Analysis of Hydraulic Transient of Pipeline System: A Review on Method of Characteristics

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Abstract

With increasing construction of pipelines, the complexity of system has become a big issue to analyze the safety, which should be guaranteed to improve the operation of whole system in service. However, due to the limit of SCADA system, Hydraulic transient is hard to be detected, which is often within short time intervals. Therefore, this paper aims to review the techniques of method of characteristics (MOC) in solving hydraulic transient of pipeline system. By combing Euler equation and conservation of mass equation, MOC is came up with to solve the motion of fluid under certain circumstances. As the development of the method, it has been improved to achieve more accurate results in such analysis.

Keywords

Hydraulic Transient, Pipeline System, Method Of Characteristic.

1. Introduction

Nowadays, pipeline is becoming one of the major methods of liquid transportation, no matter in municipal water distribution system or water injection networks in oil and gas industry [1, 2]. Under normal operation, flow rate of each pipe is maintained in steady state, as well as hydraulic grade line and pressure head at each node. However, there are many unpredictable factors that interrupt normal operation, such as the fluctuation of demand at each node, leakage of fluid during transport and even the sudden failure of water pump[3, 4, 5, 6], which would contribute to the hydraulic transient. Thus, it is of great significance to analyze transient state in order to prevent accidents for whole system.

2. Numerical Analysis

As for numerical analysis, controlling equations should be introduced as the foundation of the numerical model. Then, MOC can be written by arranging the controlling equations in Lagrange form [7, 8] and solving it under certain circumstance.

2.1 Controlling Equations

After determining the structure of pipeline system, continuity equations is written for each node to describe the discharge at the junction, which is shown as follows

$$QJ_i - \sum Q_i = 0 \tag{1}$$

Where QJ_i ——demand at node i;

$\sum Q_i$ ——total discharge flowing in and out node i;

By considering the compressibility of liquid and the elasticity of material of each pipe, conservation of mass equation is used

$$\frac{1}{\rho}\frac{dp}{dt} + a^2\frac{\partial V}{\partial s} = 0 \tag{2}$$

Considering $P = \rho g(H - z)$, then

$$\frac{dH}{dt}+\frac{a^2}{g}\frac{\partial V}{\partial s}=0 \tag{3}$$

Where H ——hydraulic grade line;

a ——wave speed.

Euler equation is used to describe the motion of fluid, which can be determined by

$$\frac{1}{g}\frac{\partial V}{\partial t}+\frac{\partial H}{\partial s}+\frac{V}{g}\frac{\partial V}{\partial s}+\frac{f}{D}\frac{V|V|}{2g}=0 \tag{4}$$

2.2 Numerical Model

During the hydraulic transient, the sudden change of discharge at one point would cause the expansion of pressure wave, which would travel along the pipe and cause the fluctuation of pressure and flow rate. Therefore, in order to describe the phenomenon, Euler equation and conservation of mass equation should be combined in Lagrange from. In order to get rid of partial differential equation [9,10], equations of MOC can be expressed as

$$C^+:\frac{a}{g}\frac{dV}{dt}+\frac{dH}{dt}+\frac{a}{g}\frac{f}{2D}V|V|=0 \tag{5}$$

Only if $\frac{ds}{dt}=V+a$

$$C^-:\frac{a}{g}\frac{dV}{dt}-\frac{dH}{dt}+\frac{a}{g}\frac{f}{2D}V|V|=0 \tag{6}$$

Only if $\frac{ds}{dt}=V+a$

As indicated in the equations, V and H are functions of time and location. However, the variation of time is much bigger than that of location [11], namely, $\frac{\partial V}{\partial s}<<\frac{\partial V}{\partial t}$, $\frac{\partial H}{\partial s}<<\frac{\partial H}{\partial t}$. Then rearrange Lagrange form and solve it out. The essence of MOC is to write the equation above in the form of fully differential equation under certain conditions[12], in which the equations remain the same, however, only if $\frac{ds}{dt}=\pm a$ respectively.

