof is over.

A similarity model based on cosine similarity, defined by Salton and McGill, is defined by the cosine of the angle between two vectors. See [17].

Define 5 C-similarity is defined as follow:

$$C(X, Y) = \frac{XY}{\|X\|_2 \|Y\|_2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2} \sqrt{\sum_{i=1}^{n} y_i^2}} \quad (5)$$

There is a defect before for the construction of similarity model, which does not fully consider the problem for vector space model, the similarity between two vectors or only consider the angle between two vectors, or only consider the two vector distance between, this will lead to the error modeling. In theory, judging whether the two vectors are similar or not, we should consider two aspects, that is, the size and direction of the vector, and that only the size or direction of the vector will lead to the one sidedness of the model. Therefore, in order to neutralize the two shortcomings, the thesis combines two methods and defines a mixed similarity method. As follows:

Definition 6 H-similarity is defined as follow:

$$H(X,Y) = \alpha * E(X,Y) + (1-\alpha) * C(X,Y)$$
$$= \alpha * \left[1 - \frac{\sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}}{n}\right] + (1-\alpha) * \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^n}\sqrt{\sum_{i=1}^{n} y_i^n}} \quad (6)$$

Here $0 \leq \alpha \leq 1$, it can be set according to the preferences of decision makers. If the mixed similarity is a valid similarity, it should satisfy the requirements of theorem 3.1, and the following is the proof process for it.

Proof:

(1) When X=Y, that is $x_i = y_i$, for any i=1,2,…….,n, there is an equation:

$$H(X,Y) = \alpha * E(X,Y) + (1-\alpha) * C(X,Y)$$
$$= \alpha * \left[1 - \frac{\sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}}{n}\right] + (1-\alpha) * \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^n}\sqrt{\sum_{i=1}^{n} y_i^n}}$$
$$= \alpha * \left[1 - \frac{\sqrt{\sum_{i=1}^{n}(y_i - y_i)^2}}{n}\right] - (1-\alpha) * \frac{\sum_{i=1}^{n} y_i y_i}{\sqrt{\sum_{i=1}^{n} y_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}$$
$$= \alpha + 1 - \alpha = 1$$

(2)$H(Y,X) = \alpha * E(Y,X) + (1-\alpha) * C(Y,X)$

$$= \alpha * \left[1 - \frac{\sqrt{\sum_{i=1}^{n}(y_i - x_i)^2}}{n}\right] + (1-\alpha) * \frac{\sum_{i=1}^{n} y_i x_i}{\sqrt{\sum_{i=1}^{n} y_i^2}\sqrt{\sum_{i=1}^{n} x_i^2}}$$
$$= \alpha * \left[1 - \frac{\sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}}{n}\right] + (1-\alpha) * \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}$$
$$= H(X,Y)$$

(3)Because $E(X,Y) \in (0,1), C(X,Y) \in (0,1)$, so $[\ \alpha * E(X,Y) + (1-\alpha) * C(X,Y)\ ] \in (0,1)$, so $H(X,Y) \in (0,1)$

So $H(X,Y)$ is proved to be a valid similarity.

3.2 Comparison of several different similarities

In order to show more clearly similarity and differences between the new similarity and the existing similarities, we use the following examples to compare the four similarity model indexes in the previous section. We use literature data in [21], the 18 vector is shown in table 3-1, then take an ideal vector, and use four kinds of similarity to calculate the similarity between 18 vectors and the ideal vector respectively, the result is showed in table. Ideal vector is

$V_p = (0.8542, 0.3460, 0.7000, 0.7890, 0.1234, 0.6534)$.

Take $\alpha = 0.1, 0.5, 0.9$ separately. The similarity between 18 vectors and the ideal vector is calculated using the formula (1) - (4), as shown in figure 1-4.

Table 1. As shown in figure

vector	attribute1	attribute2	attribute3	attribute4	attribute5	attibute6
V1	0.8452	0.9037	1.0000	1.0000	0.8003	0.9316
V2	0.4410	0.6640	0.4722	0.6227	0.5933	0.6010
V3	0.8613	0.7228	0.7569	0.3666	0.7118	0.9731
V4	1.0000	0.0000	0.3151	0.1609	0.5933	0.0432
V5	0.6810	0.9374	0.6152	0.8043	0.7031	1.0000
V6	0.1113	1.0000	0.0000	0.1963	0.3100	0.5725
V7	0.6206	0.7769	0.4711	0.5180	0.3172	0.4194
V8	0.9055	0.9256	0.6170	0.8522	1.0000	0.7818
V9	1.0000	0.8582	0.7363	0.8865	0.5421	0.8721
V10	0.9786	0.7769	0.8008	0.7156	0.5093	0.4552
V11	0.3351	0.2397	0.2104	0.2844	0.2282	0.2720
V12	0.1273	0.6164	0.4122	0.3690	0.1098	0.4389
V13	0.0000	0.2996	0.3252	0.4216	0.1683	0.2370
V14	0.4397	0.4730	0.3241	0.3034	0.4352	0.6344
V15	0.5536	0.6292	0.6152	0.6836	0.7534	0.8070
V16	0.3338	0.0813	0.4122	0.1254	0.1098	0.6344
V17	0.8311	0.8716	0.4711	0.6387	0.5933	0.6360
V18	0.3847	0.8716	0.3316	0.3034	0.4352	0.6344

