Research on Financial Risk Evaluation with Hesitant Fuzzy Information

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Abstract

The aim of this paper is to investigate the multiple attribute decision making problems for financial risk evaluation with hesitant fuzzy information. Based on the basic ideal of traditional TOPSIS, calculation steps for solving hesitant fuzzy multiple attribute decision-making problems with completely known weight information are given. The weighted Hamming distances between every alternative and positive ideal solution and negative ideal solution are calculated. Then, according to the weighted Hamming distances, the relative closeness degree to the positive ideal solution is calculated to rank all alternatives. Finally, a practical example for financial risk evaluation is used to illustrate the developed procedures.

Keywords

Multiple attribute decision-making, TOPSIS, Hesitant fuzzy numbers, weight information, financial risk evaluation.

1. Introduction

Since China's reform and opening up, China's economic development speed rapidly grow up, the market economic system gradually improve. Small and medium-sized enterprise develop rapidly in such an environment, added vigor into the market, has become the important component of the socialist market economy. According to the latest statistics, there are over10.23million registered small and medium-sized enterprises in our country, the industrial production of them is even more than a large enterprise in the creation, have played a vital role the rapid development of the socialist market economy. However, small and medium-sized enterprise assets, scale, enterprise management capacity, operation stability, still can't compared to the large listed companies, small and medium-sized enterprises also face more financial risk than the big enterprise and easier to get hurt from the dangers of financial risk.There are some characteristics of defects in operations of small and medium-sized enterprises,, for example, lack of funding, small scale, low market share, enterprise management disorder, weak sustainable management ability, weak risk awareness, etc., which will affect the health of the enterprise long-term development. To small and medium-sized enterprises, financial risk get the biggest influence among all the risk faced by enterprises. Faced with the same financial risk, small and medium-sized enterprises reaction appears more pale compared to large enterprises stability and sustainability. Therefore, small and medium-sized enterprise only can do homework on the financial risk assessment to make up for the inadequacy of their own, avoid financial risks to bring the huge losses. Find a reasonable financial risk evaluation method for the small and medium-sized enterprises enables them to adept to the risk management is an important topic of the small and medium-sized enterprise financial risk research.

Multiple attribute decision making (MADM) problems are wide spread in real life decision situations. A MADM problem is to find a best compromise solution from all feasible alternatives assessed on multiple attributes, both quantitative and qualitative[1-5]. Atanassov [6-7] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set [8]. The intuitionistic fuzzy set has received more and more attention since its appearance[9-17]. Furthermore, Torra [18] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu[19] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. They developed some hesitant fuzzy operational rules based on the interconnection between the hesitant fuzzy set and the intuitionsitic fuzzy set. In order to aggregate the hesitant fuzzy information, they proposed a series of operators under various situations and discussed the relationships among them. Moreover, they applied the developed aggregation operators to solve the decision making problems with anonymity. Xia and Xu[20] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. Gu et al.[21] utilized the hesitant fuzzy weighted averaging (HFWA) operator to investigate the evaluation model for risk investment with hesitant fuzzy information. Wei et al.[22] proposed two hesitant fuzzy Choquet integral aggregation operators: hesitant fuzzy choquet ordered averaging (HFCOA) operator and hesitant fuzzy choquet ordered geometric (HFCOG) operator. Wei[23] developed some prioritized aggregation operators for aggregating hesitant fuzzy information, and then apply them to develop some models for hesitant fuzzy multiple attribute decision making (MADM) problems in which the attributes are in different priority level. Zhu et al.[24] explored the geometric Bonferroni mean (GBM) considering both the BM and the geometric mean (GM) under hesitant fuzzy environment. Xu and Xia[25] proposed a variety of distance measures for hesitant fuzzy sets, based on which the corresponding similarity measures can be obtained. Xu and Xia[26] defined the distance and correlation measures for hesitant fuzzy information and then discuss their properties in detail. Wang et al.[27] proposed the generalized hesitant fuzzy hybrid weighted distance (GHFHWD) measure, which is based on the generalized hesitant fuzzy weighted distance (GHFWD) measure The GHFHWD measure generalizes both the GHFWD and GHFOWD measures, and reflects the importance degrees of both the given arguments and their ordered positions. Wei et al.[28] presented hesitant triangular fuzzy set based on hesitant fuzzy set and investigated the multiple attribute decision making (MADM) problems in which attribute values take the form of hesitant triangular fuzzy information, then they developed some hesitant triangular fuzzy aggregation operators: the hesitant triangular fuzzy weighted averaging (HTFWA) operator, hesitant triangular fuzzy weighted geometric (HTFWG) operator, hesitant triangular fuzzy ordered weighted averaging (HTFOWA) operator, hesitant triangular fuzzy ordered weighted geometric (HTFOWG) operator, hesitant triangular fuzzy hybrid average (HTFHA) operator and hesitant triangular fuzzy hybrid geometric (HTFHG) operator. They have also applied the hesitant triangular fuzzy weighted averaging (HTFWA) operator and hesitant triangular fuzzy weighted geometric (HTFWG) operators to multiple attribute decision making with hesitant triangular fuzzy information.