2.3 Boundary Condition

There are several kinds of boundary conditions [13, 14], in accordance with the real situation. By properly simplifying the boundary condition, restraint equations can be categorized as follows:

1) Velocity Boundary Condition

Velocity boundary condition usually appears at the valve due to the activated operation of valve to control the fluid. Assuming the velocity keeps constant when s. During the transient, the relation between velocity of fluid and time should be given.

2) Pressure Boundary Condition

Pressure boundary condition is set at the upstream where the hydraulic grade line of reservoir keeps constant during the transient. However, since the reservoir has been treated as the boundary, so that it is not seem as node anymore.

3) Velocity-Pressure Boundary Condition

For the pipeline system with pump, the curve of each pump is characterized by discharge and pressure head. So that this kind of boundary condition is often used as inner boundary condition.

3. Solutions

By applying linear differential equation to MOC, the transient of pipeline can be determined. First of all, wave speed in pipeline should be calculated according to the basic parameter of pipeline, such as diameter and material of pipe. Then each pipe should be divided into several segments where time interval is related to the number of pipe segments and wave speed as

$$\Delta t \le \frac{\Delta s}{\max|a+V|} \tag{7}$$

to ensure the stability of iteration for each time interval [15].

3.1 Approximate Approach

When ignoring the spatial variation of velocity and pressure, the slope of characteristics equation is constant and it can be solved in $s-t$ coordinate. For any point P, it can be expressed by C^+ and C^- characteristics equations, shown in Fig.1.

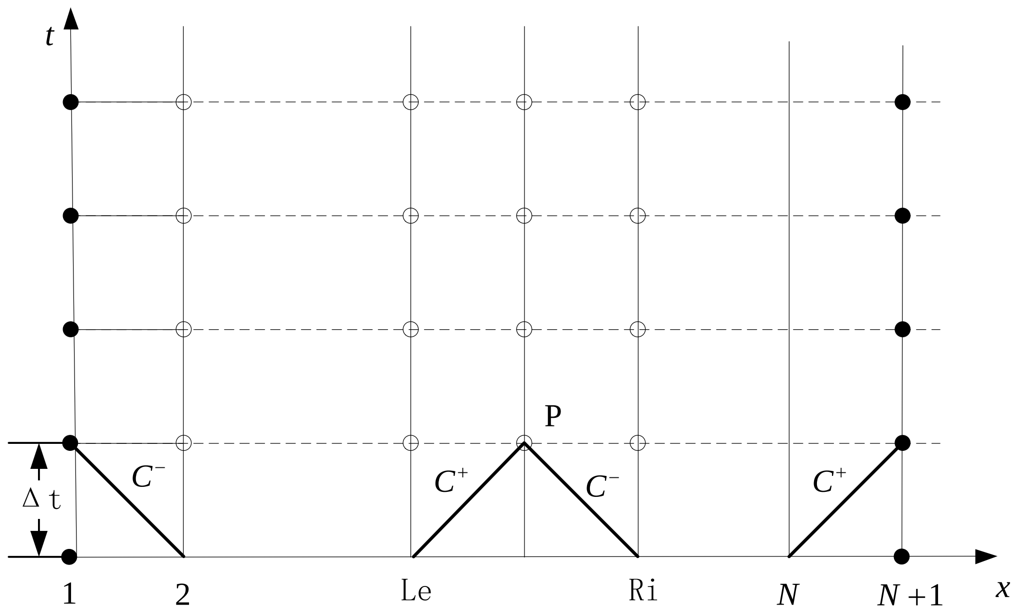

Fig.1 Schematic of approximate approach

Assume that the characteristic equations from point P intersect with coordinate at the point Le and point Ri, where the hydraulic grade line and velocity of each point can be written as H_{Le}, H_{Ri}, V_{Le} and V_{Ri} respectively. Then the velocity and hydraulic grade line of point P can be shown as

