# Model for Evaluating the Processing Technology Quality of Platycodon Cut Crude Drug with Triangular Fuzzy Information

Mei Wang, Jinglin Li, Yuanyuan Liu, Yajun Shi

College of Pharmacy, Shaanxi University of Chinese Medicine, Xianyang, Shaanxi, 712046, China

\*Corresponding author, E-mail: 282850387@qq.com

# Abstract

In this paper, we investigate the multiple attribute decision making problems for evaluating the processing technology quality of platycodon cut crude drug with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating the processing technology quality of platycodon cut crude drug in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the processing technology quality of platycodon cut crude drug is given to verify the developed approach and to demonstrate its practicality and effectiveness.

### **Keywords**

# Multiple Attribute Decision-Making (MADM), Triangular Fuzzy Information, Grey Relational Analysis (GRA), Processing Technology Quality, Platycodon Cut Crude Drug.

# 1. Introduction

Multiple attribute decision making (MADM) problems are to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In the recent years, MADM has received a great deal of attention from researchers in many disciplines [8-11]. Grey system theory is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. In grey system theory, according to the degree of information, if the system information is fully known, the system is called a white system; if the information is unknown, it is called a black system. A system with information known partially is called a grey system. The grey system theory includes five major parts: grey prediction, grey relational analysis (GRA), grey decision, grey programming and grey control. GRA is part of grey system theory, which is suitable for solving problems with complicated interrelationships between multiple factors and variables. So, GRA method has been widely used to solve the uncertainty problems under the discrete data and incomplete information [12-17]. In addition, GRA method is one of the very popular methods to analyze various relationships among the discrete data sets and make decisions in multiple attribute situations. The major advantages of the GRA method are that the results are based on the original data, the calculations are simple and straightforward, and, finally, it is one of the best methods to make decisions under business environment.

The problem of evaluating the processing technology quality of platycodon cut crude drug with triangular fuzzy information is the multiple attribute decision making (MADM) problems [1-17]. The aim of this paper is to investigate the MADM problems for the evaluating the processing technology quality of platycodon cut crude drug with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating the processing technology quality of platycodon cut crude drug in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both

the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the processing technology quality of platycodon cut crude drug is given to verify the developed approach. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to triangular fuzzy sets. In Section 3 we extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the processing technology quality of platycodon cut crude drug in triangular fuzzy setting. In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.

### 2. Preliminaries

In this section, we briefly describe some basic concepts and basic operational laws related to triangular fuzzy numbers.

Definition 1[18]. A triangular fuzzy numbers  $\tilde{a}$  can be defined by a triplet  $(a^L, a^M, a^U)$ . The membership function  $\mu_{\tilde{a}}(x)$  is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a^{L}, \\ \frac{x - a^{L}}{a^{M} - a^{L}}, & a^{L} \le x \le a^{M}, \\ \frac{x - a^{U}}{a^{M} - a^{U}}, & a^{M} \le x \le a^{U}, \\ 0, & x \ge a^{U}. \end{cases}$$
(1)

where  $0 < a^{L} \le a^{M} \le a^{U}$ ,  $a^{L}$  and  $a^{U}$  stand for the lower and upper values of the support of  $\tilde{a}$ , respectively, and  $a^{M}$  for the modal value.

Definition 2[18]. Basic operational laws related to triangular fuzzy numbers:

$$\begin{split} \tilde{a} \oplus \tilde{b} &= \left[ a^{L}, a^{M}, a^{U} \right] \oplus \left[ b^{L}, b^{M}, b^{U} \right] = \left[ a^{L} + b^{L}, a^{M} + b^{M}, a^{U} + b^{U} \right] \\ \tilde{a} \otimes \tilde{b} &= \left[ a^{L}, a^{M}, a^{U} \right] \otimes \left[ b^{L}, b^{M}, b^{U} \right] = \left[ a^{L} b^{L}, a^{M} b^{M}, a^{U} b^{U} \right] \\ \lambda \otimes \tilde{a} &= \lambda \otimes \left[ a^{L}, a^{M}, a^{U} \right] = \left[ \lambda a^{L}, \lambda a^{M}, \lambda a^{U} \right], \ \lambda > 0 \,. \end{split}$$