When α=0.1,calculating J,E,C,H-similarity separately between 18 vectors and ideal vector ,the result is showed in table 2:

Table 2α=0.1, four similarity

vector	J-similarity	E-similarity	C-similarity	H-similarity
V1	0.7671	0.8349	0.9283	0.9190
V2	0.7671	0.8763	0.8728	0.8729
V3	0.7661	0.8563	0.8812	0.8784
V4	0.5051	0.8117	0.6911	0.7031
V5	0.7628	0.8469	0.8988	0.8855
V6	0.3512	0.7726	0.5355	0.5592
V7	0.8015	0.8872	0.9008	0.8994
V8	0.7140	0.8225	0.8713	0.8665
V9	0.8530	0.8801	0.9542	0.9468
V10	0.8648	0.8939	0.9366	0.9323
V11	0.5040	0.8391	0.9411	0.9309
V12	0.5700	0.8413	0.8141	0.8168
V13	0.4152	0.8187	0.8003	0.8022
V14	0.6898	0.8643	0.8655	0.8654
V15	0.7892	0.8699	0.8843	0.8829
V16	0.5663	0.8450	0.8591	0.8577
V17	0.8029	0.8739	0.8937	0.8918
V18	0.6191	0.8362	0.7762	0.7822

When theα =0.1, 0.5, 0.75, and 0.9 ,the result is correspond to the four similarity curves, as shown in figures 1 to 4

