

P System for Resource-constrained Project Scheduling Problem based on Serial Schedule Generation Scheme

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Abstract

A P system for resource-constrained project scheduling problem(RCPSP) based on serial schedule generation scheme is proposed in this paper to provide new ideas for parallel solutions in today's big data environment. Because P system has great parallelism, it could solve hard problems, typically NP-complete problems, in polynomial time. We use membrane to conduct the whole process of computation. First, the P system is established with all its rules to solve the problem. Then, the time complexity and the descriptional complexity is discussed. Finally, an example is given to prove its feasibility. And this work represents another attempt in the application of P system.

Keywords

Membrane computing; P system; Resource-constrained project scheduling problem; Serial schedule generation scheme.

1. Introduction

In the last 60 years, change, and the need to manage change through projects, has been touching all our lives, in working and social environments. Now, over 30 percent of the global economy is project-based [1]. And the project-oriented organization is common. And a conservative estimate for the global mega-project market is between US\$6 and US\$9 trillion per year, or approximately 8% of the total global gross domestic product (GDP). However, nine out of ten mega-projects have cost overruns. And project management is critical [2]. For project management, project scheduling is a crucial component of it for today's complex business and manufacturing systems. And models and methods from project scheduling play a critical role in project management. During the last decades, the resource-constrained project scheduling problem (RCPSP) has become a standard problem for project scheduling in the literature. And it has been proved to be an NP-hard problem [3]. Since the benchmarks of RCPSP were presented, RCPSP has been studied by a great number of researchers and many exact methods and approximation algorithms have been proposed. Exact methods, such as linear programming, branch and bound, have been successful in solving small instances. However, they require a very high computing time as the size of project actives increase.

Membrane computing is a new branch of natural computing inspired from the functioning and the architecture of biological cells. It was initiated by Gh. Păun in 1998, and has developed rapidly [4]. The models obtained are distributed and parallel computing devices, called P systems. And the obtained computing devices proved to be very powerful, even equivalent with Turing machines when using restricted combinations of features, and computationally efficient. A number of applications were proposed in several areas - biology, economics, linguistics, data mining, approximate optimization, etc [5]. So far, P system is used for solving hard problems, typically NP-complete problems, in polynomial time. The purpose of this paper is the joint study of membrane computing with resource-constrained project scheduling problem (RCPSP) by using the benefit from the high parallelism of membrane computing. In this paper, we use membrane to conduct the whole process of computation. And the remainder is organized as follows: Section 2 gives a formal model of the RCPSP and an introduction of SSGS as well as an introduction of colored P System. Section 3

presents the design of the P system and analyze its computation complexity. Section 4, an example is given to prove its feasibility. We conclude the paper in Section 5.

2. Preliminaries

2.1 Resource-constrained Project Scheduling Problem Formulation

The classical Resource-constrained project scheduling problem can be stated as follows. The RCPSP considers a project with J activities which are labeled $j=1, \dots, J$. The duration of an activity j is denoted as p_j . And its start time is denoted as s_j , its finish time is denoted as c_j . We assume that once the activity starts, it may not be interrupted. The activities are interrelated by two kinds of constraints: precedence constraints and resource constraints. For precedence constraints, due to technological or logical requirements, there are precedence relations between activities. We assume that the precedence relations are only the finish-start precedence relationships with zero time-lag, which indicates that an activity j may not be started before each of its predecessors $i \in P_j$. And the project can be represented by an activity-on-the-node network $G=(V,E)$ in which V denotes the set of nodes representing the activities and E is the set of edges representing the finish-start precedence relationships with zero time-lag. For resource constraints, each activity requires certain amounts of resources. Here we assume the resources are only renewable resources which are available for their full capacity in every period. We consider K renewable resources which are labeled $k=1, \dots, K$. For each resource k , the per-period availability is assumed to be constant overtime and it is given by R_k . Activity j requires r_{jk} units of resource k for each period. The RCPSP can be formulated as follows:

$$\min c_J \quad (1)$$

s.t.

$$s_1 = 0, s_j \in \text{int}^+, j = 1, \dots, J \quad (2)$$

$$s_j \geq \max_{h \in P_j} c_h \quad (3)$$

$$\sum_{j \in A_t} r_{jk} \leq R_k \quad (4)$$

where the objective function is given as (1). The project makespan is minimized by minimizing the finishing time of the ending activity J . (3) indicates the precedence constraints. (4) Indicates the resource constraints.

