A Federated Algorithm for Personal Seamless Positioning based on GPS/INS/ Magnetometer Integration

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Abstract

This paper propose a new method of seamless positioning technologies. Firstly, a new heading solution algorithm basing on magnetometer/Gyro was proposed. Sencondly, an extended Kalman filter algorithm (EKF) model of the magnetic /INS was constructed, and the personal positioning information is solved in real time. At last, adoptting unscented Kalman filter algorithm (UKF) to fuse the location information of EKF and GPS implement the seamless positioning.

Keywords

Personal Navigation, Seamless Positioning, EKF, UKF.

1. Introduction

With the increasing demand for positioning accuracy in location information service, personal navigation and positioning technologies have been widely researched in recent years^{[1][2].} Global Positioning System (GPS) is a revolutionary tool, which provides accurate and flexible positions, but the multi-path effect and the influences of natural environment will reduce the positioning accuracy. Inertial Navigation System (INS) can determine the position of individual with strong autonomy and anti-interference ability, but its performance is time-limited by drift problems^{[3][4]}. The combination of INS and GPS can give full play to their respective advantages to improve the navigation accuracy and positioning reliability.

2. Basic theory

2.1 Pedestrian Dead Reckoning Based on Inertial device

Assuming pedestrians as a particle, then the question of pedestrian navigation is naturally transformed into a matter of movement in the plane. It is easier to calculate the positioning information of the pedestrian at any time if the distance and the heading of current step are known.

The basic PDR algorithm schematic is shown in Fig. 1.

The step length L estimated by Non - linear model is

$$L = k \sqrt{a_{\max} - a_{\min}} \tag{1}$$

The steps length can be different by adjusting the size of the factor K.

According to the Figure 1, if the previous position is $[E_{i-1}, N_{i-1}]$, then the next position $[E_i, N_i]$ will be expressed as:

$$\begin{cases} E_{i} = E_{i-1} + L_{i-1} \sin \alpha_{i-1} \\ N_{i} = N_{i-1} + L_{i-1} \cos \alpha_{i-1} \end{cases}$$
(2)

In this type, the previous step length is L_{i-1} ; the previous heading is α_{i-1} .



Fig.1 PDR Schematic

2.2 Attitude Calculation Based on Quaternion

The attitude matrix can be solved by the attitude differential equation of the triaxial angular rate, and the quaternion is a most common way to update the vector posture angle. The concrete formula is:

$$\dot{Q} = \frac{1}{2} Q \otimes \vec{w}, Q(t_0) = Q_1$$
(3)

In the type, the real matrix is $\overset{\bullet}{Q}$; the attitude angular velocity quaternion is \vec{w} ; the initial quaternion is Q_1 . The specific matrix expression is as follows:

$$\begin{bmatrix} \cdot & & & & \\ q_0 & & & \\ \cdot & & & \\ q_1 & & & \\ \cdot & q_2 & & \\ \vdots & q_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$
(4)

Then, the attitude matrix of the quaternion is obtained:

$$T_{n}^{b} = \begin{bmatrix} q_{1}^{2} + q_{2}^{2} - q_{3}^{2} - q_{4}^{2} & 2(q_{2}q_{3} + q_{1}q_{4}) & 2(q_{2}q_{4} - q_{1}q_{3}) \\ 2(q_{2}q_{3} - q_{1}q_{4}) & q_{1}^{2} - q_{2}^{2} + q_{3}^{2} - q_{4}^{2} & 2(q_{3}q_{4} + q_{1}q_{2}) \\ 2(q_{2}q_{4} + q_{1}q_{3}) & 2(q_{3}q_{4} - q_{1}q_{2}) & q_{1}^{2} - q_{2}^{2} - q_{3}^{2} + q_{4}^{2} \end{bmatrix}$$
(5)

2.3 Heading estimation by Magnetometer/Gyro

There is a limitted effect in heading which soluted by gyro alone in the case of pedestrian turning. This paper proposed a new heading solution algorithm based on magnetometer/Gyro, the specific process is as follows:

$$w_M^k = \frac{\theta_M^k - \theta_M^{k-1}}{T} \tag{6}$$

In the type, the Heading an soluted by magnetometer is θ_M ; the Intervals between moment k-1 and k is T; the average angular rate value is w_M^k .