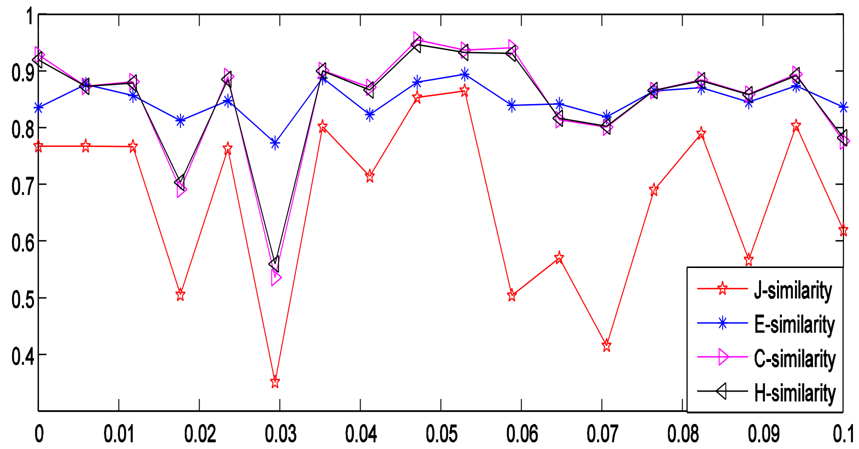


Figure 1, α=0.1, J, E, C, H-similarity curves

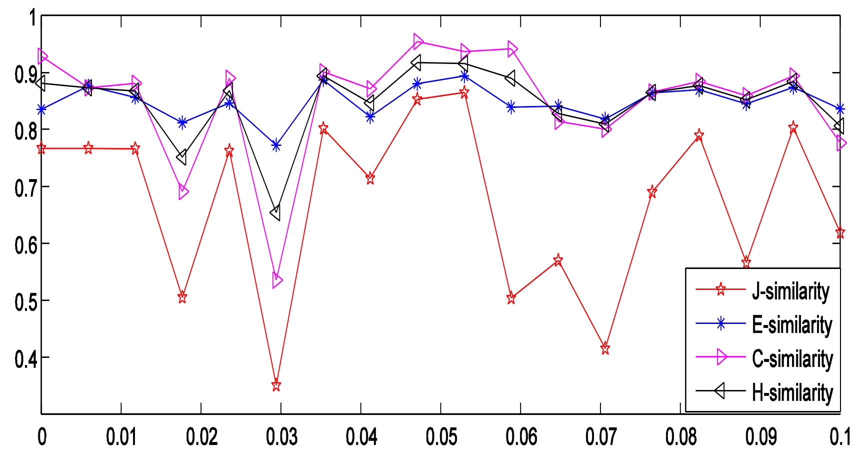


Figure 2, α=0.5, J, E, C, H-similarity curves

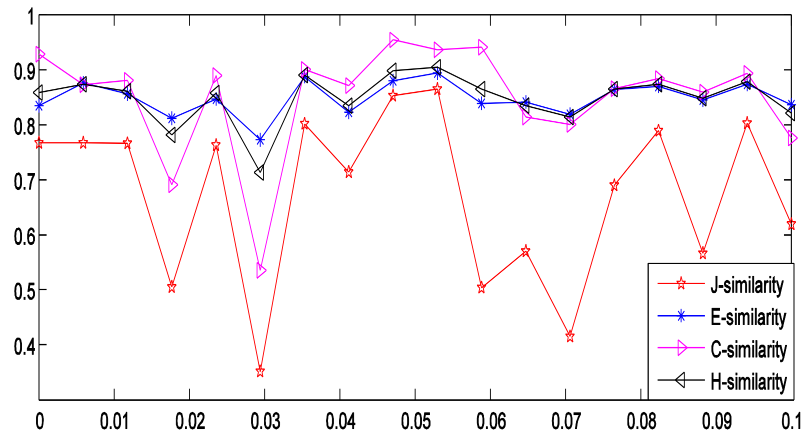


Figure 3, α=0.75, J, E, C, H-similarity curves

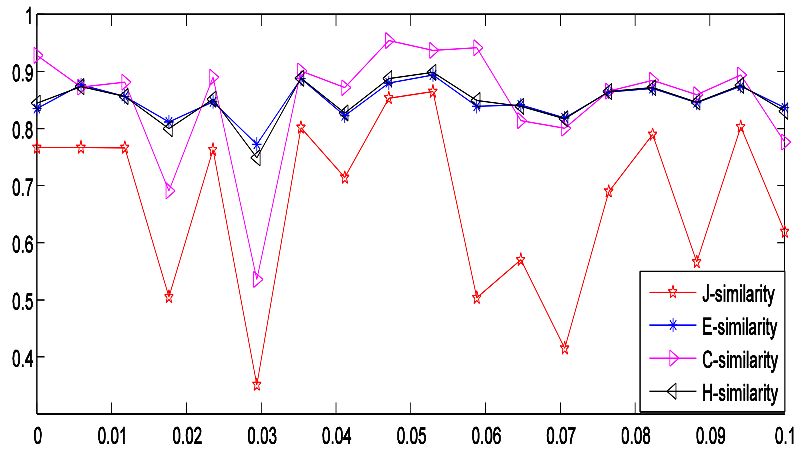


Figure 4, α=0.9, J, E, C, H-similarity curves

The above image shows theαwith the increase of image mixed similarity gradually close to the E similarity, decision makers can be determined according to the actual value of the real meaning of a vector, to achieve the results you want. In order to represent the difference between several similarities more accurately, the actual case will be used in the next section.

3.3 Comparison of several similarity measures in case analysis of emergency decision

In this section, we will take advantage of the case data in the literature [21], which is evaluated by 6 experts on the emergency management capabilities of three cities, and 18 preference vectors are obtained. The three city were C, X, and Z, 6 first class indexes are: attribute1- command emergency capability; attribute2- meteorological department detection and early warning ability; attribute3- residents emergency response capability; attribute4- power sector emergency capability; attribute5- transportation management department emergency power; attribute6- other departments of emergency response capability.

The steps of the decision problem can be divided into the following steps:

Step 1 Uses the (3-2) formula to preprocess the data of the document, as shown in table 3-1.

$$Z_{ij} = \frac{Z_{max} - Z_{ij}}{Z_{max} - Z_{min}} \tag{7}$$

$Z_{ij} \epsilon (0,1)$, for any i, $i = (1,2,\cdots,n)$, it is true.

The evaluation object is corresponding to evaluation vector: C-city$(V^1 - V^6)$, Z-city$(V^7 - V^{12})$, X-city$(V^{13} - V^{18})$.

Step 2 Constructing an ideal scheme for a preference vector :

$V^p = (0.8542, 0.346, 0.7, 0.789, 0.1234, 0.6534)$, α=0.75. The similarity between the 18 preference vectors and the ideal scheme can be obtained by using the 1 – 4 formula as shown in the following table:

Table 3 four similarity between the 18 vectors and the ideal scheme whenα=0.75

vector	J-similarity	E-similarity	C-similarity	H-similarity
V1	0.7671	0.8349	0.9283	0.8583
V2	0.7671	0.8763	0.8728	0.8734
V3	0.7661	0.8563	0.8812	0.8605
V4	0.5051	0.8117	0.6911	0.7815
V5	0.7628	0.8469	0.8898	0.8577
V6	0.3512	0.7726	0.5355	0.7133
V7	0.8015	0.8872	0.9008	0.8906

V8	0.7140	0.8225	0.8713	0.8347
V9	0.8530	0.8801	0.9542	0.8986
V10	0.8648	0.8939	0.9366	0.9046
V11	0.5040	0.8391	0.9411	0.8646
V12	0.5700	0.8413	0.8141	0.8345
V13	0.4152	0.8187	0.8003	0.8141
V14	0.6898	0.8643	0.8655	0.8646
V15	0.7892	0.8699	0.8843	0.8735
V16	0.5663	0.8450	0.8591	0.8485
V17	0.8029	0.8739	0.8937	0.8788
V18	0.6191	0.8362	0.7762	0.8212