$$V_p = \frac{1}{2}\left[(V_{Le}+V_{Ri}) + \frac{g}{a}(H_{Le}-H_{Ri}) - \frac{f\Delta t}{2D}\left(V_{Le}|V_{Le}| + V_{Ri}|V_{Ri}|\right) \right] \tag{8}$$

$$H_p = \frac{1}{2}\left[\frac{a}{g}(V_{Le}-V_{Ri}) + (H_{Le}+H_{Ri}) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{Le}|V_{Le}| - V_{Ri}|V_{Ri}|\right) \right] \tag{9}$$

3.2 Modified Approximate Approach

Modified approximate approach applies characteristic equations along the t-axis forward and backward respectively to locate the leakage point along the pipe, shown as Fig.2.

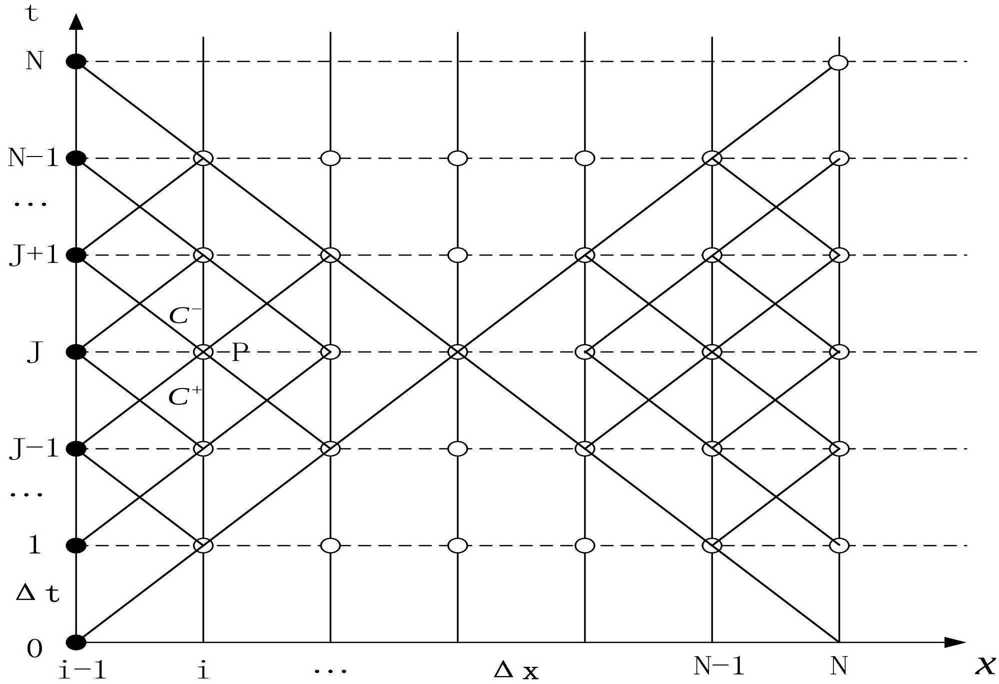

Fig.2 Schematic of modified approximate approach

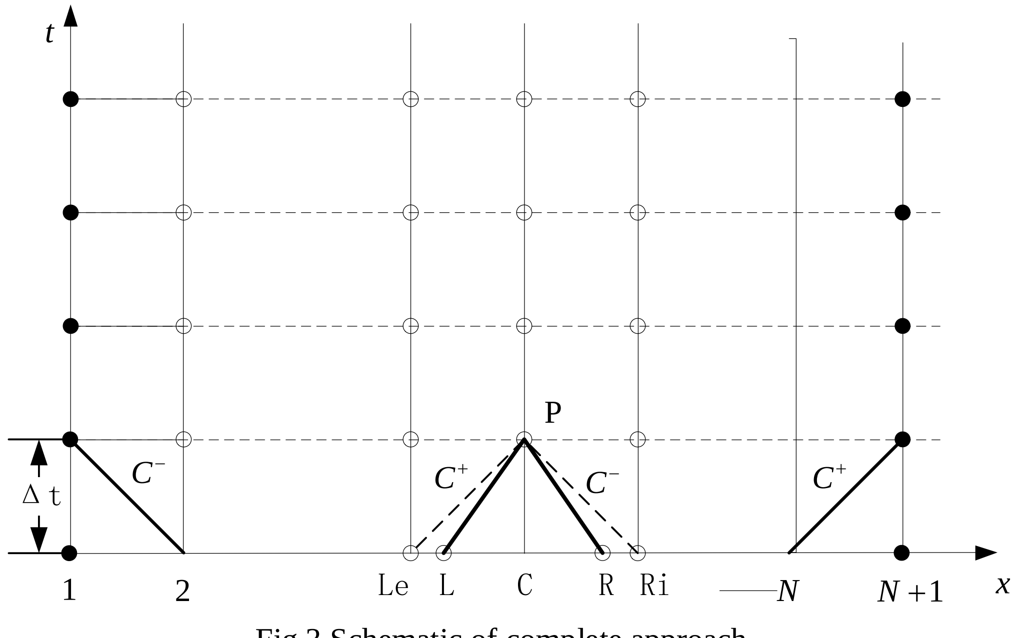

Fig.3 Schematic of complete approach

If hydraulic grade line and velocity at point $i-1$ is known at $t=J-1$, $t=J$ and $t=J+1$, then the velocity and hydraulic grade line at point i at $t=J$ can be calculated.

$$\frac{V_P-V_{J+1}}{\Delta t}-\frac{g}{a}\frac{H_P-H_{J+1}}{\Delta t}+\frac{f}{2D}V_{J+1}\left|V_{J+1}\right|=0 \tag{10}$$

$$\frac{V_P-V_{J-1}}{\Delta t}+\frac{g}{a}\frac{H_P-H_{J-1}}{\Delta t}+\frac{f}{2D}V_{J-1}\left|V_{J-1}\right|=0 \tag{11}$$

Rearrange the equations above, and they can be expressed as