Combined with the the gyro angular rate, the optimal angle rate is:

$$w_{k} = (1 - \chi) w_{Gyro}^{k} + \chi w_{M}^{k}, (0 \le \chi \le 1)$$
(7)

In the type, the parameter χ can be adjust proportional to N which can be expressed as follows:

$$N = \left| \operatorname{mod}_{M}^{k} - \operatorname{mod}_{M}^{local} \right| \tag{8}$$

In the type, the model of the magnetometer at moment k is mod_M^k ; The model of local Magnetometer is mod_M^{local} . The *N* is not equal to zero in the situation of external magnetic interference.

In some case, even if the moel of magnetometer changes a little but there is a big error in angle solved by magnetometer .So, the above derivation must meet the following condition:

$$-M < w_M^k - w_{Gyro}^k < M \tag{9}$$

3. Fusion algorithm model

3.1 Attitude Fusion Based on Extended Kalman Filter(EKF)

The EKF fusion model requires a state equation and an observation equation, and the normalized accelerometer value and the measured value of the magnetometer are taken as the observation vector. The derivation process is as follows:

State equation:

$$X_{k} = q(k) = \Phi_{k,k-1}(\omega_{k}, T_{k})X_{k-1} + W_{k}$$

=exp $(\frac{1}{2}\Omega(\omega_{k})T_{k})q_{k-1} + W_{k}$ (10)

In the type, the state transition matrix is $\Phi_{k,k-1}(\omega_k, T_k)$; the optimum triaxial angular rate is ω_k ; the state noise matrix is W_k , which covariance matrix is J(k).

Observation equatio

$$Y_{k} = \begin{bmatrix} A_{k} \\ M_{k} \end{bmatrix} = f(X_{k}) + V_{k} = \begin{bmatrix} C_{n}^{b}(q_{k}) & 0 \\ 0 & C_{n}^{b}(n) \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} + \begin{bmatrix} V_{k}^{m} \\ V_{k}^{a} \end{bmatrix}$$
(11)

In the type, the normalized accelerometer value is A_k ; the normalized magnetometer value is M_k ; the gravity acceleration normalized vector is G; the magnetic field normalized vector is H; the quaternion updated posture transformation matrix is $T_n^b(q_k)$. the noise vector is V_k^m , which covariance matrix is:

$$\delta = \begin{bmatrix} \sigma_a^2 I & 0\\ 0 & \sigma_m^2 I \end{bmatrix}$$
(12)

The jacobian matrix is used to process the measurement equations:

$$\begin{cases} H_{k,k-1}(\hat{x}_{k-1}) = \frac{\partial \Phi_{k,k-1}(X)}{\partial X} \Big|_{X=x} \\ F_{k}(\hat{x}_{k,k-1}) = \frac{\partial f_{k-1}(X)}{\partial X} \Big|_{X=x} \end{cases}$$
(13)

In the type, the $H_{k,k-1}(\hat{m}_{k-1})$ is the jacobian matrix elements to $\Phi_{k,k-1}(\omega_k,T_k)$; the $F_k(x_{k,k-1})$ is the jacobian matrix elements to $f(X_k)$.

3.2 GPS / PDR Seamless Location Based on Unscented Kalman Filter (UKF)

The UKF fusion model also requires a state equation and an observation equation, This paper takes position, step size, and heading angle into the non - linear dynamic state. dynamic state:

$$X = [E \ N \ S \ \psi] \tag{14}$$

$$Y = [Y_{GPS} \ Y_{PDR}] \tag{15}$$

In the type, the position information soluted by GPS is Y_{GPS} ; the position information soluted by EKF is Y_{PDR} ; the east direction coordinates is E; the north direction coordinates is N; the step length is S; the heading is ψ . The specific process is as follows.