The problem of financial risk evaluation with hesitant fuzzy information is the multiple attribute decision making problems. The aim of this paper is to extended the concept of TOPSIS[29-33] to develop a methodology for solving MADM problems for Financial risk evaluation with hesitant fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of hesitant fuzzy numbers.

The rest of the paper is organized as follows: next section briefly introduce some basic concepts related to intuitionistic fuzzy sets and hesitant fuzzy sets. In section 3, we introduce the classical TOPSIS method. In Section 4, we extend the TOPSIS method for hesitant fuzzy multiple attribute decision making. In Section 5, we illustrate our proposed algorithmic method with an example for financial risk evaluation. The final section concludes.

2. Preliminaries

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets.

Definition 1. Let X be an universe of discourse, then a fuzzy set is defined as:

$$A=\left\{\left\langle x,\mu_A\left(x\right)\right\rangle \middle| x\in X\right\} \tag{1}$$

Which is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A[8].

Atanassov[6-7] extended the fuzzy set to the IFS, shown as follows:

Definition 2. An IFS A in X is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

Where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1,\ \forall\, x \in X$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A[6-7].

Definition 3. For each IFS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),\ \forall\, x \in X. \tag{3}$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A[6-7].

However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra[18] proposed another generation of FS.

Definition 4[18]. Given a fixed set X, then a hesitant fuzzy set(HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$.

To be easily understood, Xu express the HFS by mathematical symbol:

$$E = (\langle x, h_E(x)\rangle | x \in X) \tag{4}$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xu call $h = h_E(x)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Given three HFEs reprented by h, h_1 and h_2, Torra[18] defined some operations in them, which can be described as follows:

$h^c = \bigcup_{\gamma \in h} \{1-\gamma\}$;

$h_1 \cup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$;

$h_1 \cap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$.

In the following, Torra [18] showed that the envelop of a HFE is an IFV.

Definition 5. Given a HFE h, we define the IFV $A_{env}(h)$ as the envelope of h, where $A_{env}(h)$ can be defined as $(h^-, 1-h^+)$, with $h^- = \min\{\gamma | \gamma \in h\}$ and $h^+ = \max\{\gamma | \gamma \in h\}$.

Then, Torra [18] gave the further study of the relationship between HFEs and IFVs:

$A_{env}(h^c) = (A_{env}(h))^c$;

$A_{env}(h_1 \cup h_2) = A_{env}(h_1) \cup A_{env}(h_2)$;

(3) $A_{env}(h_1 \cap h_2) = A_{env}(h_1) \cap A_{env}(h_2)$.

Drawing on the well-known Hamming, Xu and Xia[25] define a hesitant weighted Hamming distance.

Definition 6. Let M and N be two HFSs on $X=\{x_1,x_2,\cdots,x_n\}$, then the distance measure between M and N is defined as

$$d(M,N)=\sum_{j=1}^{n} w_j\left[\frac{1}{l_{x_j}}\sum_{k=1}^{l_{xj}}\left|h_M^{\sigma(k)}(x_j)-h_N^{\sigma(k)}(x_j)\right|\right] \tag{5}$$

where $h_M^{\sigma(k)}(x_j)$ and $h_N^{\sigma(k)}(x_j)$ are kth largest values in $h_M(x_j)$ and $h_N(x_j)$, respectively. which will be used thereafter.

3. Classical TOPSIS method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang [29], with reference to Hwang and Yoon [30]. TOPSIS is a multiple criteria method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The procedure of TOPSIS can be expressed in a series of steps:

(1) Calculate the normalized decision matrix. The normalized value b_{ij} is calculated as

$$b_{ij}=a_{ij}\Big/\sqrt{\sum_{j=1}^{n}a_{ij}^2}\ ,\quad i=1,2,\cdots,m\ ,\quad j=1,2,\cdots,n. \tag{6}$$

(2) Calculate the weighted normalized decision matrix. The weighted normalized value c_{ij} is calculated as

$$c_{ij}=w_j b_{ij}\, ,i=1,2,\cdots,m\ ,\quad j=1,2,\cdots,n \tag{7}$$

where w_j is the weight of the j th criterion G_j, and $\sum_{j=1}^{n}w_j=1$.

