The Research On The Terminal Guidance Law of High-precision Missiles Based On Nonlinear Motion

Chunyang Zhao

Beihang University, Beijing 100083, China;

cyzhao1994@163.com

Abstract

Considering the nonlinear motion model of missiles, the traditional slide guidance law was studied based on the theory of variable structure. And advanced the three dimensional nonlinear slide guidance law based on this, which has solved the high accuracy demanding for interceptor missiles in their terminal guidance stage. The nonlinear dynamic model of relative motion of the missile and target was established. Then, the commonly used proportional guidance law is expounded. And three dimensional nonlinear slide guidance law was derived from the basic slide guidance law. Based on the study of new guidance law, the missile simulation model was established to analyze the strike result of the missile under different guidance laws. The result shows that the nonlinear dynamic model is consistent with the actual operational characteristics of the missile in the atmosphere. The precision guidance method has been derivate can adapt to the maneuvers of target, which improves the deficiency of the traditional proportional guidance law. It is found that the new guidance law is obviously better than Proportional guidance law by simulation and data comparison. The whole research can provide the theoretical basis for the precision guidance of missile.

Keywords

Nonlinear motion model, theory of variable structure, slide guidance law, three dimensional nonlinear slide guidance law, proportional guidance law.

1. Introduction

The tactical ballistic missiles have been applied to the actual combat for many times. Which are the main threats to the future local wars and regional conflicts.[1] As the threat of tactical ballistic missile is becoming more and more realistic, more countries around the world begin to pay attention to the research and development of the anti-missile system. They are actively studying or purchasing and deploying missile defense systems and missile defense systems are widely spreading around the world. China also has a strong demand for ballistic missile defense. Therefore the research on interceptor missiles is particularly necessary.

The precision of striking for missile basically depends on the guidance subsystem. As the theory basic of guidance, guidance law has important influence for the hit probability and its accuracy in the process against target[2]. The guidance laws built on early concepts are often called classical guidance laws. They mainly include pure tracking method, normal value front Angle tracking method, parallel approach and proportional guidance method. The modern guidance laws are based on modern control theory and countermeasure theory. They mainly include the optimal guidance law, differential game guidance law, a variable structure guidance law, fuzzy guidance law, and the neural network guidance law, etc.[3]

The essence of proportional guidance law[4][5][6][7] is controlling the rotation of the line of sight. Although proportional guidance law and its modified forms are widely used, a large number of simulation results show that they can make the missile miss the target when the target is confrontational or maneuvering. Therefore it is necessary to further study the higher precision and practical guidance law, so as to adapt to the needs of the development.

At present, one of the good guidance law is slide guidance law [8] which is based on the theory of variable structure control. Variable structure control represents the slide variable structure control, the fundamental difference of the control strategy between which and regular control is the discontinuity control. Which means that "structure" can change at any time so that force the system to make small amplitude and high frequency vibration along the specified state trajectory. Therefore slide mode can be designed and which has no relation with the parameters and the disturbance of the system[9][10][11]. In this way, the control system with slide mode has good robustness.

In this paper, based on the three dimensional nonlinear motion model, improve the traditional slide guidance law and put forward three dimensional nonlinear slide guidance law. Which has the higher precision and is more suitable for accuracy demand in terminal guidance. This paper analyzes the characteristics of the new guidance law by comparing with the traditional proportional guidance law. Simulate the missile combat with proportional guidance law and new guidance law as guidance laws, and verify the superiority of new guidance law proposed in this paper by comparison of data. The results show that the new guidance law can greatly improve the precision of missile guidance, and provide a basis for missile guidance control theory.

2. Nonlinear Dynamic Model

2.1 Dynamic Model of Missile.

2.1.1 Dynamic differential equations.

Establish dynamics equation about the center of mass in the ground inertial coordinate system

$$m\frac{d^2\mathrm{R}}{dt^2} = \sum \mathrm{F} \tag{1}$$

Where m is the mass of the aircraft, it's the quantity that changes over time but not the constant. R is the vehicle position vector of the aircraft and $\sum \mathrm{F}$ is the resultant of all the external forces.

The resultant of all the external forces is

$$\sum \mathbf{F} = \mathbf{P} + \mathbf{G} + \mathbf{A} \tag{2}$$

Where P is thrust, G is gravity, and A is aerodynamic.

