A Research on Optimized Dividend Strategy in the Listed Companies

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Abstract

Capital asset pricing is one of the fundamental study in financial economics. The paper introduces the bonus and allotment of shares into stock price to redesign the market investment opportunities and stock dividend process referring to the Consumption Capital Asset Pricing Model and General Equilibrium Theory. It also demonstrates that we assume a bank sell out all riskless assets to buy stock. Then there will be an opportunity in a risk-aversion market. With the restriction of private property and market clearing, the paper utilizes the optimized investor utility function to count the stock price in the market. In addition, it analyzes the impact of bonus rate and quantity of shares allotment on stock prices so that it may provide preliminary references to develop dividend strategies for listed companies.

Keywords

Dividend strategy, Optimization, Equilibrium price, CCAPM.

1. Introduction

The subprime mortgage crisis, the European debt crisis and the Brexit event had great influence on the entire world economic development, which seriously shook the confidence of traders in the global stock market. Simi-larly, the shareholders were suffocated and desperate on 6.29th in 2015 when the stock market experienced dramatic fluctuations between growth and collapse in China. The increasing risk aversion in the stock market reduced stock trading volume and stock prices were affected seriously. When the administrators in listed com-panies operated daily business, they should consider some appropriate measures to attract more investors and maintain the company stock price in the valuation of normal range. Because of it, asset pricing became the inevitable problem which the administrators in listed companies had to confront. Historically, Sharpe [1], Linte [2] and Mossin [3] proposed the Capital Asset Pricing Model (CAPM) as the basic quantitative model of the shares risk and price. Ross [4] proposed the Arbitrage Pricing Theory where the model was simpler than the CAPM but the measurement scale of stock risk was more complex. Black-Scholes [5-6] explained the method of continuous-time option pricing based on the concept of risk-hedging. Merton [7] proposed Intertemporal Capital Asset Pricing Model (ICAPM) that derived a generalized continuous and multi-stage equilibrium model. In the model, when the return of the risk-free asset is a constant, the form of the excess return of ICAPM is in accordance with the conclusion of CAPM. After ICAPM, Consumption-based Capital Asset Pricing Model (CCAPM) of Lucas [8] and Breeden [9], Production-based Capital Asset Pricing Model (PCAPM) of Cox, Inger-soll and Ross [10], Money-based Capital Asset Pricing Model (MCAPM) of Lucas and Stokey [11], Three-factor Model of Fama and French [12-13], as well as Liquidity-based Capital Asset Pricing Model (LCAPM) of Holmstrem and Tirole [14] refer to relatively fixed underlying assets. Some of the above-mentioned models fuse time cost; some of them mix with the expected benefits to find the balance relationship. But in the capital market, stock as the main trading commodity owns the specific characteristics of bonus and allotment of shares. Among the above models, CCAPM is the most appropriate for the stock market analysis, so the paper intends to analyze how the bonus and the quantity of the shares allotment impact the stock price based on the CCAPM to offer the references for the companies' decision makers on the characteristics of stock and the fac-tors of bonus and allotment of shares.

2. The Model

CCAPM stands for Consumption-based Capital Asset Pricing Model. The CCAPM factors in consumption as a means of understanding and calculating an expected return on investment. The CCAPM implies that the expected risk premium on a risky asset, defined as the expected return on a risky asset less the risk free return, is proportional to the covariance of its return and consumption in the period of the return. It has become an important part of modern macroeconomics and finance and is one of the main achievements in the field of economics. The CCAPM establishes an expected utility function in which the consumer's relative risk aversion factor indicates their risk tolerance. As we know, the goal of the consumers is to maximize expected present discounted value in their life, for which the consumers need design the proportion of consumption and securities investment in the period of the return. In addition, the CCAPM may set different utility functions to describe the behavior of consumers. In it, the equilibrium price of assets, expected excess return and risk levels can be evaluated under some constrains such as the private property and market regulations.