Step 3 Using the data of table 3-3, the matrix of the evaluation object C-city can be obtained $T_1 = (t_{ij})_{6*4}$:

$$T_1 = \begin{pmatrix} 0.7671 & 0.8349 & 0.9283 & 0.8583 \\ 0.7671 & 0.8763 & 0.8728 & 0.8734 \\ 0.7661 & 0.8563 & 0.8812 & 0.8605 \\ 0.5051 & 0.8117 & 0.6911 & 0.7815 \\ 0.7628 & 0.8469 & 0.8898 & 0.8577 \\ 0.3512 & 0.7726 & 0.5355 & 0.7133 \end{pmatrix}$$

Evaluation object Z-City matrix: $T_2 = (t_{ij})_{6*4}$:

$$T_2 = \begin{pmatrix} 0.8015 & 0.8872 & 0.9008 & 0.8906 \\ 0.7140 & 0.8225 & 0.8713 & 0.8347 \\ 0.8530 & 0.8801 & 0.9542 & 0.8986 \\ 0.8648 & 0.8939 & 0.9366 & 0.9046 \\ 0.5040 & 0.8391 & 0.9411 & 0.8646 \\ 0.5700 & 0.8413 & 0.8141 & 0.8345 \end{pmatrix}$$

Evaluation object X-City matrix: $T_3 = (t_{ij})_{6*4}$:

$$T_3 = \begin{pmatrix} 0.4152 & 0.8187 & 0.8003 & 0.8141 \\ 0.6898 & 0.8643 & 0.8655 & 0.8646 \\ 0.7892 & 0.8699 & 0.8843 & 0.8735 \\ 0.5663 & 0.8450 & 0.8591 & 0.8485 \\ 0.8029 & 0.8739 & 0.8937 & 0.8788 \\ 0.6191 & 0.8362 & 0.7762 & 0.8212 \end{pmatrix}$$

The weight of the 6 decision attributes is $W = (0.162, 0.198, 0.201, 0.157, 0.174, 0.108)$, The formula of mixed weighted similarity is as follows:

$$WS = WT \tag{8}$$

Step 4 Use the formula (8) to obtain the sorting result, as shown in table 4 .

Table 4 sort results of four weighted similarity atα=0.75

	WJ-similarity	**WE-similarity**	**WC-similarity**	**WH-similarity**
C-city	0.6801	0.8391	0.8215	0.8339
Z-city	0.7277	0.8607	0.909	0.8728
X-city	0.6579	0.8536	0.853	0.8535
rank	z>c>x	z>x>c	z>x>c	z>x>c

see from table 4, the decision result of H-similarity method are: Z, X, C, the emergency management capacity of Z-city is the best as good as the E and C similarity decision results, but the decision result of J-similarity are not consistent with three other method. In order to realize the method is universal, we change the α value, to see whether the final result will change or not, such as α =0.1 and 0.5 when use formula (3-6) to calculate four similarity decision results respectively, as shown in table 5,6.

Table.5 sort results of four weighted similarity at α =0.1

	WJ-similarity	WE-similarity	WC-similarity	WH-similarity
C-city	0.6801	0.8391	0.8215	0.8231
Z-city	0.7277	0.8607	0.909	0.9041
X-city	0.6579	0.8536	0.853	0.8531
rank	z>c>x	z>x>c	z>x>c	z>x>c

Table.6 sort results of four weighted similarity at α =0.5

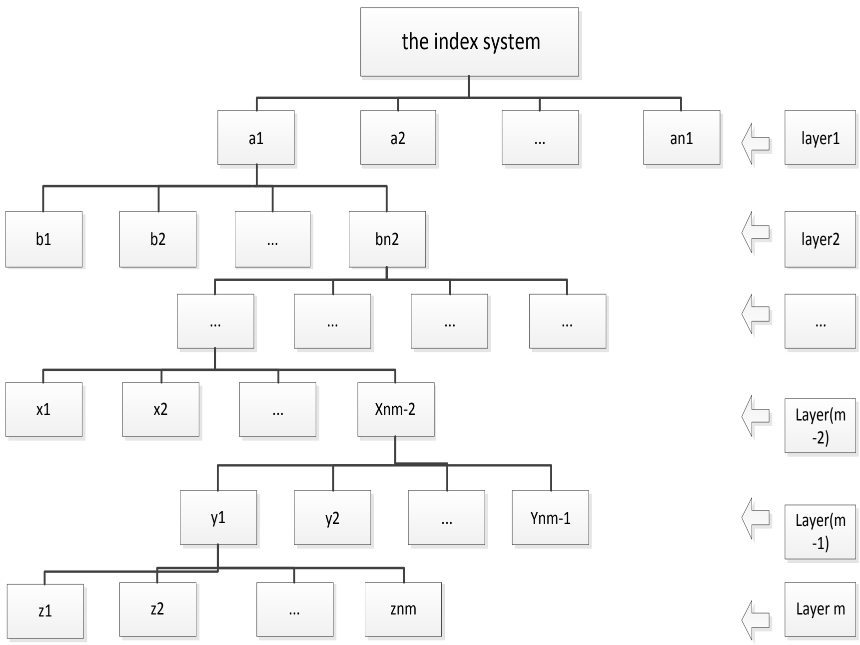
	WJ-similarity	WE-similarity	WC-similarity	WH-similarity
C-city	0.6801	0.8391	0.8215	0.8231
Z-city	0.7277	0.8607	0.909	0.9041
X-city	0.6579	0.8536	0.853	0.8531
rank	z>c>x	z>x>c	z>x>c	z>x>c

As can be seen from tables 5 and 6, the decision result is consistent with the result of table 3-4, indicating that the decision result is almost independent of the value ofα.