The RCPSP is a generalization of the job shop problem and belongs to the class of NP-hard problems. And multitude of exact and heuristic approaches were proposed [6].

2.2 Serial Schedule Generation Scheme

A priority rule based scheduling heuristic is made up of two components, a schedule generation scheme and a priority rule. And two different schemes can be distinguished, the serial schedule generation scheme (SSGS) and the parallel schedule generation scheme (PSGS) [7]. The serial schedule generation scheme was proposed by Kelley [8]. It consists of J stages, in each stage n ($n=1, 2, \dots, J$), only one activity is selected and scheduled, the start time of activity j is denoted as s_j , its finish time is denoted as c_j , the project has a partial schedule denoted as PS_n and a decision set denoted as E_n . The activity set of the project in period t is denoted as A_t , the left over capacity of the renewable resource k in period t is denoted as πR_{kt} , $\pi R_{kt} = R_k - \sum_{j \in A_t} r_{jk}$, the priority value of activity j is

denoted as $v(j)$. The serial schedule generation scheme can be formally described as follows:

Initialization: $n=1$

While $n < J+1$

 Compute $E_n, \pi R_{kt}$

$$j^* = \min_{j \in E_n} \left\{ j \mid v(j) = \max_{i \in E_n} \{v(i)\} \right\}$$

$$es_{j^*} = \max \{s_i + d_i \mid i \in P_{j^*}\}$$

$$s_{j^*} = \min \left\{ t \geq es_{j^*}, r_{j^*k} \leq \pi R_{k\tau}, \tau = t, t+1, \dots, t+d_{j^*}-1, k = 1, 2, \dots, K \right\}$$

$$PS_{n+1} = PS_n \cup \{j^*\}$$

$n=n+1$
End
Stop

It has been proved that the time complexity for the SSGS is $O(J^2, K)$.

2.3 Colored P System

A colored P system is a construct of the form [9]:

$$\Pi = (O, K, \mu, M_0, \dots, M_m, R_0, \dots, R_m, \rho, i_0)$$

where O is the alphabet of objects, μ is a membrane structure with m membranes, K which is the number of the colors is a natural number and with each set $R_i, 1 \leq i \leq m$, written in the form

$$R_i = \bigcup_{j=1}^K R_{i,j}. \text{The rules in sets } R_{i,j} \text{ for a given } j, \text{ are considered of the same color. At each step, only rules}$$

of the same color can be used, in this way, on one hand, the parallelism in each region is restricted, on the other hand, a cooperation between regions is ensured[10]. M_1, \dots, M_m are strings over O indicating the multisets of objects at the beginning present in the m regions of μ, ρ is the priority of the rules, i_0 is the output region of the P system Π .

3. P System for RCPSP based on SSGS

3.1 The P System based on SSGS

In this section, a P system for RCPSP is proposed. Its structure is depicted in Figure 1. Here we assume that the duration of every activity except is positive integer.

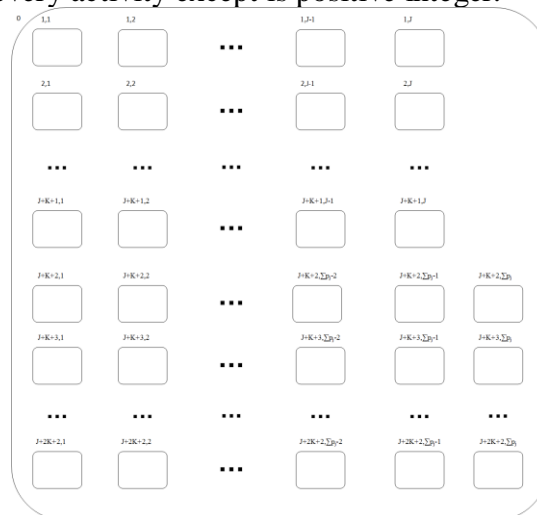


Figure 1. The P System for RCPSP Based On SSGS

The P system for RCPSP is defined as follows:

$$\Pi = (O, L, \mu, M_0, M_{1,1}, \dots, M_{J+K+1,J}, M_{J+K+2,1}, \dots, M_{J+2K+2, \sum p_j}, R_0, R_{1,1}, \dots, R_{J+K+1,J}, R_{J+K+2,1}, \dots, R_{J+2K+2, \sum p_j}, \rho, i_0)$$