Definition 3[18]. A fuzzy set  $\tilde{A}$  of the universe of discourse X is convex if and only if for all  $x_1, x_2$  in X,

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda) x_2) \ge Min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \lambda \in [0, 1]$$

$$\tag{2}$$

Definition 4 [18]. Let  $\tilde{a} = [a^L, a^M, a^U]$  and  $\tilde{b} = [b^L, b^M, b^U]$  be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$\|\tilde{a} - \tilde{b}\| = \sqrt{\frac{1}{3} \left[ \left( a^{L} - b^{L} \right)^{2} + \left( a^{M} - b^{M} \right)^{2} + \left( a^{U} - b^{U} \right)^{2} \right]}$$
(3)

# **3.** An Approach to for Evaluating the Processing Technology Quality of Platycodon Cut Crude Drug with Triangular Fuzzy Information

The problem of evaluating the processing technology quality of platycodon cut crude drug with triangular fuzzy information is the multiple attribute decision making (MADM) problems. In this section, consider a multiple attribute decision making problems to evaluate the processing technology quality of platycodon cut crude drug with triangular fuzzy information: Let  $A = \{A_1, A_2, \dots, A_m\}$  be a

discrete set of alternatives. Let  $G = \{G_1, G_2, \dots, G_n\}$  be a set of attributes. The information about attribute weights is completely known. Let  $\omega = (\omega_1, \omega_2, \dots, \omega_n)$  be the weight vector of attributes, where  $\omega_j \ge 0$ ,  $j = 1, 2, \dots, n$ . Suppose that  $A = (\tilde{a}_{ij})_{m \times n} = [a_{ij}^L, a_{ij}^M, a_{ij}^U]_{m \times n}$  is the decision making matrix, where  $\tilde{a}_{ij}$  is a preference value, which take the form of triangular fuzzy numbers, given by the decision maker, for the alternative  $A_i \in A$  with respect to the attribute  $G_j \in G$ .

In the following, we shall extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the processing technology quality of platycodon cut crude drug in triangular fuzzy setting. The method involves the following steps:

Step 1. Normalize each attribute value  $\tilde{a}_{ij}^{(k)}$  in the matrix *A* into a corresponding element in the matrix  $\tilde{R} = (\tilde{r}_{ij})_{m \times n} (\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^M, r_{ij}^U])$  using the following formulas:

$$\begin{cases} r_{ij}^{L} = a_{ij}^{L} / \sum_{i=1}^{m} a_{ij}^{U} \\ r_{ij}^{M} = a_{ij}^{M} / \sum_{i=1}^{m} a_{ij}^{M} , & \text{for benefit attribute } G_{j}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, k = 1, 2, \cdots, t . \ (4) \\ r_{ij}^{U} = a_{ij}^{U} / \sum_{i=1}^{m} a_{ij}^{L} \\ r_{ij}^{L} = \left(\frac{1}{r_{ij}^{H}}\right) / \left(\sum_{i=1}^{m} \frac{1}{r_{ij}^{H}}\right) \\ r_{ij}^{M} = \left(\frac{1}{r_{ij}^{H}}\right) / \left(\sum_{i=1}^{m} \frac{1}{a_{ij}^{H}}\right), \text{ for benefit attribute } G_{j}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, k = 1, 2, \cdots, t . \ (5) \\ r_{ij}^{U} = \left(\frac{1}{r_{ij}^{L}}\right) / \left(\sum_{i=1}^{m} \frac{1}{a_{ij}^{H}}\right) \end{cases}$$