4. Ranking algorithm based on multi-attribute hierarchical relation

4.1 Aggregations of preference vectors based on hierarchical relations

Suppose in a natural disaster evaluation system, there are m evaluation indexes, and $n_i(i = 1,2,\cdots,m)$ evaluation indexes in each layer, evaluation index of each object is same; the index system is as follow:



Definition 4.1 experts go to evaluate and get the value of evaluation $v_j(j = 1,2,\cdots,n)$, just call vector $V_j = (v_1, v_2, \cdots, v_n)$ as evaluation vector of evaluation object. The evaluation vectors of evaluation objects constitute the matrix$T^j = [V_1, V_2, \cdots, V_p]$, called the evaluation vector matrix.

Evaluation index of each layer can be regarded as an evaluation object of next layer, each index has an evaluation vector, then all indicators can form a vector matrix, then the evaluation vector matrix of the first evaluation object in fourth layer:

$$T_{m-1}^{1}=\begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n_{m-1}} \\ t_{21} & t_{22} & \cdots & t_{2n_{m-1}} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n_m1} & t_{n_m2} & \cdots & t_{n_mn_{m-1}} \end{pmatrix} \tag{9}$$

The evaluation vector of the first evaluation object in (m-2)th layer is

$$\begin{aligned} V_{m-2}^{1} &= W_m T_{m-1}^{1} \\ &= \left(w_m^1, w_m^2, \cdots, w_m^{n_m}\right)\begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n_{m-1}} \\ t_{21} & t_{22} & \cdots & t_{2n_{m-1}} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n_m1} & t_{n_m2} & \cdots & t_{n_mn_{m-1}} \end{pmatrix} \\ &= \left(v_{m-1}^{11}, v_{m-1}^{12}, \cdots, v_{m-1}^{1n_{m-1}}\right) \end{aligned} \tag{10}$$

According to the above formula (4-2), you can get n_{m-2} evaluation vectors in (m-2)th layer, they can form evaluation matrix of the (m-2)th layer. You can get evaluation vector of the (m-3)th layer by taking the weight of indexes on the (m-2) layer. Then evaluation vectors of the (m-3)th layer form evaluation matrix of the (m-3)th layer, and so on. Finally get the evaluation target layer vector, and then take on the attribute weights of each index, we can get the evaluation results.

4.2 Evaluation model algorithm based on attribute hierarchical relation

Suppose that there is an evaluation problem, I evaluation object, denoted as $a^i (i = 1,2,\cdots,l)$ there are m evaluation indexes in the evaluation system. The number of evaluation indexes is same, the specific steps are as follows:

Step 1 data preprocessing

For raw data, a nonlinear transformation is performed:

$$Z_{ij}=\frac{v^{ij}-\overline{v^j}}{\max(v^j)-\overline{v^j}}(1-\beta)+\beta \tag{11}$$

Here, $i = 1,2,\cdots,l; j = 1,2,\cdots,n_k, \overline{v^j} = \frac{\sum_{i=1}^{l} v_{ij}}{l}$, β is the mean of the evaluation value which is set. The value of βmay be between 0.5 and 0.75.

Step 2 Calculate the weight of index

Firstly, the n_k indexes are arranged from high to low according to importance, and the weight vector is calculated by using formula (2-1) - (2-4): $W_k = \left(w_k^1, w_k^2, \cdots, w_k^{n_k}\right)$.After traversing each layer index, we can get the index weight vector of each layer.

Step 3 Evaluate vector aggregations.

Accumulation starts from the bottom, and find the (m − 1)th layer evaluation vector matrix$T_k^i (i = 1,2,\cdots,n_{m-1})$, According to the formula (4-1) above, the evaluation vector matrix of m-2 evaluation indexes can be obtained. Then, the evaluation vector matrix of the upper layer can be gradually obtained by iteration.