Where:

$$1) O = \{a, a', b, b', c_1, c_2, \dots, c_{\sum p_j}, c'_1, c'_2, \dots, c'_{\sum p_j}, d_1, d_2, \dots, d_{\sum p_j}, d'_1, d'_2, \dots, d'_{\sum p_j}, e, e', k_1, k_2, \dots, k_J, s_1, s_2, \dots, s_{\sum p_j}, f_1, f_2, \dots, f_{\sum p_j}\}$$

O represents the collection of objects in the P system.

$$2) L = 4$$

L represents the number of the color in the P system.

$$3) \mu = [0[1,1]1,1[1,2]1,2 \dots [J+K+1,J]J+K+1,J [J+K+2,1]J+K+2,1 \dots [J+2K+2,\sum p_j]J+2K+2,\sum p_j]0$$

μ represents the membrane structure of the P system.

$$4) M_0 = \{\lambda\}; M_{i,j} = \{a^{1-a_j}, b^{a_j}\} (1 \leq i \leq J, 1 \leq j \leq J); M_{i,j} = \{c_j^{v_j}, d_{p_j}\} (i = J+1, 1 \leq j \leq J);$$

$$M_{i,j} = \{a^{r_{j-j-1}}\} (J+2 \leq i \leq J+K+1, 1 \leq j \leq J);$$

$$M_{i,j} = \{a^{r_{i-j-K+1}}\} (J+K+2 \leq i \leq J+2K+1, 1 \leq j \leq \sum p_j);$$

$$M_{i,j} = \{\lambda\} (i = J+2K+2, 1 \leq j \leq \sum p_j);$$

$M_{i,j}$ represents the collection of initial objects in each membrane. And $A = (a_{ij})$ is the adjacency matrix of G ; $V = (v_j)$ is the priority-matrix representation of activities, but different from normal, in P system, the smaller priority, the more prioritized it is, p_j is the duration of activity j ; $R = (r_{ij})$ is the resource-demand-matrix representation of activities; $RR = (rr_i)$ is the resource-constant-matrix representation of the project.

5) The rules in R_0 :

$$r_{1,1} = \{c_1^{g_1} c_2^{g_2} \dots c_J^{g_J} \rightarrow c_1^{g_1-1} c_2^{g_2-1} \dots c_J^{g_J-1}\} \cup \{c_1^{g_1} c_2^{g_2} \dots c_J^{g_J} \rightarrow d_h e \mid h \in \{1, 2, \dots, J\}\}_{-c_h}$$

$$r_{1,2} = \{e \rightarrow k_1\}_{-k_2, \dots, k_J} \cup \{k_g e \rightarrow k_{g+1} \mid 2 \leq g < J\}$$

$$r_{1,3} = \{k_g d_h \rightarrow (a')_{in(h,1)} (a')_{in(h,2)} \dots (a')_{in(h,J)} (b')_{in(1,h)} (b')_{in(2,h)} \dots (b')_{in(J,h)} (e)_{in(J+2,h)} (e)_{in(J+3,h)} \dots (e)_{in(J+K+1,h)} (e')_{in(J+1,h)} k_g d'_h \mid g, h \in \{1, 2, \dots, J\}\}$$

$$r_{3,1} = \{c_1^{g_1} c_2^{g_2} \dots c_{\sum p_j}^{g_{\sum p_j}} \rightarrow c_1^{g_1-1} c_2^{g_2-1} \dots c_{\sum p_j}^{g_{\sum p_j}-1}\} \cup \{c_1^{g_1} c_2^{g_2} \dots c_{\sum p_j}^{g_{\sum p_j}} \rightarrow s_h \mid h \in \{1, 2, \dots, \sum p_j\}\}_{-d_h}$$

$$r_{3,2} = \{s_g d'_h \rightarrow (s_g)_{in(J+1,h)} \mid g \in \{1, 2, \dots, \sum p_j\}, h \in \{1, 2, \dots, J\}\}$$