The attitude dynamics equation established in the ontology coordinate system is

$$J\frac{d\omega}{dt} + \omega \times J = \sum \mathbf{M} \tag{3}$$

Where J is the inertia matrix, $\boldsymbol{\omega}$ is the angular velocity vector and M is the resultant of all external moment of the aircraft.

The attitude equation of the aircraft is

$$\frac{d\mathbf{q}}{dt} = \frac{1}{2}\tilde{\omega}\mathbf{q} \tag{4}$$

Where q is four-order quaternion that describes the attitude of aircraft.

$$\mathbf{q} = q_0 + \mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 \tag{5}$$

Thrust is

$$\mathbf{P} = L_{gb}(\mathbf{P})_b \tag{6}$$

Where Lgb is the coordinate transformation matrix from ontology coordinate system to inertial coordinate system

$$L_{gb}{}^T = L_{bg} = L(\mathbf{q}) \tag{7}$$

(P)b represents the component description of thrust in the ontology coordinate system.

Aerodynamic is

$$\mathbf{A} = L_{gb}L_{ba}(\mathbf{A})_a \tag{8}$$

Where L_{ba} represents the coordinate transformation matrix from the pneumatic coordinate system to the ontology coordinate system

$$L_{ba} = L(\alpha, \beta) \tag{9}$$

(A)a represents the component description of aerodynamic forces in pneumatic coordinate systems.

$$\begin{aligned}(\mathbf{A})_a &= (-D, C, L)^T \\ D &= C_D \cdot \frac{1}{2}\rho V^2 S \\ C &= C_C \cdot \frac{1}{2}\rho V^2 S \\ L &= C_L \cdot \frac{1}{2}\rho V^2 S\end{aligned} \tag{10}$$

Where ρ is the air density, V is the speed of aircraft, S is reference area for aerodynamic, Cd is the drag coefficient, Cc is the lateral force coefficient, and CL is the lift coefficient. They are the function about f speed, altitude, angle of attack, angle of sideslip and control of the rudder angle.

2.1.2 Calculation of Resultant of All External Moment.

The resultant of all external moment

$$\begin{aligned}\sum \mathbf{M} &= (M_x M_y M_z)^T \\ M_x &= m_x \frac{1}{2}\rho V^2 SL \\ M_y &= m_y \frac{1}{2}\rho V^2 SL \\ M_z &= m_z \frac{1}{2}\rho V^2 SL\end{aligned} \tag{11}$$

Where mx, my, mz are moment coefficients. They are functions about speed, altitude, angle of attack, angle of sideslip and control of rudder angle.

2.2 The Nonlinear Relative Motion Model between Missile and Target.

In order to study conveniently, the relative motion between missile and target is shown in figure 1. Suppose that the missile and the target simultaneously move in the same vertical plane and they are all seen as a particle. At some point, the position of target is represented by a T point and the position of missile is represented by M point. The velocity vector of target is represented by Vt and the velocity vector of the missile is represented by V. The line between missile and target is called the line of sight of target and the distance between the two is represented by R. Angle between MT and the reference axis AX is called the angle of sight line, which is represented by q.

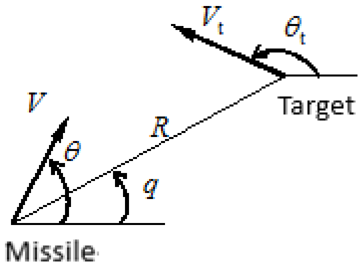

Fig. 1 The relative movement of the missile and target

The equation of motion in the ground coordinate system is

$$T\left(x_t, y_t, z_t, t, \vec{V}_t, \theta_t, \varphi_t\right) = 0 \tag{12}$$

Therefore the relative movement of the missile and target is

$$\begin{cases} \dfrac{d\vec{R}}{dt} = \left|\vec{V}_t\right| \cos\eta_t - \left|\vec{V}\right| \cos\eta_m \\ q_t = \sigma_t + \eta_t \\ q_t = \sigma_m + \eta_m \\ \end{cases} \tag{13}$$