The Arrow-Debreu General Equilibrium Theory tells us rational investors pursue the optimistic allocation of resources of space and time in the market activities. As long as all rational investors estimate the possible opportunity of profit and loss, they will reach a consensus. In this case, each of them will achieves the optimistic allocation of resources when the market is general equilibrium. Also, the equilibrium price can be calculated when the market is clearing.

The paper cites the framework ideas of Consumption Capital Asset Pricing Model and General Equilibrium Theory. We assume there are only the risky stock and the riskless bond (the term 'bond' is the one traditionally used to describe the riskless security) in the general discrete market models. The introduction mentions the current traders are lack of the confidence in the market, so we suppose that all traders have risk-aversion preference in the model, the utility function $U(\cdot)$ denoted their process of trading. The traders through the opportunity which can short-selling the riskless bond and buy the risky stock to earn expected excess return, they have the same expected profits are finite in the market and only trade at the discrete time regardless of the costs. In addition, we define the number of risk assets is fixed as constant M. The bond interest rate is r $(r>0)$ and the supply of bonds is perfectly elastic. It is that the behavior of traders will not have an impact on bonds price.

Assuming by that the process of stock dividends subject to the following rules:

$$D_t = \alpha_t P_t + \beta P_{t+1} \ (0 < \alpha_t < 1) \tag{1}$$

In Equation (1), P_t stands for the stock price. D_t is a sequence of random variables concerning about the bonus distribution $\alpha_t P_t$ and the allotment of shares βP_{t+1} .The model implies that traders receive the bonus as well as shares at time t , and then trade shares at time $t+1$. In the $\alpha_t P_t$ part, to make bonus distribution obvious, α_t is separated into τ and $(P_{t+1}-P_t)/P_t$, formulizing $\alpha_t = \tau(P_{t+1}-P_t)/P_t$ is the rate of return at time t and τ $(0<\tau<1)$ is a constant as the bonus factor. $\alpha_t P_t$ is vary according to the changes of the rate of return $(P_{t+1}-P_t)/P_t$. Moreover, in the βP_{t+1} part, β $(\beta>0)$ is a fixed constant standing for the allotment factor that decides the new shares distribution. Company information is fully disclosed, which means the company's profit and loss are shown in the stock price thoroughly.

We write Equation (1) at time $t=1,\ 2,\ \cdots,\ T+1$, respectively as following:

$$\begin{aligned} &D_1 = -\tau P_1 + (\tau+\beta)P_2 \\ &D_2 = -\tau(1+\beta)P_2 + (1+\beta)(\tau+\beta)P_3 \\ &\vdots \\ &D_{T+1} = -\tau(1+\beta)^T P_{T+1} + (1+\beta)^T(\tau+\beta)P_{T+2} \end{aligned} \tag{2}$$

The number of traders is h_t at t time, which is a random variable. We denote $\eta_t = 1/h_t, (\eta_t, h_t > 0)$ for easy calculation purpose.

Now, we suppose the market is the probability space $\{\Omega, F, \mathrm{P}\}$, which has no arbitrage and is complete, the completeness impose the traders know the entire history information about the price and dividend. Let F_t be the information of traders which is defined as $F_t = \sigma\{P_\tau, D_\tau / \tau \le t\} \subset F$. $E_t(\cdot)$ represents the calculation of the conditional expectation of F_t.