$$r_{4,1} = \{k_J \rightarrow halt\}$$

The rules in $R_{i,j} (1 \leq i \leq J, 1 \leq j \leq J)$:

$$r_{1,4} = \{a \rightarrow (a)_{in(J+1,j)} a'\}$$

$$r_{1,5} = \{bb' \rightarrow (b)_{in(J+1,i)} b\}$$

$$r_{4,2} = \{a'e \rightarrow a\}$$

$$r_{4,3} = \{a' \rightarrow \lambda\} \cup \{b' \rightarrow \lambda\}$$

The rules in $R_{i,j} (i = J+1, 1 \leq j \leq J)$:

$$r_{1,6} = \{a^J c_j^{p_j} \rightarrow (c_j^{p_j})_{in 0} c_j^{p_j}\} \cup \{a^J e \rightarrow (c_j^{J+1})_{in 0} e\}$$

$$r_{1,7} = \{a^g \rightarrow (c_j^{J+1})_{in 0} \mid h < g, h = 1, 2, \dots, J-1, g = 1, 2, \dots, J-1\}$$

$$r_{1,8} = \{a^h \rightarrow (c_j^{J+1})_{in 0} \mid h < g, h = 1, 2, \dots, J-1, g = 1, 2, \dots, J-1\}$$

$$r_{1,9} = \{d_g e' \rightarrow (d_g)_{in(J+2K+2,1)} (d_g)_{in(J+2K+2,2)} \dots (d_g)_{in(J+2K+2,\sum p_j)} d_g\}$$

$$r_{1,10} = \{f_g b \rightarrow (b)_{in(J+2K+2,1)} (b)_{in(J+2K+2,2)} \dots (b)_{in(J+2K+2,g)} f_g \mid g \in \{1, 2, \dots, \sum p_j\}\}$$

$$r_{3,3} = \{s_g d_h \rightarrow (e)_{in(1,1)} \cdots (e)_{in(J,J)} (e)_{in(J+2K+2,1)} (e)_{in(J+2K+2,2)} \cdots (e)_{in(J+2K+2,\sum p_j)} \\ (c_j)_{in(J+2K+2,g)} (c_j)_{in(J+2K+2,g+1)} \cdots (c_j)_{in(J+2K+2,g+h-1)} f_{g+h-1} e\}$$

The rules in $R_{i,j}$ ($J+2 \leq i \leq J+K+1$, $1 \leq j \leq J$):

$$r_{1,11} = \{a^{r_{j,j-1}} c_g \rightarrow (b^{r_{j,j-1}})_{in(i+K,g)} a^{r_{j,j-1}} \mid g \in \{1,2,\dots,\sum p_j\}\}$$

$$r_{1,12} = \{a^{r_{j,j-1}} e \rightarrow (b^{r_{j,j-1}})_{in(i+K,1)} (b^{r_{j,j-1}})_{in(i+K,2)} \cdots (b^{r_{j,j-1}})_{in(i+K,\sum p_j)} a^{r_{j,j-1}}\}$$

The rules in $R_{i,j}$ ($J+K+2 \leq i \leq J+2K+1$, $1 \leq j \leq \sum p_j$):

$$r_{1,13} = \{ab \rightarrow a'\}$$

$$r_{1,14} = \{b \rightarrow (b)_{in(J+2k+2,j)}\}$$

$$r_{3,1} = \{b \rightarrow \lambda\} \cup \{a' \rightarrow a\}_{-b}$$

The rules in $R_{i,j}$ ($i=J+2K+2$, $1 \leq j \leq \sum p_j$):

$$r_{1,15} = \{c_g \rightarrow (c_j)_{in(J+2,j)} (c_j)_{in(J+3,j)} \cdots (c_j)_{in(J+K+1,j)} c'_g \mid g \in \{1,2,\dots,J\}_{-e}$$

$$r_{1,16} = \{d_g b \rightarrow (b)_{in(J+2K+2,j-g+1)} (b)_{in(J+2K+2,j-g+2)} \cdots (b)_{in(J+2K+2,j)} d'_g \mid g \in \{p_1, p_2, \dots, p_J\}\}$$

