Optimal Control of Train Energy Saving Operation Based on PSO

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Abstract

In order to find the time interval $H$ that minimizes the total energy consumed by all trains, we build a multi-train energy-saving control optimization model, based on the kinetic model of train operation, a train energy-saving optimization algorithm based on particle swarm optimization. Taking the example of Problem D(two) in Chinese Mathematical Modeling 2015, we can get the basic energy consumption of a single train as $5.6530\times 10^5$ kJ and the regenerative energy as $3.3108\times 10^5$ kJ. The minimum basic energy consumption of 100 trains is $5.6530\times 10^7$ kJ, and the total regenerative energy is $4.6285\times 10^6$ kJ.

Keywords

Energy-saving control model  PSO  optimal control

1. Introduction

The rapid and rapid development of China's railways has alleviated many problems concerning people's livelihood and the economic construction of the country and has greatly promoted the development of the national economy. However, with the increase of the railway scale and the volume of transportation, the total amount of energy consumed by railway transportation is very huge. Therefore, how to save energy has become an important issue in the operation and management of trains. Domestic and foreign scholars have carried out research on energy-saving operation of trains, and the main research results have focused on the optimization and control of trains. Howlett [1] summarizes the theoretical foundations of the Metromiser system and suggests that Metromiser systems in suburban railways achieved over 13% fuel savings in normal operation. P. Howlett [2] uses the Pontryagold principle to find the nature of the optimal strategy, but the solution process is tedious. Ding Yong et al [3] discussed the energy-saving maneuvering problem under train traction characteristics and studied energy-saving maneuvering methods under fluctuating ramp conditions. Jin Weidong[4] gives a local optimization and optimization of the global optimization of the computing structure used to generate the train optimization of the speed control mode curve. Lv Jidong [5] proposed a hierarchical model of train control system including system layer, scene layer, component layer and function layer, which improved the efficiency of modeling based on the consistency of system model. Wang Zili[6] gives the optimization of the train energy-saving operation sequence and optimization strategy to optimize the train energy-saving operation possible, but also speed up the process of optimization. Xiong Chaobin[7] analyzed and established the train energy-saving manipulation optimization model, and optimized the key steps in the solution process to get a better optimization performance of the algorithm. In order to achieve better performance, [8] proposed a timeline for the optimization of integration, the simulation results show that the energy of the entire route is reduced by 14.5%. Miaojian[9] and so on to build a multi-objective model of train operation, and based on the train operation dynamics model to get the train local operation control strategy formulation method. Yu Jin[10] established a multi-objective optimization model for train operation process and studied the problem with hybrid particle swarm optimization in binary and real numbers. The optimal train control strategy was obtained. Qing[11] established the basic maneuver strategy and judgment function through the research on the train optimization control method, built the fuzzy expert system to guide train operation on-line and re-optimized train operation sequence by
using offline genetic algorithm. In practical application Achieved good results. The above algorithm has the characteristics of fast convergence and good effect of optimization. However, the model is simpler. In this paper, a new optimization control model is proposed and solved by PSO to obtain satisfactory results.

2. **Optimal control model for multi-trains in energy-saving operation**

2.1 **Train dynamics model**

Train in operation, in line with Newton’s law of kinematics. The forces can be divided into four categories: gravity G in the vertical orbit of the force component and by the orbit offset, parallel to the track direction can be classified into the total train running resistance W. Train traction is F, train braking force is B. The following is a description of these forces [12]: train traction F (kN) formula is:

\[ F = \mu F_{max} \]  

(1)

Where \( \mu \) is the actual output of the traction acceleration and the percentage of maximum acceleration, \( F_{max} \) is the maximum traction force (KN) and satisfies:

\[
F_{max} = \begin{cases} 
203 & 0 \leq v \leq 51.5 \text{km/h} \\
-0.002032v^3 + 0.4928v^2 & -42.13v + 1343 \\
51.5 < v \leq 80 \text{km/h} 
\end{cases}
\]  

(2)