As we know, the traders are lack of the confidence in the current market, so we suppose that the utility function $U(\cdot)$ which implies the traders in this model are risk averse is strictly increasing and strictly concave. In the meanwhile, the traders are different in the risk tolerance ϕ_i. The expected return of ith trader at time t can be written as:

$$E_t[u(\omega_{i,t+1})] = E_t(-\exp(-\frac{\omega_{i,t+1}}{\phi_i})) \tag{3}$$

3. The Equilibrium

In order to obtain market equilibrium, the trading strategies are required as self-financing, which implies that every change in the value result entirely from net gains (or losses) realised on the investments. We describe the investment opportunity firstly, the initial wealth of every traders is assumed as zero as well as he finances by selling his all bonds to purchase one unit of the stock. we set the undiscounted cash flows of the portfolio as Φ_t, so at time $t+1$, there is：

$$\begin{aligned} \Phi_{t+1} &= P_{t+1} + D_{t+1} - P_t(1+r) \\ &= \hat{e}_{\Phi,t} + \nu_{\Phi,t} \end{aligned} \tag{4}$$

where $\hat{e}_{\Phi,t} = E_t[\Phi_{t+1}]$ stands for the conditional expectation of the excess return at time t, $\nu_{\Phi,t} = \Phi_{t+1} - \hat{e}_{\Phi,t}$ is the residual variance (The "residual" means the gap between the actual payment of the company and expected payment in the market model at a certain period, so the residual variance measures the reflection of the company to the uncertain events). Hereby, we assume $\nu_{\Phi,t}$ is the normal distribution and the variance $Var_t(\nu_{\Phi,t+1}) = \sigma_v^2$ is irrelevant to the time t.

Then we are to solve the profit maximization of the ith trader:

$$\max_{\lambda_{i,t}} E_t(-\exp(-\frac{\omega_{i,t+1}}{\phi_i})) \tag{5}$$

The constraint condition is:

$$\omega_{i,t+1} = (1+r)\omega_{i,t} + \lambda_{i,t}\hat{e}_{\Phi,t} + \lambda_{i,t}\nu_{\Phi,t} \tag{6}$$

$\lambda_{i,t}$ in the above formula illustrates the shareholdings of the ith trader.

$$\begin{aligned} & E_t(-\exp(-\frac{\omega_{i,t+1}}{\phi_i})) \\ &= (-e^{-\frac{(1+r)\omega_{i,t}}{\phi_i}}) \cdot (-e^{-\frac{\lambda_{i,t}\hat{e}_{\Phi,t}}{\phi_i}}) \cdot E_t(-e^{-\frac{\lambda_{i,t}}{\phi_i}\nu_{\Phi,t}}) \\ &= (-e^{-\frac{(1+r)\omega_{i,t}}{\phi_i}}) \cdot (-e^{-\frac{\lambda_{i,t}\hat{e}_{\Phi,t}}{\phi_i}}) \cdot -E_t(e^{it(i\frac{\lambda_{i,t}}{t\phi_i}\nu_{\Phi,t})}) \\ &= -e^{-\frac{(1+r)\omega_{i,t}}{\phi_i} - \frac{\lambda_{i,t}\hat{e}_{\Phi,t}}{\phi_i} + \frac{1}{2}\sigma_v^2\frac{\lambda_{i,t}{}^2}{\phi_i^2}} \end{aligned}$$

Therefore, calculate the optimized problem as following:

$$\min_{\lambda_{i,t}} e^{-\frac{\lambda_{i,t}\hat{e}_{\Phi,t}}{\phi_i} + \frac{1}{2}\sigma_v^2\frac{\lambda_{i,t}{}^2}{\phi_i^2}} \tag{7}$$

Then the optimal investment strategy is:

$$\lambda_{i,t} = \frac{\phi_i\hat{e}_{\Phi,t}}{\sigma_v^2} \tag{8}$$

Because the number of the traders is $\eta_t = 1/h_t$, the market clearing condition is equivalent to:

$$\sum_{i=1}^{h_t}\lambda_{i,t}=M \tag{9}$$

Combine Equations (8) (9), we obtain:

$$\sum_{i=1}^{h_t}\frac{\phi_i\hat{e}_{\Phi,t}}{\sigma_v^2}=M \tag{10}$$

In order to recognize the relation between the factors, we use the average degree of risk tolerance constant ϕ instead of the personality risk tolerance ϕ_i .Under the circumstance there may be a little loss. We will discuss it later $\phi_i=\phi(h_i)$ will change along with the number of shareholders. so we have:

$$\hat{e}_{\Phi,t}=\frac{\sigma_v^2M}{h_t\phi} \tag{11}$$

Remark 1. Equation (11) is shown that the expected return $\hat{e}_{\Phi,t}$ is inversely proportional with the current average degree of risk tolerance ϕ, is inversely proportional with the quantity of temporary shareholders and is directly proportional with supply of the shares in the market.