In the three-dimensional plane, θ_l and ψ_l are respectively the inclination of sight and the declination of sight. The target parameters are denoted by the subscript "t". The nonlinear motion equation of the missile and target is

$$\dot{R} = V_t[\cos\theta_1 \cos\theta_t \cos(\psi_1 - \psi_{ct}) + \sin\theta_1 \sin\theta_t] - V[\cos\theta_1 \cos\theta \cos(\psi_1 - \psi_c) + \sin\theta_1 \sin\theta)] \tag{14}$$

$$R\dot{\theta}_1 = V_t[\cos\theta_1 \sin\theta_t - \sin\theta_1 \cos\theta_t \cos(\psi_1 - \psi_{ct})] - V[\cos\theta_1 \sin\theta - \sin\theta_1 \cos\theta \cos(\psi_1 - \psi_c)] \tag{15}$$

$$-R\dot{\psi}_1 \cos\theta_1 = V_t \cos\theta_t \sin(\psi_1 - \psi_{ct}) - V \cos\theta \sin(\psi_1 - \psi_c) \tag{16}$$

3. Guidance Law of Missile

3.1 Traditional Proportional Guidance Law.

The traditional proportional guidance law means that the rotation angular velocity of the missile velocity vector is proportional to the angular velocity of the target line in the process of approaching the target. And which can be expressed as

$$\frac{d\theta}{dt} = K\frac{dq}{dt} \tag{17}$$

Where K is the proportional coefficient of the proportional guidance law. The proportional guidance law is widely used, but the shortcomings in the guide process lead to the low precision for missile guidance. The shortcoming of proportional guidance law is: when the missile hit the target, the overload required is directly related to the velocity and the direction of the missile at the hitting point.

3.2 Derivation of the New Guidance Law.

3.2.1 Variable Structure Theory.

Consider the general situation, in the state space of the system [12], there is a hyperplane,

$$\text{x=f(x)} \qquad x \in R^n \tag{18}$$

which divides the space into two parts, S>0 and S< 0. The movement points on the hyperplane include the usual point, starting point and termination point[13]. Usually point and starting point are almost meaningless, but there are special meanings for the end point. Because if all the points in a switching surface are the termination point, once the motion state to converge with this area and it will be "attracted" to move in the area.

The design of slide variable structure control system is composed of designing switch function and designing control function. The selection methods of switch function include the pole collocation method[14], the quadratic optimal method[15] and the feature vector method[16]. And the design of conventional slide control include constant switch control, function switch control and proportional switch control.[17] The requirements for variable structure control is that the state trajectory of the system should be kept as small as possible during the approach period and at the same time the control quantity should not be too large when the system reaches the sliding surface. Therefore, the choice of control law of variable structure system is to choose the appropriate approach law.[18]

3.2.2 Slide Guidance Law.

Suppose the slide surface $S = \dot{q}$ and design control law to make S=0, therefore

$$\dot{S} = \ddot{q} = \frac{-2\dot{r}\dot{q} - a_m \cos(\theta - q) + a_t \cos(\theta_t - q)}{r} \tag{19}$$

Make arrival law is

$$\dot{S} = -\frac{K}{r}S - \frac{W}{r}\operatorname{sgn}(S), K > 0, W > \alpha + \mu, \mu > 0 \tag{20}$$

$$\operatorname{sgn}(S)=\begin{cases}1 & S>0\\ 0 & S=0\\ -1 & S<0\end{cases} \tag{21}$$

The target sight acceleration is regarded as bounded disturbance quantity. And by comparing $\dot{S}$ in the two formulas, the instruction acceleration is given as

$$a_c=\frac{-2\dot{r}\dot{q}+K\dot{q}+W\operatorname{sgn}(\dot{q})}{\cos(\theta-q)} \tag{22}$$

Bring the value of the instruction acceleration into the expression of and we can get

$$\dot{V}=\frac{\dot{q}[-K\dot{q}-W\operatorname{sgn}(\dot{q})+a_t\cos(\theta_t-q)]}{r} \tag{23}$$

Beacause of $W\geq\alpha+\mu,\mu,K>0$, we can get

$$\begin{aligned}\dot{V}\leq\frac{\dot{q}[-K\dot{q}-\mu\operatorname{sgn}(\dot{q})-(\alpha\operatorname{sgn}(\dot{q})-a_t\cos(\theta_t-q))]}{r}&=\\ \frac{[-K\dot{q}^2-\mu\dot{q}\operatorname{sgn}(\dot{q})-(\alpha\dot{q}\operatorname{sgn}(\dot{q})-a_t\dot{q}\cos(\theta_t-q))]}{r}&\\ =\frac{[-K\dot{q}^2-\mu|\dot{q}|-(\alpha|\dot{q}|-a_t\dot{q}\cos(\theta_t-q))]}{r}&\end{aligned} \tag{24}$$