$$r_{2,1} = \{d_g \rightarrow d'_g \mid g \in \{p_1, p_2, \dots, p_J\}\}$$

$$r_{3,4} = \{d'_g \rightarrow (c'_j)_{in0} \mid g \in \{p_1, p_2, \dots, p_J\}\}_{-b} \cup \{d'_g b \rightarrow (c'^{1+\sum p_j})_{in0} \mid g \in \{p_1, p_2, \dots, p_J\}\}$$

$$r_{3,5} = \{c'_g e \rightarrow c_g e\} \cup \{b \rightarrow \lambda\}$$

$$r_{4,4} = \{e \rightarrow \lambda\}_{-c'_1, c'_2, \dots, c'_J}$$

$$\rho = \{r_{i,j} > r_{j,i} \mid 1 \leq i < j \leq 4\} \cup \{r_{i,i} > r_{j,i} \mid 1 \leq i < j \leq 16\}$$

$$6) i_0 = \{(J+2K+2,1), (J+2K+2,2), \dots, (J+2K+2, \sum p_j)\}$$

i_0 represents the output regions of the P system.

3.2 An Overview of Computation

Now, we are focused on how the rules designed above guide the process of solving RCPSP.

Firstly, through rule $r_{1,4}$, objects a are sent into membrane $(J+i,i)$ for the action of rules $r_{1,6}, r_{1,7}, r_{1,8}$. and objects a' are generated for the action of rules $r_{4,2}$. And through rule $r_{1,10}$, objects c_j are sent into membranes for the action of rules $r_{1,11}$. Secondly, through rules $r_{1,6}, r_{1,7}, r_{1,8}$, objects $c_i^{p_i}$ are sent into membrane 0 in order to select eligible nodes for the action of rules $r_{1,1}, r_{1,2}, r_{1,3}$. And through rules $r_{1,13}$, objects a' are generated in order to compute πR_{kt} . Thirdly, through rules $r_{1,1}, r_{1,2}, r_{1,3}$, objects a', b', e, e' are sent into membranes for the action of rules $r_{1,5}, r_{4,2}, r_{1,12}, r_{1,9}$. Fourthly, through rules $r_{1,12}, r_{1,13}, r_{1,14}$, πR_{kt} has been computed, objects b are sent into membranes in order to close the time point which the activity cannot be operated. And through rules $r_{1,9}, r_{1,16}, r_{2,1}, r_{3,4}$, the time points which the activity can be started is chosen. Fifthly, through rules $r_{3,1}, r_{3,2}, r_{3,3}$, the finish time of the activity is decided, and objects e are generated for the action of $r_{3,5}$. Finally, if there is none of object K_j , through rules $r_{4,2}, r_{4,3}, r_{4,4}$, objects a are generated for the action of rule $r_{1,4}$. And if there is object K_j , through rule $r_{4,1}$, the P system is halt, the outputs of the P system are in membranes $(J+2K+2,1), (J+2K+2, 2), \dots, (J+2K+2, \sum p_j)$.

For solving RCPSP through the serial schedule generation scheme, the time complexity is $O(J^2, K)$. The time complexity for solving RCPSP using proposed P system is $O(J)$. However, the descriptonal complexity for the algorithm is rather high: $(J^2+(K+1)*(J+\sum p_j))$ membranes and the usage of rules of unbounded weight.

4. Example and discussion

To illustrate how the P system show a project problem specifically, the following simple example is considered: the project with 7 activities and 1 resource shown in Table 1. And we assume the priority matrix through priority rule MINSLK is $V=(5,1,4,6,2,3,7)$.

Table 1. The project problem

Job	Duration(p_j)	Resource demand(r_j)	Successors
1	3	2	4
2	3	3	3,5
3	1	2	4
4	2	3	
5	2	4	6
6	3	2	
7	3	1	
Resource availability	4		

And the adjacency matrix A of the project is:

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} .$$

And the collection of initial objects in the P system is shown in Figure 2.

Step of the P system are listed in Table 2. Because the steps are almost the same, only parts of them are listed here.

In the end, the result of the problem is $S=(6,0,5,9,3,5,0)$, $F=(9,3,6,11,5,8,3)$, the makespan of the project is 11. So the right result of the project problem is gained.