The total train running resistance W (N) formula is:

\[
W = (w_0 + w_1) \times g \times M / 1000 \\
w_0 = A + Bv + Cv^2 \\
w_1 = w_c + w_i \\
w_i = i \\
w_c = c / R 
\]  

(3)

Where A, B and C are the resistance polynomial coefficients, v is the train speed, R is the radius of curvature (m), c is the empirical constant (often 600) that comprehensively reflects many factors that affect the resistance of the curve, M is the mass of the train, g is the acceleration of gravity. Train braking force B (KN) formula is:

\[ B = \mu B_{max} \]  

(4)

Where, \( B_{max} \) is the maximum braking force, and satisfy:

\[
B_{max} = \begin{cases} 
166 & 0 \leq v \leq 77 \text{km/h} \\
0.1343v^2 - 25.07v + 1300 & 77 < v \leq 80 \text{km/h} 
\end{cases}
\]  

(5)

2.2 **Model establishment**

Based on Newtonian kinetic knowledge and energy consumption law can be obtained train energy consumption formula:

\[ E = \int_0^T \mu_r v(t)F(t)dt \]  

(6)

The differential equation is:
\[
\begin{align*}
\frac{ds}{dt} &= v \\
\frac{dv}{dt} &= \mu_f F(v) - \mu_g B(v) - (w_0 + w_i)g
\end{align*}
\] (7)

Meanwhile, the constraints of each parameter involved in the operation of the train are as follows:

\[
\begin{align*}
L &= \int_0^T v(t)dt \\
v(0) &= v(T) = 0 \\
0 &\leq \mu_f \leq 1 \\
0 &\leq \mu_g \leq 1 \\
0 &\leq v \leq v_{max}
\end{align*}
\] (8)

In summary, the establishment of multi-train energy-saving control optimization model is as follows:

\[
\min \ E_i = E + E_{used}
\]

\[
\sum_{k=1}^{99} h_k = T_0
\]

\[
H_{min} \leq h_k \leq H_{max}, k = 1, 2, ..., 99
\]

\[
s.t. \quad V_{limit} = \min(V_{line} \sqrt{2LB_v})
\]

\[
D_{min} \leq D_i \leq D_{max}, i = 2, 3, ..., 13
\]

\[
T_1 = T + D = T + \sum_{i=2}^{13} D_i
\]

Where \( E \) represents the energy consumption caused by the four forces when the train is running, and in this question, it is considered that the energy consumption of each train is the same, which is expressed as \( E = \sum_{j=1}^{100} E_j = 100E_j \), \( E_{used} \) represents the regenerative energy utilized by the multi-train train during braking, while only 95% of the regenerative energy generated during operation can be utilized, expressed as:

\[
\begin{align*}
E_{used} &= E_{reg} \cdot \frac{t_{overlap}}{t_{brake}} \\
E_{reg} &= (E_{mech} - E_f) \cdot 95%
\end{align*}
\] (10)

Among them, Emeech said the braking process changes in the amount of mechanical energy, mechanical energy here for the train, the main consideration of train energy and train gravity potential. For time to overlap, which is mainly affected by train braking and train acceleration time, which is affected by the train departure time \( H \) of each train, which satisfies:

\[
\sum_{k=1}^{99} h_k = T_0
\]

\[
H_{min} \leq h_k \leq H_{max}, k = 1, 2, ..., 99
\]

Trains in tracking operation, in order to ensure safety, in the original maximum operating speed limit, for tracking the train (after the car) also has the speed limit to prevent rear-end collision occurred, the specific formula is expressed as:

\[
V_{limit} = \min(V_{line} \sqrt{2LB_v})
\] (12)
When establishing an optimization model to find the interval $H$ that minimizes the total energy consumption of all trains, we find that there are totally 99 kinds of $H$, ie, $h_1, h_2, ..., h_{99}$, and the interval $H$ has a certain limit. When the second is the research Hypothesis is $H_{max} - H_{min} = n$, then $H$ has a total of $N = \sum_{i=1}^{99} n_i$ kinds of combinations, $n_i$ said the number of $h_i$ in seconds for the unit of study.