Step 4 Calculate the evaluation vector matrix of second layers according to the formula

$$T_2=\begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n_1} \\ t_{21} & t_{22} & \cdots & t_{2n_1} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n_21} & t_{n_22} & \cdots & t_{n_2n_1} \end{pmatrix}$$

Then the first layer's evaluation vector is:

$$V_1 = W_2T_2 = (w_2^1, w_2^2, \cdots, w_2^{n_2}) \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n_1} \\ t_{21} & t_{22} & \cdots & t_{2n_1} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n_21} & t_{n_22} & \cdots & t_{n_2n_1} \end{pmatrix} = (v_1^1, v_1^2, \cdots, v_1^{n_1})$$

The outcome of the evaluation object a_1 is:

$$O_1 = W_1(V_1)' = (w_1^1, w_1^2, \cdots, w_1^{n_1}) \begin{pmatrix} v_1^1 \\ v_1^2 \\ \cdots \\ v_1^{n_1} \end{pmatrix} \quad (12)$$

Step 5 Set $i = i + 1$,execute step 3-4,until $i > l$,The evaluation result of l evaluation objects is $o_1, o_2, \cdots, o_l$,form vector $O = (o_1, o_2, \cdots, o_l)$,here $0 \le o_i \le 1$,if there is one of them $o_i > 1$,normalize O.

Step 6 Establish evaluation level interval set as$\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_s\}$, If the object value is $\frac{s-1}{s} \le \alpha_i \le \frac{s-2}{s}$, then its rating belongs to α_2.The level of the evaluation object is shown in table 4-1.

Table 7 rating hierarchy

rating	1	2	…	i	…	s
Interval value	$\left(1, \frac{s-1}{s}\right)$	$\left(\frac{s-1}{s}, \frac{s-2}{s}\right)$	…	$\left(\frac{s-i+1}{s}, \frac{s-i}{s}\right)$	…	$\left(0, \frac{1}{s}\right)$
set	α_1	α_2	…	α_i	…	α_s

Step 7 Judge result belongs to which interval.

Determine the range according to the size of the O_i, and then judge the grade of the evaluated object.

5. Case analysis

Take the annual natural disasters situation (2002-2014, see Table 6 and table 7) in China environmental statistics yearbook of 2015 as an example, assesse the severity of natural disasters from 2002 to 2014 for government to better understand the disaster as a reference. So in these decision problems, disasters every year is the evaluation object, the evaluation index has two layers, geological disasters, earthquake disaster, marine disasters and forest fires as the first level indicators, second level indicators are disaster, casualties and direct economic losses.

Table 8 disaster situation throughout the country (2002-2014 years)

year	geological disasters			earthquake disasters		
	number of disasters	casualties	direct economic losses	number of disasters	casualties	direct economic losses
2002	40246	2759	509740	5	362	13100
2003	15489	1333	504325	21	7465	466040
2004	13555	1407	408828	11	696	94959
2005	17751	1223	357678	13	882	262811
2006	102804	1227	431590	10	229	79962
2007	25364	1123	247528	3	422	201922
2008	26580	1598	326936	17	446293	85949594
2009	10580	845	190109	8	407	273782
2010	30670	3445	638509	12	13795	2361077
2011	15804	410	413151	18	540	6020873

2012	14675	636	625253	12	1279	828757
2013	15374	929	1043568	14	15965	9953631
2014	10937	637	567027	20	3666	3326078

Table 9 continued - national natural disasters over the years (2002-2014 years)

year	marine disasters			forest fires		
	number of disasters	casualties	direct economic losses	number of disasters	casualties	direct economic losses
2002	126	124	65.9	7527	98	3610
2003	172	128	80.5	10463	142	37000
2004	155	140	54.2	13466	252	20213
2005	176	371	332.4	11542	152	15029
2006	180	492	218.5	8170	102	5375
2007	163	161	88.4	9260	94	12416
2008	128	152	206.1	14144	174	12594
2009	132	95	100.2	8859	110	14511
2010	120	137	132.8	7723	108	11611
2011	114	76	62.1	5550	91	20173
2012	138	68	155	3966	21	10802
2013	115	121	163.5	3929	55	6062
2014	100	24	136.1	3703	112	42513

Step1 Data preprocessing

According to the formula (4-3), the data in Table 8 and table 9 are standardized and the β is 0.6, and the results of non-linear variations of the upper table are shown in table 10 and table 11.

Table 10 vector of natural disaster assessment

year	geological disasters			earthquake disasters		
	number of disasters	casualties	direct economic losses	number of disasters	casualties	direct economic losses
2002	0.6736	0.8689	0.6199	0.2367	0.5633	0.5565
2003	0.5444	0.5964	0.616	1	0.5702	0.5588
2004	0.5343	0.6106	0.548	0.5229	0.5636	0.5569
2005	0.5562	0.5754	0.5116	0.6183	0.5638	0.5578
2006	1	0.5762	0.5642	0.4752	0.5632	0.5568
2007	0.5959	0.5563	0.4331	0.1413	0.5633	0.5574
2008	0.6023	0.6471	0.4897	0.8092	1	1
2009	0.5188	0.5032	0.3922	0.3798	0.5633	0.5578
2010	0.6236	1	0.7115	0.5706	0.5764	0.5686
2011	0.5461	0.4201	0.5511	0.8569	0.5635	0.5875
2012	0.5402	0.4632	0.7021	0.5706	0.5642	0.5607
2013	0.5438	0.5192	1	0.6661	0.5786	0.6078
2014	0.5207	0.4634	0.6606	0.9523	0.5665	0.5736