Figure 2. The P System for Solving the Problem with 7 Activities

	$c^2]_{11,1}[11,2c^2]_{11,2}[11,3bc^2]_{11,3}[11,4b^2c^5]_{11,4}[11,5b^2c^5]_{11,5}[11,6b^2c^3c^6]_{11,6}[11,7b^2c^1c^6]_{11,7}[11,8c^1c^6]_{11,8}[11,9c^1]_{11,9}[11,10c^4]_{11,10}[11,11c^4]_{11,11}[11,12]_{11,12}[11,13]_{11,13}[11,14]_{11,14}[11,15]_{11,15}[11,16]_{11,16}[11,17]_{11,17}0$
87	$[ok7d^7s_1[1,1a^1]_{1,1}[1,2a^1]_{1,2}[1,3a^1]_{1,3}[1,4ab^1]_{1,4}[1,5a^1]_{1,5}[1,6a^1]_{1,6}[1,7ab^1]_{1,7}][7,1a^2]_{7,1}[7,2a^2]_{7,2}[7,3a^2]_{7,3}[7,4a^2]_{7,4}[7,5a^2]_{7,5}[7,6a^2]_{7,6}[7,7a^2b^1]_{7,7}[8,1c^1f_9e]_{8,1}[8,2c^2f_3e]_{8,2}[8,3c^3f_6e]_{8,3}[8,4c^4f_{11}e]_{8,4}[8,5c^5f_5e]_{8,5}[8,6c^6f_8e]_{8,6}[8,7c^7d_3]_{8,7}[9,1a^2]_{9,1}[9,2a^3]_{9,2}[9,3a^2]_{9,3}[9,4a^3]_{9,4}[9,5a^4]_{9,5}[9,6a^2]_{9,6}[9,7a^1]_{9,7}[10,1a^4]_{10,1}[10,2a^4]_{10,2}[10,3a^4]_{10,3}[10,4a^4]_{10,4}[10,5a^4]_{10,5}[10,6a^4]_{10,6}[10,7a^4]_{10,7}[10,8a^4]_{10,8}[10,9aa^3]_{10,9}[10,10a^4]_{10,10}[10,11a^4]_{10,11}[10,12a^3a]_{10,12}[10,13a^3a]_{10,13}[10,14a^3a]_{10,14}[10,15a^3a]_{10,15}[10,16a^3a]_{10,16}[10,17a^3a]_{10,17}[11,1c^2]_{11,1}[11,2c^2]_{11,2}[11,3bc^2]_{11,3}[11,4b^2c^5]_{11,4}[11,5b^2c^5]_{11,5}[11,6b^2c^3c^6]_{11,6}[11,7b^2c^1c^6]_{11,7}[11,8c^1c^6]_{11,8}[11,9c^1]_{11,9}[11,10c^4]_{11,10}[11,11c^4]_{11,11}[11,12]_{11,12}[11,13]_{11,13}[11,14]_{11,14}[11,15]_{11,15}[11,16]_{11,16}[11,17]_{11,17}0$
88	$[ok7[1,1a^1]_{1,1}[1,2a^1]_{1,2}[1,3a^1]_{1,3}[1,4ab^1]_{1,4}[1,5a^1]_{1,5}[1,6a^1]_{1,6}[1,7ab^1]_{1,7}][7,1a^2]_{7,1}[7,2a^2]_{7,2}[7,3a^2]_{7,3}[7,4a^2]_{7,4}[7,5a^2]_{7,5}[7,6a^2]_{7,6}[7,7a^2b^1]_{7,7}[8,1c^1f_9e]_{8,1}[8,2c^2f_3e]_{8,2}[8,3c^3f_6e]_{8,3}[8,4c^4f_{11}e]_{8,4}[8,5c^5f_5e]_{8,5}[8,6c^6f_8e]_{8,6}[8,7c^7d_3s_1]_{8,7}[9,1a^2]_{9,1}[9,2a^3]_{9,2}[9,3a^2]_{9,3}[9,4a^3]_{9,4}[9,5a^4]_{9,5}[9,6a^2]_{9,6}[9,7a^1]_{9,7}[10,1a^4]_{10,1}[10,2a^4]_{10,2}[10,3a^4]_{10,3}[10,4a^4]_{10,4}[10,5a^4]_{10,5}[10,6a^4]_{10,6}[10,7a^4]_{10,7}[10,8a^4]_{10,8}[10,9aa^3]_{10,9}[10,10a^4]_{10,10}[10,11a^4]_{10,11}[10,12a^3a]_{10,12}[10,13a^3a]_{10,13}[10,14a^3a]_{10,14}[10,15a^3a]_{10,15}[10