$$V_P=\frac{1}{2}\left[\left(V_{J-1}+V_{J+1}\right)+\frac{g}{a}\left(H_{J-1}-H_{J+1}\right)-\frac{f\Delta t}{2D}\left(V_{J-1}\left|V_{J-1}\right|-V_{J+1}\left|V_{J+1}\right|\right)\right] \tag{12}$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_{J-1} - V_{J+1}\right) + \left(H_{J-1} + H_{J+1}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{J-1}\left|V_{J-1}\right| - V_{J+1}\left|V_{J+1}\right|\right)\right] \tag{13}$$

Once the velocity and hydraulic grade line at one end is known, then the parameters at another end can be calculated by this way. By comparison the results with experimental data, this approached has proved to achieve high accurate with high convergence speed [16].

3.3 Complete Approach

When considering the spatial term of velocity and pressure, the slope of characteristic equations are not constant anymore. By assuming the slope at each time interval is constant, shown as Fig.3, these lines would intersect at L and R. Instead of calculating Le and Ri, the parameters of L and R should be determined first.

Then find out the ratio relation between intercept $L-C$ and $Le-C$, by assuming $L-C$ as Δx and $Le-C$ as Δs. Therefore, the relation can be shown as [17]

$$\frac{\Delta x}{\Delta s} = \frac{V_L - V_C}{V_{Le} - V_C} = \frac{H_L - H_C}{H_{Le} - H_C} \tag{14}$$

where $\dfrac{\Delta x}{\Delta t} = a + V_L$.