Beacause of $\dot{q}^2$ >0, $|\dot{q}|$ >0, get

$$|a_t|\leq\alpha \Rightarrow \alpha|\dot{\theta}|-a_t\dot{\theta}\cos(\gamma_t-\theta)\geq 0 \tag{25}$$

Suppose $K=-K'\dot{r}$, $\mu=-K'\rho\dot{r}$, and we can get the guidance law is

$$a_c=\frac{-N\dot{r}\dot{q}+W\operatorname{sgn}(\dot{q})}{\cos(\theta-q)} \tag{26}$$

$W\geq\alpha-K'\rho\dot{r}$, $N=K'+2$

Considering the shake, the final guidance equation is

$$a_c=\frac{-N\dot{r}\dot{q}+W\dfrac{\dot{q}}{|\dot{q}|+\delta}}{\cos(\theta-q)} \tag{27}$$

Supposing θ_l and ψ_l are respectively the inclination of sight and the declination of sight, we can get the guidance law equation in the lateral plane.

$$a_{myc}=\frac{N|\dot{r}|\dot{\theta}_l+W\dfrac{\dot{\theta}_l}{|\dot{\theta}_l|+\delta}}{\cos(\theta-\theta_l)} \tag{28}$$

$$a_{mzc}=\frac{-N|\dot{r}|\dot{\psi}_l-W\dfrac{\dot{\psi}_l}{|\dot{\psi}_l|+\delta}}{\cos(\psi-\psi_l)} \tag{29}$$

3.2.3 Three Dimensional Nonlinear Slide Guidance Law.

For the time variant vector R, we can make the differentiation about scalar time. R can be written as IRR, where IR is the unit vector in R direction

$$\frac{d\mathbf{R}}{dt}=\frac{d}{dt}(\mathbf{I}_R R)=\frac{d\mathbf{I}_R}{dt}R+\mathbf{I}_R\frac{dR}{dt} \tag{30}$$

If ω_R represents the angular velocity of R, so the tangent velocity of R can represent the angular velocity of R. Therefore the tangent velocity of R can be expressed as

$$R\frac{dI_R}{dt} = \omega_R \times R \tag{31}$$

The second derivative of position vector is

$$\begin{aligned}\frac{d^2\mathrm{R}}{dt^2} &= \mathrm{I}_R\frac{d^2R}{dt^2} + (\omega_R \times \mathrm{I}_R)\frac{dR}{dt} + \dot{\omega}_R \times \mathrm{R} + \omega_R \times (\mathrm{I}_R\frac{dR}{dt} + \omega_R \times \mathrm{R}) \\ &= \mathrm{I}_R\frac{d^2R}{dt^2} + 2(\omega_R \times \mathrm{I}_R)\frac{dR}{dt} + \dot{\omega}_R \times \mathrm{R} + \omega_R \times (\omega_R \times \mathrm{R})\end{aligned} \tag{32}$$

The angular velocity of the sight line is

$$\omega = \dot{\theta} + \dot{\phi} \tag{33}$$

Beacause

$$\dot{R} = v_{xl},\ R\dot{\theta} = v_{yl},\ R\dot{\phi}\cos\theta = -v_{zl} \tag{34}$$

$$\omega_R \times (\omega_R \times \mathbf{R}) = (\dot{\phi} + \dot{\theta}) \times [(\dot{\phi} + \dot{\theta}) \times \mathbf{R}] \tag{35}$$

Therefore

$$\ddot{R} = R\dot{\phi}^2\cos^2\theta + R\dot{\theta}^2 + a_{tx_l} - a_{mx_l} \tag{36}$$

$$\ddot{\theta} = \frac{-R\dot{\phi}^2\sin\theta\cos\theta - 2\dot{\theta}\dot{R} + a_{ty_l} - a_{my_l}}{R} \tag{37}$$

$$\ddot{\phi} = \frac{-2\dot{R}\dot{\phi}\cos\theta + 2R\dot{\phi}\dot{\theta}\sin\theta - a_{tz_l} + a_{mz_l}}{R\cos\theta} \tag{38}$$