,16a^3a]_{10,16}[10,17a^3a]_{10,17}[11,1c^2]_{11,1}[11,2c^2]_{11,2}[11,3bc^2]_{11,3}[11,4b^2c^5]_{11,4}[11,5b^2c^5]_{11,5}[11,6b^2c^3c^6]_{11,6}[11,7b^2c^1c^6]_{11,7}[11,8c^1c^6]_{11,8}[11,9c^1]_{11,9}[11,10c^4]_{11,10}[11,11c^4]_{11,11}[11,12]_{11,12}[11,13]_{11,13}[11,14]_{11,14}[11,15]_{11,15}[11,16]_{11,16}[11,17]_{11,17}0$
89	$[ok7[1,1a^1]_{1,1}[1,2a^1]_{1,2}[1,3a^1]_{1,3}[1,4ab^1]_{1,4}[1,5a^1]_{1,5}[1,6a^1]_{1,6}[1,7ab^1]_{1,7}][7,1a^2]_{7,1}[7,2a^2]_{7,2}[7,3a^2]_{7,3}[7,4a^2]_{7,4}[7,5a^2]_{7,5}[7,6a^2]_{7,6}[7,7a^2b^1]_{7,7}[8,1c^1f_9e]_{8,1}[8,2c^2f_3e]_{8,2}[8,3c^3f_6e]_{8,3}[8,4c^4f_{11}e]_{8,4}[8,5c^5f_5e]_{8,5}[8,6c^6f_8e]_{8,6}[8,7c^7f_3e]_{8,7}[9,1a^2]_{9,1}[9,2a^3]_{9,2}[9,3a^2]_{9,3}[9,4a^3]_{9,4}[9,5a^4]_{9,5}[9,6a^2]_{9,6}[9,7a^1]_{9,7}[10,1a^4]_{10,1}[10,2a^4]_{10,2}[10,3a^4]_{10,3}[10,4a^4]_{10,4}[10,5a^4]_{10,5}[10,6a^4]_{10,6}[10,7a^4]_{10,7}[10,8a^4]_{10,8}[10,9aa^3]_{10,9}[10,10a^4]_{10,10}[10,11a^4]_{10,11}[10,12a^3a]_{10,12}[10,13a^3a]_{10,13}[10,14a^3a]_{10,14}[10,15a^3a]_{10,15}[10,16a^3a]_{10,16}[10,17a^3a]_{10,17}[11,1c^2]_{11,1}[11,2c^2]_{11,2}[11,3bc^2]_{11,3}[11,4b^2c^5]_{11,4}[11,5b^2c^5]_{11,5}[11,6b^2c^3c^6]_{11,6}[11,7b^2c^1c^6]_{11,7}[11,8c^1c^6]_{11,8}[11,9c^1]_{11,9}[11,10c^4]_{11,10}[11,11c^4]_{11,11}[11,12]_{11,12}[11,13]_{11,13}[11,14]_{11,14}[11,15]_{11,15}[11,16]_{11,16}[11,17]_{11,17}0$
90	$[ok7[1,1a^1e]_{1,1}[1,2a^1e]_{1,2}[1,3a^1e]_{1,3}[1,4abe]_{1,4}[1,5a^1e]_{1,5}[1,6a^1e]_{1,6}[1,7abe]_{1,7}][7,1a^2e]_{7,1}[7,2a^2e]_{7,2}[7,3a^2e]_{7,3}[7,4a^2e]_{7,4}[7,5a^2e]_{7,5}[7,6a^2e]_{7,6}[7,7a^2be]_{7,7}[8,1c^1f_9e]_{8,1}[8,2c^2f_3e]_{8,2}[8,3c^3f_6e]_{8,3}[8,4c^4f_{11}e]_{8,4}[8,5c^5f_5e]_{8,5}[8,6c^6f_8e]_{8,6}[8,7c^7f_3e]_{8,7}[9,1a^2]_{9,1}[9,2a^3]_{9,2}[9,3a^2]_{9,3}[9,4a^3]_{9,4}[9,5a^4]_{9,5}[9,6a^2]_{9,6}[9,7a^1]_{9,7}[10,1a^4]_{10,1}[10,2a^4]_{10,2}[10,3a^4]_{10,3}[10,4a^4]_{10,4}[10,5a^4]_{10,5}[10,6a^4]_{10,6}[10,7a^4]_{10,7}[10,8a^4]_{10,8}[10,9aa^3]_{10,9}[10,10a^4]_{10,10}[10,11a^4]_{10,11}[10,12a^3a]_{10,12}[10,13a^3a]_{10,13}[10,14a^3a]_{10,14}[10,15a^3a]_{10,15}[10,16a^3a]_{10,16}[10,17a^3a]_