Table 11 further - natural disaster assessment vector

year	geological disasters			earthquake disasters		
	number of disasters	casualties	direct economic losses	number of disasters	casualties	direct economic losses
2002	0.461	0.5557	0.4513	0.5447	0.5463	0.4063
2003	0.9202	0.5605	0.4813	0.7467	0.6759	0.9159
2004	0.7505	0.575	0.4272	0.9533	1	0.6597
2005	0.9601	0.8539	1	0.821	0.7054	0.5806
2006	1	1	0.7655	0.5889	0.5581	0.4333
2007	0.8303	0.6004	0.4976	0.6639	0.5345	0.5407
2008	0.481	0.5895	0.7399	1	0.7702	0.5434
2009	0.5209	0.5207	0.5219	0.6363	0.5816	0.5727
2010	0.4012	0.5714	0.589	0.5582	0.5758	0.5284
2011	0.3413	0.4977	0.4435	0.4086	0.5257	0.6591
2012	0.5808	0.4881	0.6347	0.2996	0.3194	0.5161
2013	0.3512	0.5521	0.6522	0.2971	0.4196	0.4438
2014	0.2015	0.4435	0.5958	0.2816	0.5875	1

Step2 Calculate the weight of the index

Sort the second layer index according to their importance, the result is casualties ,direct economic losses, the number of disasters, then calculate the weight of indexes according to formula(2-1)-(2-2),the weight of second layer is 0.5690, 0.3333, 0.0976, so Then index weight vector of the second layer is $W_2 = (0.0976{,}0.5690{,}0.3333)$. Then the first layer index: earthquake disasters, geological disasters, marine disasters, forest disaster, their index weight are respectively 0.4522, 0.3272, 0.1727, 0.0478, then the first layer for weight vector $W_1 = (0.3272{,}0.4522{,}0.1727{,}0.0478)$.

Step3 Firstly evaluate 2002 year, the evaluation vector matrix of second layer of 2002year is $T_2 = \begin{pmatrix} 0.6736 & 0.2367 & 0.461 & 0.5447 \\ 0.8689 & 0.5633 & 0.5557 & 0.5463 \\ 0.6199 & 0.5565 & 0.4513 & 0.4063 \end{pmatrix}$

Step4 Calculate the evaluation vector of the first layer evaluation index of 2002 year is

$$V_1 = W_2 T_2 = (0.0976{,}0.5690{,}0.3333)\begin{pmatrix} 0.6736 & 0.2367 & 0.461 & 0.5447 \\ 0.8689 & 0.5633 & 0.5557 & 0.5463 \\ 0.6199 & 0.5565 & 0.4513 & 0.4063 \end{pmatrix} = (0.7668{,}0.5291{,}0.5116{,}0.4994)$$

According to the formula (4-4), the evaluation result of the evaluation object of 2002 year is

$$O_{2002} = W_1 (V_1)' = \left(w_1^1, w_1^2, \cdots, w_1^{n_1}\right)\begin{pmatrix} v_1^1 \\ v_1^2 \\ \cdots \\ v_1^{n_1} \end{pmatrix} = (0.3272{,}0.4522{,}0.1727{,}0.0478)\begin{pmatrix} 0.7668 \\ 0.5291 \\ 0.5116 \\ 0.4994 \end{pmatrix} = 0.6024$$

Step5 According to the above method, the evaluation vectors can be obtained from 2003 to 2014 respectively, and an evaluation vector matrix is formed, which is denoted as T:

$$T_{03-08} = \begin{pmatrix} 0.5978 & 0.5822 & 0.5522 & 0.6153 & 0.5190 & 0.5902 \\ 0.6083 & 0.5573 & 0.5671 & 0.5524 & 0.5201 & 0.9813 \\ 0.5692 & 0.5428 & 0.9129 & 0.9217 & 0.5885 & 0.6290 \\ 0.7627 & 0.8819 & 0.6750 & 0.5195 & 0.5491 & 0.7170 \end{pmatrix}$$

$$T_{09-14} = \begin{pmatrix} 0.4677 & 0.8670 & 0.4760 & 0.5503 & 0.6818 & 0.5347 \\ 0.5435 & 0.5732 & 0.6001 & 0.5636 & 0.5968 & 0.6065 \\ 0.5211 & 0.5606 & 0.4643 & 0.5460 & 0.5658 & 0.4706 \\ 0.5839 & 0.5582 & 0.5587 & 0.3830 & 0.4157 & 0.6951 \end{pmatrix}$$

the evaluation results of 2003 to 2014 is

$$O_{03-08} = W_1(V_1)' = (0.6054 \quad 0.5784 \quad 0.6270 \quad 0.6345 \quad 0.5329 \quad 0.7798)$$

$$O_{09-14} = W_1(V_1)' = (0.5167 \quad 0.6664 \quad 0.5340 \quad 0.5475 \quad 0.6105 \quad 0.5637)$$