Step 2. Defining the triangular fuzzy positive-ideal solution (TFPIS,Y+) and triangular fuzzy negative-ideal solution (TFNIS,Y-) as

$$\tilde{R}^+ = \left[\tilde{r}_1^+, \tilde{r}_2^+, \cdots, \tilde{r}_n^+\right], \tilde{R}^- = \left[\tilde{r}_1^-, \tilde{r}_2^-, \cdots, \tilde{r}_n^-\right]$$

where

$$\tilde{r}_{j}^{+} = (\max_{i} r_{ij}^{L}, \max_{i} r_{ij}^{M}, \max_{i} r_{ij}^{U}), \tilde{r}_{j}^{-} = (\min_{i} r_{ij}^{L}, \min_{i} r_{ij}^{M}, \min_{i} r_{ij}^{U}).$$

Step 3. Calculating the fuzzy grey relational coefficient of each alternative from TFPIS and TFNIS using the following equation, respectively:

$$\xi_{ij}^{+} = \frac{\min_{1 \le i \le m} \min_{1 \le j \le n} \left\| \tilde{r}_{j}^{+} - \tilde{r}_{ij} \right\| + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} \left\| \tilde{r}_{j}^{+} - \tilde{r}_{ij} \right\|}{\left\| \tilde{r}_{j}^{+} - \tilde{r}_{ij} \right\| + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} \left\| \tilde{r}_{j}^{+} - \tilde{r}_{ij} \right\|}$$
(6)

$$\xi_{ij}^{-} = \frac{\min_{1 \le i \le m} \min_{1 \le j \le n} \left\| \tilde{r}_{ij} - \tilde{r}_{j}^{-} \right\| + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} \left\| \tilde{r}_{ij} - \tilde{r}_{j}^{-} \right\|}{\left\| \tilde{r}_{ij} - \tilde{r}_{j}^{-} \right\| + \rho \max_{1 \le i \le m} \max_{1 \le j \le n} \left\| \tilde{r}_{ij} - \tilde{r}_{j}^{-} \right\|}$$
(7)

where the identification coefficient  $\rho = 0.5$ .

Step 4. Calculating the degree of fuzzy grey relational coefficient of each alternative from TFPIS and TFNIS using the following equation, respectively:

$$\xi_i^+ = \sum_{j=1}^n w_j \xi_{ij}^+, \xi_i^- = \sum_{j=1}^n w_j \xi_{ij}^-, i = 1, 2, \cdots, m.$$
(8)

Step 5. Calculating the fuzzy relative relational degree of each alternative from TFPIS using the following equation,

$$\xi_{i} = \xi_{i}^{+} / (\xi_{i}^{-} + \xi_{i}^{+}), i = 1, 2, \cdots, m.$$
(9)

Step 6. According to the fuzzy relative relational degree, the ranking order of all alternatives can be determined. If any alternative has the highest  $\xi_i$  value, then, it is the most important alternative.

### 4. Numerical example

In this section, we present an empirical case study of evaluating the processing technology quality of platycodon cut crude drug. The project's aim is to evaluate the best processing technology quality of platycodon cut crude drug from the different processing technologies. There are five possible processing technologies of platycodon cut crude drug  $A_i$  (i = 1, 2, 3, 4, 5) is evaluated according to the following four attributes: (1)G1 is the production process; (2)G2 is the phamaceutical value; (3) G3 is the market value; (4)G4 is the market developing prospect. The five possible processing technologies of platycodon cut crude drug  $A_i$  ( $i = 1, 2, \dots, 5$ ) are to be evaluated by using triangular fuzzy numbers by the the decision makers under the above four attributes (whose weighting vector  $w = (0.20, 0.15, 0.25, 0.40)^T$ ), and construct, respectively, the decision matrix as follows  $\tilde{R} = (\tilde{r}_{ij})_{s_{id}}$ :