Interval time $H$ mainly affects the utilization of regenerative energy. After the first combination of 99 $h_i$ is determined, there are also 99 cases of corresponding utilized $E_{used}$, and then the combination is repeated to determine the minimum energy consumption interval $H$.

3. Algorithm

Model (9) is an optimal control problem, subject to many restrictions. It is difficult to solve the problem using the Pontryaginian maximum principle. Therefore, the model (9) is discretized and solved by evolutionary algorithm. In this paper, particle swarm optimization algorithm [13] to solve the model. Particle swarm optimization algorithm is an evolutionary computation technique. From the behavior of bird foraging behavior. Particle swarm optimization algorithm's basic idea is to find the optimal solution through the collaboration and information sharing among individuals in the group.

Algorithm flow is as follows:
1) Initialize a population of particles (population size $N$), including random position and velocity;
2) evaluate the fitness of each particle;
3) For each particle, compare its fitness value with its best position $p_{best}$, and if it is better, use it as the current best position $p_{best}$;
4) For each particle, compare its fitness value with its best position $g_{best}$, if it is better, then use it as the current best position $g_{best}$;
5) According to the formula $x_i = x_i + v_i$
   $$v_i = \omega \times v_i + c_1 \times \text{rand()} \times (p_{best} - x_i) + c_2 \times \text{rand()} \times (g_{best} - x_i)$$

Adjust particle velocity and position;
6) did not reach the end of the condition then turn to step 2).

The iterative termination condition is generally chosen according to the specific problem as the maximum number of iterations $G_k$ or (and) the optimal location searched for by the population of particles to meet the predefined minimum adaptation threshold.

4. Case study

Taking the third question of Question D in National Mathematical Modeling Contest 2015 as an example [12], it is assumed that 100 trains depart from station A1 with interval $H = \{h_1, ..., h_{99}\}$, track and run through A2, A3, ... Reach the A14 station, stop in the middle of each station at least $D_{min}$ seconds, up to $D_{max}$ seconds. The variation range of each component of the interval $H$ is $H_{min}$ seconds to $H_{max}$ seconds. Please build an optimization model and look for the interval $H$ that minimizes the total energy consumed by all trains. It is required that the interval between the departure time of the first train and the departure time of the last train be $T_0 = 63900$ seconds, and that the total running time from A1 station to A14 station is unchanged, both of which are 2086s (including the stopping time). "Train parameters .xlsx" and "Line parameters .xlsx", see attached. After the current train runs between 14 stations, it is considered for safety. The latter column has speed limits. Take a train as an example, its speed limit is expressed as follows:
There is speed limit between trains in the stations. At the same time, when trains run between stations in the trains, there are speed restrictions for safety reasons. The running speed of the trains should meet the requirements of two speed limits. The concrete manifestation is the following figure:

Fig.1 A rear train speed limit diagram

After determining the basic energy consumption of the train, in order to obtain the regenerative energy due to the departure interval, the PSO Particle Swarm Optimization (PSO) algorithm is used to simulate the distribution of time intervals. The fitness curve is as follows:

Fig.2 Multi-train speed time limit chart

Finally get a schematic diagram of the interval time H, as shown below:

Fig.3 Evolutionary algebra and fitness diagram
Starting from the total energy consumption of single train, the basic energy consumption of a single train is $5.6530 \times 10^5$ kJ and the regenerative energy is $3.3108 \times 10^5$ kJ when the departure intervals of all trains are the same, and then considering the actual departure time interval, different time interval affects the regeneration energy through the formula corresponding to toverlap. Under this optimization result, the minimum basic energy consumption of 100 trains is $5.6530 \times 10^7$ kJ, and the total renewable energy is $4.6285 \times 10^6$ kJ.

5. Conclusion

In this paper, multi-train optimization algorithm for energy-saving train conducted in-depth study, the construction of a multi-train energy control optimization model. Using PSO Particle Swarm Optimization algorithm to control the train global operation control strategy. By PSO Particle Swarm Optimization, you get a set of time intervals that are as energy-efficient as possible. The results show that the optimization method used in this paper can get a satisfactory control strategy.

References