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Status of the initial conditions:

$$X_0 = E(X_0) \tag{16}$$

$$P_0 = E[(X_0 - \dot{X_0})(X_0 - \dot{X_0})^T]$$
(17)

Solution of the sigma points:

$$\xi_0 = \hat{X}_k \tag{18}$$

$$\xi_i = \hat{X}_k + \left[\sqrt{(N+\lambda)P_k}\right]_i \quad i = 1, \dots, N$$
(19)

$$\xi_{i+N} = \hat{X}_k - \left[\sqrt{(N+\lambda)P_k}\right]_i \quad i = N+1, \dots, 2N$$
⁽²⁰⁾

Calculation of the weighting factor:

$$W_0 = \frac{\lambda}{N + \lambda} \tag{21}$$

$$W_i = \frac{1}{2(N+\lambda)} \tag{22}$$

Equation of time update:

$$\lambda_{k|k-1} = f(\lambda_{k-1}, u_{k-1})$$
(23)

$$\hat{X}_{k|k-1} = \sum_{i=1}^{2N+1} \delta_i^m \lambda_{i,k|k-1}$$
(24)

$$P_{k|k-1} = \sum_{i=1}^{2N+1} \delta_i^c [\lambda_{i,k|k-1} - \hat{X}_{k|k-1}^-] [\lambda_{i,k|k-1} - \hat{X}_{k|k-1}^-]^T + R_v$$
(25)

$$Y_{k|k-1} = H(\lambda_{k|k-1}) \tag{26}$$

$$\hat{Y}_{k|k-1} = \sum_{i=1}^{2N+1} \delta_i^m Y_{i,k|k-1}$$
(27)

Equation of state update:

$$P_{\bar{Y}_{k}} = \sum_{i=1}^{2N+1} \delta_{i}^{c} [Y_{i,k} - \bar{Y}_{k}] [Y_{i,k} - \bar{Y}_{k}^{-}]^{T} + R_{n}$$
(28)

$$P_{X_k Y_k} = \sum_{i=1}^{2N+1} \delta_i^c [\lambda_{i,k|k-1} - \hat{X}_{k|k-1}] [Y_{i,k} - \overline{Y}_k]^T$$
(29)

$$K_{k} = P_{X_{k}Y_{k}} P_{Y_{k}}^{-1}$$
(30)

$$\hat{X}_{k} = \hat{X}_{k,k-1} + K_{k}(Y_{k} - \hat{Y}_{k})$$
(31)

$$P_k = P_k^- - K_k P_{\overline{Y}_k \overline{Y}_k} K_k^T$$
(32)

According to the state equation(14) and the observation equation(15), the time system state estimate \hat{X}_k can be obtained by setting a certain filter initial value.

4. Experimental result and analysis

In order to verify the validity of algorithm, the paper conducts an indoor static experiment and an outdoor experiment. The static test can verify the system error which is equal to the coordinate subtracted from the theoretical coordinates. The comparison between GPS and EKF/UKF in static experiment is shown in Fig. 2.



Fig.2 The comparison between GPS and EKF/UKF

The table 1 collects the values of the error of Fig 2.

		Maximum error	Average error
longgitude	GPS	17.64m	8.82m
	EKF/UKF	5.26m	4.57m
latitude	GPS	16.83m	7.58m
	EKF/UKF	6.84m	3.45m

Then, an outdoor experiment was conduct to verify the validity of seamless positioning algorithm. Fig. 3 is the real walking route, and fig. 4 represent the graphical results of there positioning methods. Each step is represented by a symbol (green, red or blue), being completely coloured when positioning has been applied.



Fig. 3 Real walking route



Fig. 4 Trajectory reproduction of three method

In Fig. 3 and Fig. 4, it can be seen that, there are 20% interior area out of the whole route, EKF did not return to the origin due to umulative error,GPS won't provid a continuously positioning, the precision of UKF/EKF is higher than that of other two methods. The absolute errors are 5.8 m,6.2m and 3.1m (EKF, GPS and EKF/UKF).

5. Conclusion

The EKF/UKF algorithm has been adapted to be integrated in a GPS/INS/ magnetometer system, and is investigated for bridging the navigation gap during GPS outages. On the one hand, INS/ geomagnetic can provide pedestrian location information without GPS .on the other hand, the GPS will provide the location information and eliminate the inertia sensor cumulative error.

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