Applying the equation above into point L and R, then point P can be expressed as

$$V_p = \frac{1}{2}\left[\left(V_L + V_R\right) + \frac{g}{a}\left(H_L - H_R\right) - \frac{f\Delta t}{2D}\left(V_L\left|V_L\right| + V_R\left|V_R\right|\right)\right] \tag{15}$$

$$H_p = \frac{1}{2}\left[\frac{a}{g}\left(V_L - V_R\right) + \left(H_L + H_R\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_L\left|V_L\right| - V_R\left|V_R\right|\right)\right] \tag{16}$$

3.4 Implicit-MOC Approach

With a combination of implicit approach and MOC equations, the stability of statistics can be guaranteed as well as maintaining the simplicity of calculation [18]. By applying MOC at each end and implicit approach between both ends, time interval along the pipe can be minimize while still achieving the accuracy of the solution, shown as Fig.4.

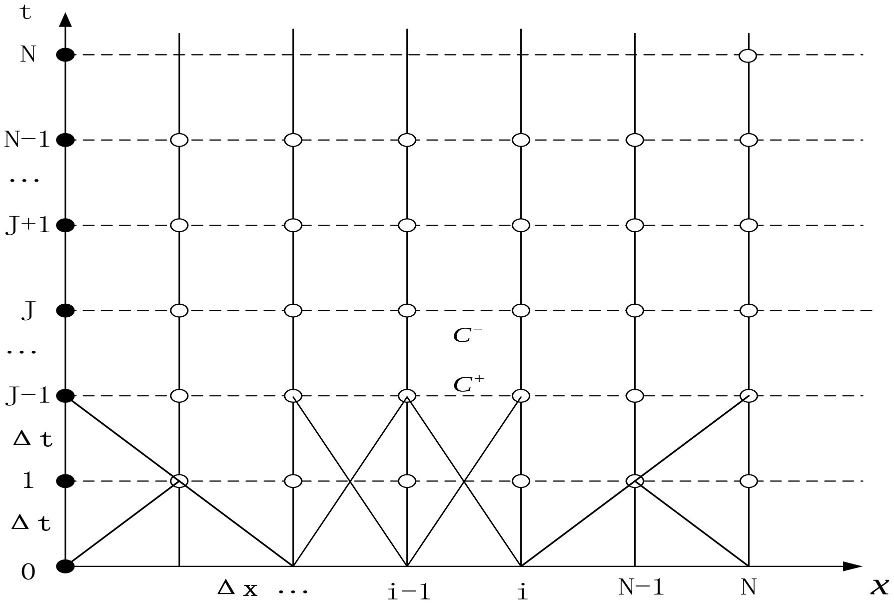

Fig.4 Schematic of implicit-MOC approach

3.5 Simpson Approach

During transient state, friction factor is actually the function of time and flow rate. So by introducing Simpson approach, the friction factor can be properly treated by considering the influence of points at each time interval and this approach has three order accuracy [19].