$$\begin{array}{cccc} G1 & G_2 & G_3 & G_4 \\ \\ A_I \begin{bmatrix} 0.65, 0.70, 0.75 \end{bmatrix} \begin{bmatrix} 0.57, 0.61, 0.65 \end{bmatrix} \begin{bmatrix} 0.76, 0.78, 0.81 \end{bmatrix} \begin{bmatrix} 0.47, 0.50, 0.53 \end{bmatrix} \\ A_2 \begin{bmatrix} 0.70, 0.75, 0.78 \end{bmatrix} \begin{bmatrix} 0.52, 0.62, 0.68 \end{bmatrix} \begin{bmatrix} 0.45, 0.52, 0.50 \end{bmatrix} \begin{bmatrix} 0.54, 0.57, 0.60 \end{bmatrix} \\ \tilde{A} = A_3 \begin{bmatrix} 0.80, 0.82, 0.83 \end{bmatrix} \begin{bmatrix} 0.75, 0.78, 0.80 \end{bmatrix} \begin{bmatrix} 0.62, 0.64, 0.67 \end{bmatrix} \begin{bmatrix} 0.78, 0.80, 0.81 \end{bmatrix} \\ A_4 \begin{bmatrix} 0.67, 0.70, 0.73 \end{bmatrix} \begin{bmatrix} 0.82, 0.83, 0.85 \end{bmatrix} \begin{bmatrix} 0.83, 0.84, 0.85 \end{bmatrix} \begin{bmatrix} 0.82, 0.84, 0.85 \end{bmatrix} \\ A_5 \begin{bmatrix} 0.63, 0.64, 0.66 \end{bmatrix} \begin{bmatrix} 0.63, 0.64, 0.60 \end{bmatrix} \begin{bmatrix} 0.68, 0.70, 0.76 \end{bmatrix} \begin{bmatrix} 0.79, 0.82, 0.80 \end{bmatrix}$$

In the following, we shall extend the grey relational analysis (GRA) procedure to solve the triangular fuzzy multiple attribute decision making for evaluating the processing technology quality of platycodon cut crude drug in triangular fuzzy setting. To get the most desirable possible processing technology of platycodon cut crude drug, the following steps are involved:

Step 1. Calculate the normalized decision matrix  $\tilde{R}$ :

$$\tilde{\mathbf{R}} = \begin{bmatrix} 0.1733, 0.1939, 0.2174 \end{bmatrix} \begin{bmatrix} 0.1592, 0.1753, 0.1976 \end{bmatrix} \begin{bmatrix} 0.2117, 0.2241, 0.2425 \end{bmatrix} \begin{bmatrix} 0.1309, 0.1416, 0.1559 \end{bmatrix} \\ \begin{bmatrix} 0.1867, 0.2078, 0.2261 \end{bmatrix} \begin{bmatrix} 0.1453, 0.1782, 0.2067 \end{bmatrix} \begin{bmatrix} 0.1253, 0.1494, 0.1497 \end{bmatrix} \begin{bmatrix} 0.1504, 0.1615, 0.1765 \end{bmatrix} \\ \begin{bmatrix} 0.2133, 0.2271, 0.2406 \end{bmatrix} \begin{bmatrix} 0.2095, 0.2241, 0.2432 \end{bmatrix} \begin{bmatrix} 0.1727, 0.1839, 0.2006 \end{bmatrix} \begin{bmatrix} 0.2173, 0.2266, 0.2382 \end{bmatrix} \\ \begin{bmatrix} 0.1787, 0.1939, 0.2116 \end{bmatrix} \begin{bmatrix} 0.2291, 0.2385, 0.2584 \end{bmatrix} \begin{bmatrix} 0.2312, 0.2414, 0.2545 \end{bmatrix} \begin{bmatrix} 0.2284, 0.2380, 0.2500 \end{bmatrix} \\ \begin{bmatrix} 0.1680, 0.1773, 0.1913 \end{bmatrix} \begin{bmatrix} 0.1760, 0.1839, 0.1824 \end{bmatrix} \begin{bmatrix} 0.1894, 0.2011, 0.2275 \end{bmatrix} \begin{bmatrix} 0.2201, 0.2323, 0.2353 \end{bmatrix} \end{bmatrix}$$