From the coordinate transformation relation, we can get a_{mx_1}, a_{my_1} and a_{mz_1} 。

$$\dot{v}_{y_t} = \dot{R}\dot{\theta} + \dot{R}\ddot{\theta} = a_{ty_t} - a_{my_t} - \dot{R}\dot{\theta} - \dot{R}\dot{\phi}^2\cos\theta\sin\theta \tag{39}$$

$$\dot{v}_{z_t} = a_{tz_t} - a_{mz_t} + \dot{R}\dot{\phi}\cos\theta \tag{40}$$

According to the lyapunov function

$$V = \frac{1}{2}(R\dot{\theta})^2 + \frac{1}{2}(R\dot{\phi})^2 + \frac{1}{2\alpha_1}\tilde{f}_1^2 + \frac{1}{2\alpha_2}\tilde{f}_2^2 \tag{41}$$

Get

$$a_{my_l} = A_1|\dot{R}|\dot{\theta} + R\dot{\phi}^2\sin\theta\cos\theta + \hat{f}_1\frac{\dot{\theta}}{|\dot{\theta}|+\delta} \tag{42}$$

$$a_{mz_l} = -A_2|\dot{R}|\dot{\phi}\cos\theta - 2R\dot{\phi}\dot{\theta}\sin\theta - \hat{f}_2\frac{\dot{\phi}}{|\dot{\phi}|+\delta} \tag{43}$$

$$\hat{f}_1 = \hat{f}_1(0) + \alpha_1\int_0^t|R\dot{\theta}|d\xi \tag{44}$$

$$\hat{f}_2 = \hat{f}_2(0) + \alpha_2\int_0^t|R\dot{\phi}|d\xi \tag{45}$$

Where $\hat{f}_1(0)$ and $\hat{f}_2(0)$ can be taken any value between zero and the absolute value of the maximum acceleration of the target. Where α_1 and α_2 are the adaptive rate, A1 and A2 are the navigation ratio.

Compared with the traditional proportional guidance law, the compensation control item for precluding target maneuver is introduced to the new guidance law. In the next chapter, the performance of the new guidance law is analyzed through the simulation comparison between the classical proportional guidance law and the three dimensional nonlinear slide guidance law.

4. Simulation Analysis

4.1 Simulation Model of Missile.

The simulation model of missile considered the actual situation. And the kinematics and dynamics model of the missile were established. The simulation program was written by C/C++ and divided into three levels: base layer, physical object layer and presentation layer. And considering the portability and extensibility of the program, the entity object layer is encapsulated with the class of C ++.

Simplify the rudder system as an inertial link

$$\frac{d\delta_i}{dt} = \frac{1}{T_i}(K_i S_i - \delta_i) \quad i = x, y, z \tag{46}$$

Where Ti, Ki are respectively the time constant and gain coefficient of the ith rudder, δi is the ith rudder Angle, Si is the control signal for the ith rudder. The rudder control signal is superimposed by the feedback signal of the autopilot and the control signal of the seeker. The feedback signal of the autopilot includes the angular velocity feedback signal and the linear acceleration feedback signal, the following

$$\mathbf{S}_a = K_w \omega + K_a \frac{d^2\mathbf{R}}{dt^2} \tag{47}$$

Where Kw is the angular velocity feedback matrix and Ka is the linear velocity feedback matrix.

When the missile with the slide guidance law, the control signal of seeker is

$$S_{Ci} = a_{mci} g / | v_{mi} | \quad i = y, z \tag{48}$$

4.2 Simulation Condition.

In the ground coordinate system, the initial position of the missile is (0, 0, 10000) and the initial velocity is (0, 1449, -388). And The initial position of the target is (15,2023 7,14968) and the initial velocity is (-4, 2900, -767). And the parameter is set to: $\hat{f}_1(0) = \hat{f}_2(0) = 0$, $\alpha_1 = \alpha_2 = 0.002$, $\delta = 0.002$, a= 0.6, b= 0.4, c= 0.01.

4.3 Simulation Result.

4.3.1 Target without Mobility.

At this time a_{ty} = 0, a_{tz} = 0.

(1) Proportional guidance law.