(3) Determine the positive ideal and negative ideal solution.

$$\begin{aligned}C^{+}&=\{c_1^{+},\cdots,c_n^{+}\}\\&=\left\{\left(\max_i c_{ij}\,|\,j\in I\right),\left(\min_i c_{ij}\,|\,j\in J\right)\right\}\end{aligned}, \tag{8}$$

$$\begin{aligned}C^{-}&=\{c_1^{-},\cdots,c_n^{-}\}\\&=\left\{\left(\min_i c_{ij}\,|\,j\in I\right),\left(\max_i c_{ij}\,|\,j\in J\right)\right\}\end{aligned}, \tag{9}$$

where I is associated with benefit criteria, and J is associated with cost criteria.

(4) Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$$d_i^{+}=\sqrt{\sum_{j=1}^{n}\left(c_{ij}-c_j^{+}\right)^2},\quad i=1,\cdots,m. \tag{10}$$

Similarly, the separation from the negative ideal solution is given as

$$d_i^{-}=\sqrt{\sum_{j=1}^{n}\left(c_{ij}-c_j^{-}\right)^2},\quad i=1,\cdots,m. \tag{11}$$

(5) Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^{+} is defined as

$$R_i = d_i^- \big/ \left(d_i^- + d_i^+\right), \quad i = 1, \cdots, m. \tag{12}$$

Since $d_i^- \geq 0$ and $d_i^+ \geq 0$, then, clearly, $R_i \in [0,1]$.

(6) Rank the preference order. For ranking DMUs using this index, we can rank DMUs in decreasing order.

The basic principle of the TOPSIS method is that the chosen alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal solution. The TOPSIS method introduces two "reference" points, but it does not consider the relative importance of the distances from these points.

4. TOPSIS method for financial risk evaluation with hesitant fuzzy multiple attribute decision making

In this section, we apply the TOPSIS method to multiple attribute decision making for financial risk evaluation. Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be the set of attributes, $w = (w_1, w_2, \cdots, w_n)$ is the weighting vector of the attribute $G_j\ (j = 1, 2, \cdots, n)$, where $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. If the decision makers provide several values for the alternative A_i under the attribute G_j with anonymity, these values can be considered as a hesitant fuzzy element h_{ij}. In the case where two decision makers provide the same value, then the value emerges only once in h_{ij}. Suppose that the decision matrix $H = \left(h_{ij}\right)_{m \times n}$ is the hesitant fuzzy decision matrix, where $h_{ij}\ (i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n)$ are in the form of HFEs.

In the following, we apply TOPSIS method to solve hesitant fuzzy MADM for financial risk evaluation with completely known weight information.

Step 1. Determine the positive ideal and negative ideal solution based on hesitant fuzzy numbers.

$$A^+ = \{1\}\ ,\quad A^- = \{0\} \tag{13}$$

Step 2. Calculate the weighted hamming distances.

The weighted hamming distances of each alternative from the ideal solution is given as

$$d\left(A_i, A^+\right) = \sum_{j=1}^{n} w_j \left[\frac{1}{l_{G_{ij}}} \sum_{k=1}^{l_{G_{ij}}} \left|h_M^{\sigma(ik)}\left(G_j\right) - 1\right|\right], i = 1, 2, \cdots, m. \tag{14}$$

Similarly, the weighted hamming distances from the negative ideal solution is given as

$$d\left(A_i, A^-\right) = \sum_{j=1}^{n} w_j \left[\frac{1}{l_{G_{ij}}} \sum_{k=1}^{l_{G_{ij}}} \left|h_M^{\sigma(ik)}\left(G_j\right) - 0\right|\right], i = 1, 2, \cdots, m. \tag{15}$$

Step 3. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to $\tilde{r}^+$ is defined as

$$c\left(A_i, A^-\right) = d\left(A_i, A^-\right) \big/ \left(d\left(A_i, A^-\right) + d\left(A_i, A^+\right)\right), i = 1, 2, \cdots, m. \tag{16}$$

Step 4. Rank all the alternatives $A_i\ (i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with $c\left(A_i, A^-\right)(i = 1, 2, \cdots, m)$.

Step 5. End.