{10,17}[11,1c^2c_7e]_{11,1}[11,2c^2c_7e]_{11,2}[11,3c^2c_7e]_{11,3}[11,4c^5e]_{11,4}[11,5c^5e]_{11,5}[11,6c^3c^6e]_{11,6}[11,7c^1c^6e]_{11,7}[11,8c^1c^6e]_{11,8}[11,9c^1e]_{11,9}[11,10c^4e]_{11,10}[11,11c^4e]_{11,11}[11,12e]_{11,12}[11,13e]_{11,13}[11,14e]_{11,14}[11,15e]_{11,15}[11,16e]_{11,16}[11,17e]_{11,17}0$
91	$[ok7[1,1a]_{1,1}[1,2a]_{1,2}[1,3a]_{1,3}[1,4ab]_{1,4}[1,5a]_{1,5}[1,6a]_{1,6}[1,7a]_{1,7}][7,1a]_{7,1}[7,2a]_{7,2}[7,3a]_{7,3}[7,4a]_{7,4}[7,5a]_{7,5}[7,6a]_{7,6}[7,7a]_{7,7}[8,1c^1f_9e]_{8,1}[8,2c^2f_3e]_{8,2}[8,3c^3f_6e]_{8,3}[8,4c^4f_{11}e]_{8,4}[8,5c^5f_5e]_{8,5}[8,6c^6f_8e]_{8,6}[8,7c^7f_3e]_{8,7}[9,1a^2]_{9,1}[9,2a^3]_{9,2}[9,3a^2]_{9,3}[9,4a^3]_{9,4}[9,5a^4]_{9,5}[9,6a^2]_{9,6}[9,7a^1]_{9,7}[10,1a^4]_{10,1}[10,2a^4]_{10,2}[10,3a^4]_{10,3}[10,4a^4]_{10,4}[10,5a^4]_{10,5}[10,6a^4]_{10,6}[10,7a^4]_{10,7}[10,8a^4]_{10,8}[10,9aa^3]_{10,9}[10,10a^4]_{10,10}[10,11a^4]_{10,11}[10,12a^3a]_{10,12}[10,13a^3a]_{10,13}[10,14a^3a]_{10,14}[10,15a^3a]_{10,15}[10,16a^3a]_{10,16}[10,17a^3a]_{10,17}[11,1c_2c_7e]_{11,1}[11,2c_2c_7e]_{11,2}[11,3c_2c_7e]_{11,3}[11,4c_5e]_{11,4}[11,5c_5e]_{11,5}[11,6c_3c_6e]_{11,6}[11,7c_1c_6e]_{11,7}[11,8c_1c_6e]_{11,8}[11,9c_1e]_{11,9}[11,10c_4e]_{11,10}[11,11c_4e]_{11,11}[11,12e]_{11,12}[11,13e]_{11,13}[11,14e]_{11,14}[11,15e]_{11,15}[11,16e]_{11,16}[11,17e]_{11,17}0$

5. Conclusion

This paper constructs a P system to solving RCPSP through the serial schedule generation scheme. This P system is suitable for solving RCPSP by example test, but it needs to be further studied whether it is suitable for solving RCPSP of large amount of data. Speaking from a theoretical point of view, owing to the great parallelism that P system has, it can reduce the time complexity of computing and increases the computational efficiency. The application in RCPSP proposed in this paper is only one example, There are many method to solving RCPSP and this paper only use the serial schedule generation scheme. Membrane computing can be applied to a variety of other methods to solving RCPSP.

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