Remark 2. While the number of shareholders is increasing, the impact of the shareholders' behavior on the equilibrium expected return is reducing. In other words, the impact of an individual stock trading behavior on the market price is reducing.

The main results are represented by Theorem 1.

Theorem 1. In the framework of model, we can find the equilibrium price P_t, such that:

$$P_t=-\frac{\sigma_v^2M}{(1-r)\phi}\left(\sum_{j=0}^{\infty}x_{t+j}\eta_{t+j+1}+\eta_t\right)$$

Where the x_{t+j} is a polynomial about τ and β. Furthermore, the number of traders is related to the bonus factor τ and the allotment factor β, which satisfy the following condition:

$$\lim_{j\to\infty}\frac{\eta_{t+j+1}}{\eta_{t+j}}<\frac{\tau(1+\beta)}{\tau+\beta}$$

Proof: See Appendix A.

Corollary 1. When the growth rate of traders is the constant α, which can be written as $\eta_{t+j+1}/\eta_{t+j}=\alpha$. Then we have:

If $\frac{\alpha\beta}{1-\alpha+\beta}<\tau<1$ and $\alpha>0$, P_t increases along with the reduction of the bonus factor τ.

If $\beta<\frac{\tau(1-\alpha)}{\alpha-\tau}$ and $\alpha<0$, P_t increases along with the reduction of the allotment factor β.

Proof: we assume that the growth rate of the traders is $\lim_{j\to\infty}\eta_{t+j+1}/\eta_{t+j}=\alpha$, $(\alpha>0)$. Thus, we have:

$$\alpha<\tau(1+\beta)/(\tau+\beta) \tag{12}$$

Converting Equation (12) to the expression of bonus factor τ and allotment factor β, such that :

$$\frac{\alpha\beta}{1-\alpha+\beta}<\tau<1 \text{ and } \beta<\frac{\tau(1-\alpha)}{\alpha-\tau}$$

First, we try to find how does the bonus factor τ impact on P_t :

$$\frac{\partial P_t}{\partial\tau}=\frac{\sigma_v^2M}{(1-r)\phi}\sum_{i=0}^{\infty}\frac{\partial(-x_{t+i})}{\partial\tau}\eta_{t+i+1}+\eta_t$$
$$=\frac{\sigma_v^2M}{(1-r)\phi}\left[\sum_{k=0}^{\infty}\frac{\partial(-x_{t+2k})}{\partial\tau}\eta_{t+2k+1}-\sum_{k=0}^{\infty}\frac{\partial(-x_{t+2k-1})}{\partial\tau}\eta_{t+2k}\right]$$

From Appendix A, we know:

$$-x_{t+2k}=\frac{(1+\beta)^{t+2k-1}}{(1+r)}[(\tau(1+\beta)-1)x_{t+2k-1}-(\tau+\beta)x_{t+2k-2}]$$ So, we get:

$$\frac{\partial(-x_{t+2k})}{\partial(-x_{t+2k-1})} = (1+r)^{-1}(1-\tau(1+\beta)^{t+2k}).$$

When $k \to \infty$, the above equation becomes:

$$\lim_{k\to\infty}\frac{\partial(-x_{t+2k})}{\partial(-x_{t+2k-1})} = \lim_{k\to\infty}(1+r)^{-1}(1-\tau(1+\beta)^{t+2k})$$

Because $\frac{\eta_{t+2k}}{\eta_{t+2k+1}} = \frac{1}{\alpha}$, so we have:

$$\lim_{k\to\infty}\frac{\partial(-x_{t+2k})}{\partial(-x_{t+2k-1})} < \frac{\eta_{t+2k}}{\eta_{t+2k+1}}$$

So we know $\frac{\partial P_t}{\partial \tau} < 0$ is permanent establishment.