5. Numerical example

Since the 1980s, the international banking industry happened a series of crisis and scandals, especially since the economic crisis in 2007, the international financial industry is facing the most severe challenge since the 1930s. In less than one year, the world goes to all-round unrest from Great Moderation in which economy grows fast and inflation is moderate. Some of the original financial institutions went bankrupt and some been taken over, the government had to intervene the core financial institutions which could cause systematic risk on a large scale. This suggests further that excess liquidity and low interest environment for a long time have caused over leveraged behavior of financial institutions, lacking of the capacity to resist unexpected loss, which means not reserve enough capital to release capital slowly, so both the theory field and the practice field are aware of the importance of capital management of commercial banks and the problems in management, in 1998 the Basle Committee made "Basle Agreement" which built the frame of outside management of capital for the international banking industry. Thereafter, the western banking industry gradually and more and more treats the problem of capital management from the aspect of their own business but not from the aspect of being regulated, the position of economic capital management in commercial banking management is improved gradually, commercial banks in different countries are studying the economic capital management system which suits their own characteristic, a great deal of advanced risk measure technology, methods of distributing economic capital are applied in the risk management process of commercial banking industry, economic capital consumption has been an important index of checking the operation achievements of commercial banks, economic capital consumption has been an important factor which constrains business development. More and more advanced financial technology and the continually emerging financial innovative products as well as the deepen of marketization have made the business and its risk combination of our commercial banks more and more complicated, harder to recognize and measure, traditional method which depends on expert judgments and simple risk measurement to recognize and manage risk can not meet the severe situation faced by Chinese commercial banking industry. Even through the shock in our commercial banking industry by the economic crisis started in 2007 is slight which due to our low open level of commercial banks, small volume of derivative trade, but it cannot suggest that our management of commercial banks is better than abroad, on the contrary, there is a big gap between our management of commercial banks and abroad banks. Chinese commercial banks must go in the direction of international banking risk management, combined with the reality of our country, from the aspect of internal management of commercial banks, study how to use economic capital management to resist all-round risk, improve the level of management and capacity of resistance, enhance the capacity of coping with significant disadvantageous matters. In the following, we shall investigate the multiple attribute decision making problems for financial risk evaluation with hesitant fuzzy information. There is a panel with five possible commercial banks $A_i\left(i=1,2,3,4,5\right)$ to evaluate. The team of experts must take a decision according to the following four attributes: ①G1 is the market risk; ②G2 is the risk of enterprise operation and management; ③G3 is the risk of enterprise assets structure; ④ G4 is the environmental risk. In order to avoid influence each other, the decision makers are required to evaluate the five possible commercial banks $A_i\left(i=1,2,\cdots,5\right)$ under the above four attributes in anonymity and the decision matrix $H=\left(h_{ij}\right)_{m\times n}$ is presented in Table 1, where $h_{ij}\left(i=1,2,3,4,5,\ j=1,2,3,4\right)$ are in the form of HFEs.

Table 1.Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.2,0.4,0.5)	(0.3,0.6)	(0.2,0.4,0.6,0.7)	(0.5,0.8)
A_2	(0.3,0.7)	(0.1,0.2,0.3,0.6)	(0.6,0.8)	(0.2,0.4,0.7)
A_3	(0.2,0.3,0.7,0.9)	(0.4,0.5,0.6)	(0.2,0.6)	(0.6,0.7)
A_4	(0.5,0.6,0.7)	(0.2,0.4)	(0.8,0.9)	(0.3,0.4,0.5,0.8)
A_5	(0.2,0.3)	(0.2,0.5,0.6)	(0.7,0.8)	(0.1,0.3,0.4)

The information about the attribute weights is known as follows: $w=(0.2,0.3,0.4,0.1)$.

The calculation results and the ordering of the alternatives are shown in Table 2. As we can see, the best commercial banks is A4.

Table 2. Calculation results

	$d(A_i,A^+)$	$d(A_i,A^-)$	$c(A_i,A^+)$	order
A_1	0.4792	0.5208	0.5208	4
A_2	0.4867	0.6000	0.5521	2
A_3	0.4650	0.5350	0.5350	3
A_4	0.3950	0.6050	0.6050	1
A_5	0.5750	0.4467	0.4372	5

6. Conclusion

With financial market of our country opened completely in 2006, the commercial banks faced fierce unprecedented competition. Since the start of the non-performing loans divestiture in 1999, the quality of banks' credit assets has been improved to a certain extent basing on the improvement of their own credit risk control and the gradual enhancement of exterior supervision and control. But because of relative lag in the credit risk control technology, the banking could not keep up with the variety of increasingly complicated macro economy and credit operation environment in many respects. So it is imminent to energetically reinforce the research on bank credit risk control to our country commercial banks, especially the operation of credit risk measurement technology and the enhancement of inner control management. In this paper, we have investigated the problem of hesitant fuzzy multiple attribute decision-making problems for financial risk evaluation with completely known attribute weight information. A modified TOPSIS analysis method is proposed. Based on the basic ideal of the traditional TOPSIS, the weighted Hamming distances between every alternative and positive ideal solution and negative ideal solution are calculated. Then, according to the weighted Hamming distances, the relative closeness degree to the positive ideal solution is calculated to rank all alternatives. Finally, an illustrative example for financial risk evaluation is given. In the future, we shall continue working in the application of the hesitant fuzzy multiple attribute decision-making to other domains[34-41].

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