Step 2. Determining TFPIS and TFNIS as:

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\tilde{R}^{+} = \begin{bmatrix} [0.2133, 0.2271, 0.2406] & [0.2291, 0.2385, 0.2584] & [0.2312, 0.2414, 0.2545] & [0.2284, 0.2380, 0.2500] \end{bmatrix}
\tilde{R}^{-} = \begin{bmatrix} [0.1680, 0.1773, 0.1913] & [0.1453, 0.1753, 0.1824] & [0.1253, 0.1494, 0.1497] & [0.1309, 0.1416, 0.1559] \end{bmatrix}
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Step 3. Calculating the fuzzy grey relational coefficient of each processing technology of platycodon cut crude drug from TFPIS and TFNIS

$$\xi^{+} = \left(\xi_{ij}^{+}\right)_{5\times4} = \begin{bmatrix} 0.6058 & 0.4384 & 0.7534 & 0.3449 \\ 0.7085 & 0.4312 & 0.3333 & 0.3993 \\ 1.0000 & 0.7535 & 0.4714 & 0.8157 \\ 0.6094 & 1.0000 & 1.0000 & 1.0000 \\ 0.5118 & 0.4486 & 0.5778 & 0.8307 \\ 0.5118 & 0.4486 & 0.5778 & 0.8307 \\ \end{bmatrix}$$
$$\xi^{-} = \left(\xi_{ij}^{-}\right)_{5\times4} = \begin{bmatrix} 0.7360 & 0.8092 & 0.3730 & 1.0000 \\ 0.6370 & 0.7814 & 1.0000 & 0.7167 \\ 0.5118 & 0.4642 & 0.5300 & 0.3740 \\ 0.7556 & 0.4031 & 0.3333 & 0.3449 \\ 1.0000 & 0.7328 & 0.4358 & 0.3686 \\ \end{bmatrix}$$

Step 4. Calculating the degree of fuzzy grey relational coefficient of each processing technology of platycodon cut crude drug from TFPIS and TFNIS

$$\xi_1^+ = 0.5132, \xi_2^+ = 0.4494, \xi_3^+ = 0.7572, \xi_4^+ = 0.9219, \xi_5^+ = 0.6464$$
  
$$\xi_1^- = 0.7618, \xi_2^- = 0.7813, \xi_3^- = 0.4541, \xi_4^- = 0.4329, \xi_5^- = 0.5663.$$

Step 5. Calculating the fuzzy relative relational degree of each processing technology of platycodon cut crude drug from TFPIS

$$\xi_1 = 0.4025, \xi_2 = 0.3652, \xi_3 = 0.6251, \xi_4 = 0.6805, C_5 = 0.5330.$$

Step 6. According to the fuzzy relative relational degree, the ranking order of the five possible processing technologies of platycodon cut crude drug is:  $A_4 > A_3 > A_5 > A_1 > A_2$ . Thus, the most desirable possible processing technology of platycodon cut crude drug is  $A_4$ .

#### 5. Conclusion

In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the processing technology quality of platycodon cut crude drug with triangular fuzzy information. Then, we extend the grey relational analysis (GRA) procedure for triangular fuzzy multiple attribute decision making for evaluating marine service industry in triangular fuzzy setting. According to the concept of the GRA, a fuzzy relative relational degree is defined to determine the ranking order of all alternatives by calculating the degree of fuzzy grey relational coefficient to both the triangular fuzzy positive-ideal solution (TFPIS) and triangular fuzzy negative-ideal solution (TFNIS) simultaneously. Finally, an illustrative example for evaluating the processing technology quality of platycodon cut crude drug is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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