Step6 Establish evaluation rating interval set: $\alpha = (\text{first level}, \text{second level}, \cdots, \text{sixth level})$, among, the first level disaster is an extremely large type of disaster, the second level is a major disaster, the third level is a major one, the fourth one is medium, the fifth is small, and the sixth is a minor one. The corresponding evaluation criteria are as follows:

Table 12 rating levels

rating	**1**	**2**	**3**	**4**	**5**	**6**
Interval value	(1,0.83)	(0.83,0.67)	(0.67,0.5)	(0.5,0.33)	(0.33,0.17)	(0.17,0)
rating level	first level	Second level	Third level	Fourth level	Fifth level	Sixth level

See from this table,2008 year is the most serious, it belongs to the second level disaster, 2009 year is the lowest level, however, a few of other year is equal.

6. Conclusion

This paper discusses the problem of large group decision making based on multi-attribute hierarchical relation. Firstly, it introduces the background, significance and research status of large group decision making. Secondly, it gives a new attribute weight determination method. The weight of attributes is determined by trigonometric function, which considers both the hierarchy of attributes and the weight of attributes objectively. Again, considering the shortcomings of the existing similarity model, proposes a mixed similarity model, the model not only takes distance between vectors into consideration, but also takes direction angle between two vectors. It is a supplement to the existing similarity model, in order to show the advantages of the mixed similarity, this paper illustrates the effectiveness of this method from the numerical and practical cases. Of course, this paper also has a lot of shortcomings, which are the directions that will be improved in the future.

References

[1] Zahir S. :Clusters in a group: Decision making in the vector space formulation of the analytic hi erarchy Process(European Journal of Operational Research), Vol.3(1999) No.22,p.620-634.

[2] Srdjevic B.: Linking analytic hierarchy Proeess and social choice methods to support group dec ision-making in water management(Decision Support Systems), Vol.42 (2007)No.4,p. 2261-2273 .

[3] Saaty T.L., Shang J.S.:Group decision-making : Head-count versus intensity of preference (Socio- Economic Planning Sciences),Vol.41(2007)No.1,p.22-37.

[4] Hoffmann S., Fisehbeek P., KruPniek A., et al, Elicitation from Large, Heterogeneous Expert P anels: Using Multiple Uncertainty Measures to Charaeterize Information Quality for Decision Analysis(Decision Analysis),Vol.4(2007)No.2,p. 91-109.

[5] Xuanhua Xu, Xiaohong Chen. Research on the group clustering method based on vector space (Systems Engineering and Electronics), Vol.27(2005)No.6,p. 1034-1037.

[6] Lihui Hu, Guosong Luo. Improved algorithm for group clustering based on vector space(Systems Engineering and Electronics),Vol.29(2007)No.3,p. 472-474.

[7] Xiaohong Chen, Rong Liu. Improved clustering algorithm and its application in complex huge group decision-making (Systems Engineering and Electronics),Vol.28 (2006) No.11,p.1695-1699.

[8] Rong Liu. Multi-attribute complex huge group-decision methods research basing on clustering algorithm(Ph.D.,Doctoral Dissertation of Central South University, China 2007),p.1-28.

[9] Xiaohong Chen, Xuanhua Xu, Hongliang Zeng. Method of multi-attribute large group decision making based on entropy weight (Systems Engineering and Electronics),Vol.7 (2007) ,p.1305-1309.

[10] Xuanhua Xu, Xiaohong Chen. Research of a kind of method of multi-attributes and multi-schemes large group decision making (JOURNAL OF SYSTEMS ENGINEERING),Vol.23 (2017) No.2,p. 137-141.

[11] Moreno-Jimenez J.M, Polasek W., E-Democracy and knowledge: A Multicriteria Framework for the New Democratic Era (Journal Multi-criteria Decision Analysis),Vol.12 (2003) No.1,p.163-176.

[12] Jaccaxd P. Distribution de la flore alpine dans leBassin des Drouces et dans quelques regions voisines(Bull de la SociteVaudoise des Sciences Naturelles),Vol.37(1901),p. 241-272.

[13] W.J. Wang . New similarity measures on fuzzy sets and on elements (Fuzzy Sets and Systems),Vol.85(1997),p.305-309.

[14] Z. Liang, Shi P. Similarity measures on intuitionistic fuzzy sets(Pattern Recognition Letters),Vol.24(2003),p. 2678-2693.

[15] Hung W.L., Yang M.S. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance(Pattern Recognition Letters),Vol.25(2004),p. 1603-1611.

[16] Salton G., McGill M. J. Introduction to modern information retrieval(New York: McGraw-Hill), Vol.24(1987),p.1-2.

[17] Xuanhua Xu, Fang Li. Evaluation of emergency management capability of major snow disaster-Take Hunan Province as an example (JOURNAL OF CATASTROPHOLOGY),Vol.26 (2011)No.2,p.130-137.