Therefore, when $\frac{\alpha\beta}{1-\alpha+\beta} < \tau < 1$ and $\alpha > 0$, P_t grows along with the decrease of τ.

Similarly, we can get the following conclusion:

when $\beta < \frac{\tau(1-\alpha)}{\alpha-\tau}$ and $\alpha < 0$, P_t increases along with the reduction of β.

Corollary 2. Expected excess return is reduced with the growth of traders.

Proof: we assume the numbers of the shareholders are h_1 and h_2, respectively and achieve the expected excess return in each condition based on Equation (11) is:

$\hat{e}_{\Phi,t}(h_1) = \sigma_v^2 M / \sum_{i=1}^{h_1}\phi_i$ and $\hat{e}_{\Phi,t}(h_2) = \sigma_v^2 M / \sum_{i=1}^{h_2}\phi_i$

Consequently, we denote the difference between the expected excess return in the above two conditions as the liquidity premium $LP(h_1, h_2)$ between the number of shareholders h_1 and the number of shareholders h_2.

$$LP(h_1,h_2) = \hat{e}_{\Phi,t}(h_1) - \hat{e}_{\Phi,t}(h_2)$$

So $LP(h_1,h_2) = \sigma_v^2 M(1/\sum_{i=1}^{h_1}\phi_i - 1/\sum_{i=1}^{h_2}\phi_i)$

It signifies the expected excess return decreases along with the growth of traders.

4. Conclusion

In the study we have characterized the optimized dividend strategies citing the ideas of CCAPM and General Equilibrium Theory to redesign market investment opportunities and stock dividend process. Then we calculate the bonus factor τ and the allotment factor β related to the number of traders and the stock price. The model indicates that in order to solve the dilemma in dividend choices the company administrators who make decision on how are the bonus and dividends distributed. On the contrary, most of administrators in the company considers the expected profits and financial budget by the Artificial Intelligence (AI) and Moving average method (MA) in a general dividend strategy. However, dividend strategies should be vary from time to time rather than there is only one general dividend strategy, the first of which impact on the internal price of the stocks at any time in some extent. The paper finds that when the growth rate of traders is satisfied the particular condition, the expected stock price is reverse fluctuated along with the bonus factor and the allotment factor, as long as the traders of risk-aversion generating the market recession. The listed companies can determine what dividend strategies should be adopted, keeping it the healthy growth so as to appeal the excess returns.

5. Appendix A

Proof of Theorem 1. When the market equilibrium, we can get the stock price P_t is:

$$\begin{aligned} P_t &= (1+r)^{-1}[E_t(P_{t+1}+D_{t+1}) - \frac{\sigma_v^2 M}{h_t\phi}] \\ &= (1+r)^{-1}[E_t(P_{t+1}+D_{t+1}) - \frac{\sigma_v^2 M\eta_t}{\phi}] \end{aligned} \tag{13}$$

Taking the process of stock dividends into Equation (13), we achieve:

$$\begin{aligned} P_t &= (1+r)^{-1}[(1-\tau(1+\beta)^t)E_tP_{t+1} \\ &\quad + (\tau+\beta)(1+\beta)^t E_tP_{t+2} - \frac{\sigma_v^2 M\eta_t}{\phi}] \end{aligned}$$

We denote:

$(1+r)^{-1}(1-\tau(1+\beta)^t) = m_t$,

$(1+r)^{-1}(\tau+\beta)(1+\beta)^t = n_t$,

$(1+r)^{-1} = A$, $\frac{\sigma_v^2 M}{\phi} = B$

Then we can rewritten P_t is:

$$P_t = m_tE_tP_{t+1} + n_tE_tP_{t+2} - AB\eta_t \tag{14}$$

By the iterative method, we get:

$$P_{t+1} = m_{t+1}E_{t+1}P_{t+2} + n_{t+1}E_{t+1}P_{t+3} - AB\eta_{t+1} \tag{15}$$

$$P_{t+2} = m_{t+2}E_{t+2}P_{t+3} + n_{t+2}E_{t+2}P_{t+4} - AB\eta_{t+2} \tag{16}$$

Taking Equations (15) (16) into Equation (14), according to conditional expectation smoothness, we get:

$$\begin{aligned} P_t &= [m_{t+2}(m_tm_{t+1}+n_t) + m_tn_{t+1}]E_tP_{t+3} + \\ &\quad (m_tm_{t+1}+n_t)n_{t+2}E_tP_{t+4} \\ &\quad - AB[(m_tm_{t+1}+n_t)\eta_{t+2} + m_t\eta_{t+1} + \eta_t] \end{aligned} \tag{17}$$

Then we denote the r.v. (x_{t+j}), (y_{t+j}), (z_{t+j}) substitute for the r.v. (m_t), (n_t), the initial condition is $x_t = m_t$, $y_t = n_t$, $z_t = 1$, at the time $t+i$, we have:

$$\begin{aligned} x_{t+i} &= m_{t+i}x_{t+i-1} + y_{t+i-1} \\ y_{t+i} &= n_{t+i}x_{t+i-1} \\ z_{t+i} &= x_{t+i-1} \end{aligned} \tag{18}$$

Next we can rewritten Equation (17) as:

$P_t = x_{t+i}E_tP_{t+i} + n_{t+i}x_{t+i-1}E_tP_{t+i+1} - AB(\sum_{j=0}^{i-1} x_{t+j}\eta_{t+j+1} + \eta_t)$ When time $i\to\infty$, we generally consider the expected return approaches to zero in the markets. Hence, as $i\to\infty$

$\lim_{i\to\infty} x_{t+i}E_tP_{t+i} = 0$, $\lim_{i\to\infty} y_{t+i}E_tP_{t+i+2} = 0$,

$$P_t = -AB(\sum_{j=0}^{\infty} x_{t+j}\eta_{t+j+1} + \eta_t) \tag{19}$$

By Equation (18), we have:

$$x_{t+j} = m_{t+j}x_{t+j-1} + n_{t+j-1}x_{t+j-2} \tag{20}$$

Similarly, we used the iterative method again, the r.v. (x_{t+j}) is a polynomial about β and τ.

From the definition of the equilibrium price P_t, we know that P_t is a positive series and convergence. So we have:

$-x_{t+j+1}\eta_{t+j+2} > 0$

$-x_{t+j}\eta_{t+j+1} / -x_{t+j-1}\eta_{t+j} < 1$

By the definition of η_t, we have $\eta_{t+j} > 0$, then we get:

$$-x_{t+j+1} = A(\tau(1+\beta)^{t+j+1}-1)x_{t+j} - A(\tau+\beta)(1+\beta)^{t+j}x_{t+j-1} \quad \frac{(\tau+\beta)(1+\beta)^{t+j}}{\tau(1+\beta)^{t+j+1}-1}\cdot\frac{\eta_{t+j+1}}{\eta_{t+j}} < \frac{-x_{t+j}\eta_{t+j+1}}{-x_{t+j-1}\eta_{t+j}} < 1$$

When $j \to \infty$, there is:

$$\frac{\tau+\beta}{\tau(1+\beta)}\cdot\lim_{j\to\infty}\frac{\eta_{t+j+1}}{\eta_{t+j}} < 1$$

So the r.v. (η_t) is satisfied with the condition:

$$\lim_{j\to\infty}\frac{\eta_{t+j+1}}{\eta_{t+j}} < \frac{\tau(1+\beta)}{\tau+\beta}$$

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