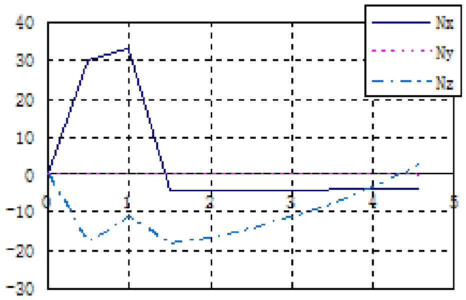

Fig. 2 Overload changes over time

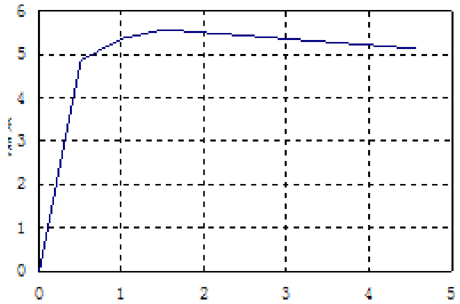

Fig. 3Mach number changes over time

(2) Nonlinear slide guidance law

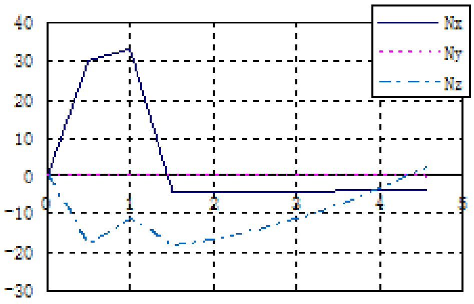

Fig. 4 Overload changes over time

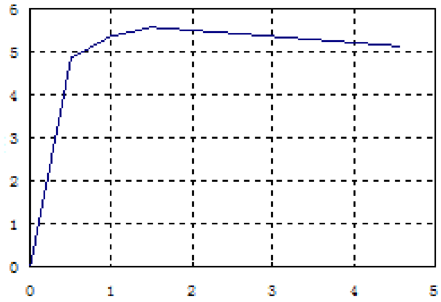

Fig. 5 Mach number changes over time

4.3.2 Target is Maneuvering in the Longitudinal Plane.

At this time $a_{ty}=0$, $a_{tz}=20$.

(1) Proportional guidance law.

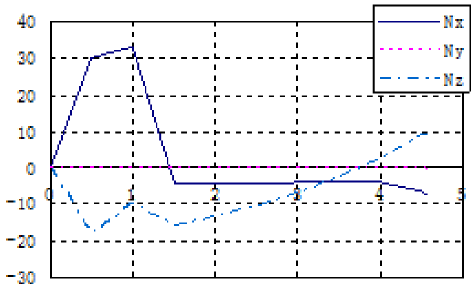

Fig. 6 Overload changes over time

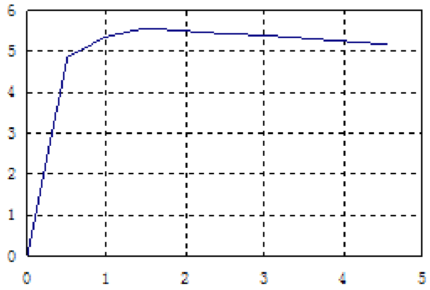

Fig. 7 Mach number changes over time

(2) Nonlinear slide guidance law.

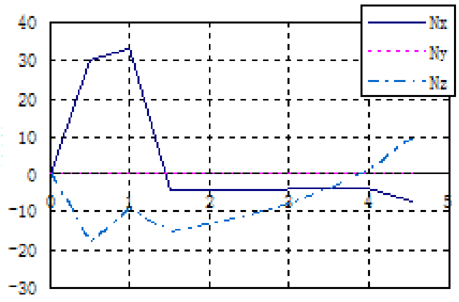

Fig. 8 Overload changes over time

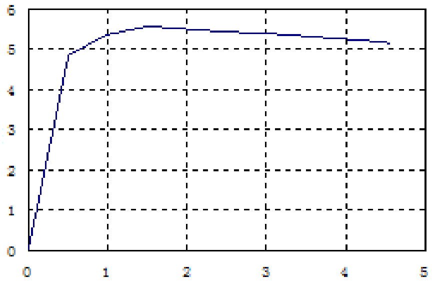

Fig. 9 Mach number changes over time

5. Conclusion

The simulation results of guidance law show that the performance of the three dimensional slide guidance law has more advantageous than the proportional guidance law. By establishing the nonlinear motion model of the relation between missile and target, prove that modification of the guidance law can obviously improve the accuracy of the missile's strike.

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