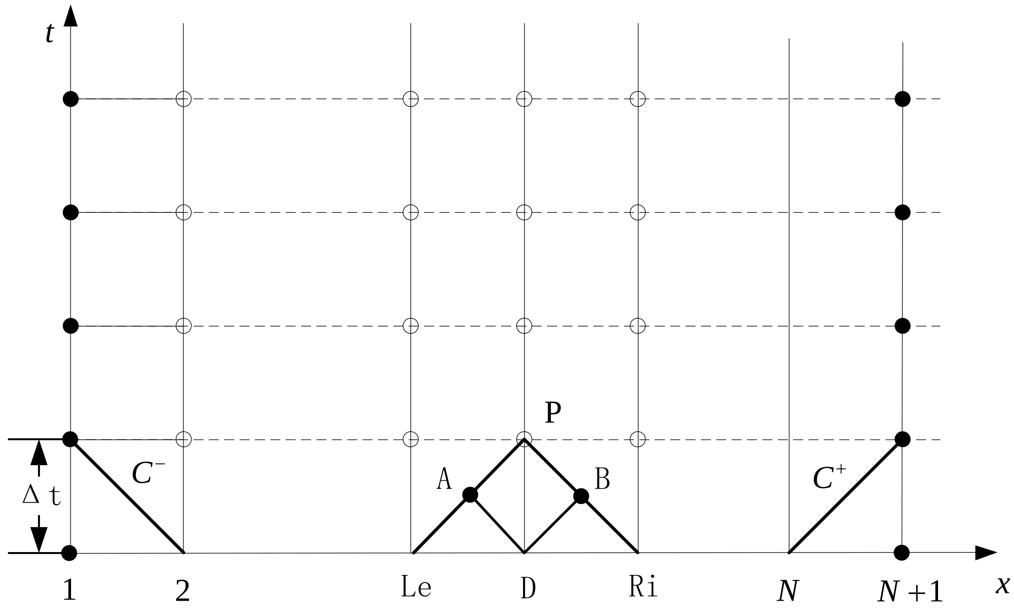

Fig.5 Schematic of Simpson approach

According to Simpson approach,

$$h_L = \int_{t_1}^{t_2} V|V|dt \approx \frac{1}{6}\left(V_1|V_1| + 4V_X|V_X| + V_2|V_2|\right)\Delta t \tag{17}$$

Where Δt——$t_2 - t_1$;

V_X——velocity at $t = \frac{t_1 + t_2}{2}$.

Then characteristic equations can be written as

$$\frac{V_p - V_{Le}}{\Delta t} + \frac{g}{a}\frac{H_P - H_{Le}}{\Delta t} + \frac{1}{12D}\left(f_{Le}V_{Le}|V_{Le}| + 4f_A V_A|V_A| + f_P V_P|V_P|\right) = 0 \tag{18}$$

$$\frac{V_p - V_{Ri}}{\Delta t} - \frac{g}{a}\frac{H_P - H_{Ri}}{\Delta t} + \frac{f}{12D}\left(f_{Ri}V_{Ri}|V_{Ri}| + 4f_B V_B|V_B| + f_P V_P|V_P|\right) = 0 \tag{19}$$

Before solving the equations above, velocity V_A and V_B need to be determined. As for point A, it can be expressed by line $Le - A$ and line $D - A$

$$f_{t_A}^{t_L} V|V|dt \approx V_A|V_{Le}|\Delta t' \tag{20}$$

$$f_{t_D}^{t_A} V|V|dt \approx V_A|V_D|\Delta t' \tag{21}$$

Then the equations at point A can be written as

$$V_A = \frac{\frac{g}{a}(H_{Le} - H_D) + (V_{Le} + V_D)}{\left[2 + \frac{f\Delta t'}{2D}\left(|V_{Le}| + |V_D|\right)\right]} \tag{22}$$

$$H_A = \frac{1}{2}(H_{Le} + H_D) + \frac{a}{g}\frac{f\Delta t'}{4D}V_A\left(|V_D| - |V_{Le}|\right) + \frac{a}{2g}(V_{Le} - V_D) \tag{23}$$

Instead of assuming friction factor as constant, Simpson approach consider the influence of velocity during each time interval.

4. Conclusion

As the construction of pipeline networks, safety is always the main issue to ensure the integrity of whole system. However, due to the potential risk caused by hydraulic transient, it is of great importance to analyze the phenomenon.

This paper investigates the numerical solution for hydraulic transient of pipeline system. In accordance, controlling equations are used to describe the motion of fluid and the conservation of mass law. By introducing method of characteristics, controlling equations can be solved under certain circumstances, where partial equation can be replaced by fully differential equations along slopes of the characteristic equations. Then by applying linear differential equations, MOC equations can be solved by each time interval. Further researches could focus on the application of MOC in hydraulic transient of pipeline system and compare the results with the field data in order to valid the numerical solution.

Acknowledgements

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