

## Optimal Design of Three Stage Planetary Gear Reducer for Cutterhead Drive System of Shield Tunneling Machine

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### Abstract

In this paper an optimization method is introduced on the three-stage planet gear reducer for in shield machine cutter driver. Firstly, the optimization model for a single-stage planetary gear is established to obtain the design parameters for minimum mass (i.e. volume for a certain density): the objective function of the mentioned optimization model is the total volume of the sun gear and all planet gears; the main parameters to influence the mechanical capacity of a planet gear reducer is regarded design variables, such as the velocity ratio, the modulus, the number of teeth, the breadth of tooth; the constraint conditions, which can be suitable for the design variables, will be deduced by the planetary gear transmission principle. And then, the optimization model for the multi-stage planet gear reducer is proposed. Finally, the model mentioned model in this paper is used realized the optimization design of a three-stage planet gear reducer for cutterhead drive system of shield tunneling machine, and the results show that the mass of the planet gear reducer is significantly lightened.

### Keywords

Three-Stage Planet Gear Train; Light-Weight; Optimization Design; Planetary Gear Transmission Principle.

### 1. Introduction

In recent years, with the rapid development of information technology and computer technology, the optimization design has become an important development direction for mechanical product design. the planetary gear transmission isn an advanced mechanical transmission, and it has the characteristics of high torque, large transmission ratio, high efficiency and high reliability, so it is used in mining machinery, engineering machinery, wind driven generator, ships and warships, helicopter etc. [1]. Light-weight has important significance for the planet gear reducer to improve performance and reduce costs [2].

Because of excellent performance of the planetary gear transmission, the application, production and research on it are paid a great attention to in some advanced industrialized countries in the world. The further and systematic researches on the planetary gear transmission are realized in our country, and many innovative research results have been obtained [3]. There are many literatures on the light-weight design on a single planetary gear transmission used in the different industry, and they have the greatest impact on improving the precision and efficiency of the planetary gear transmission.

Firstly, the optimization model for a single-stage planetary gear is established to obtain the design parameters for minimum mass in this paper. Then, a recursion model for the multi-stage planet gear transmission used in cutterhead of shield machine driving system are deduced. Finally the optimization results are obtained by the optimization toolbox of MATLAB, and by comparing with those obtained by the traditional design method, the rationality and effectiveness of the optimization model are effectively verified.

**2. Optimizaion model of a three stage planetary gear reducer**

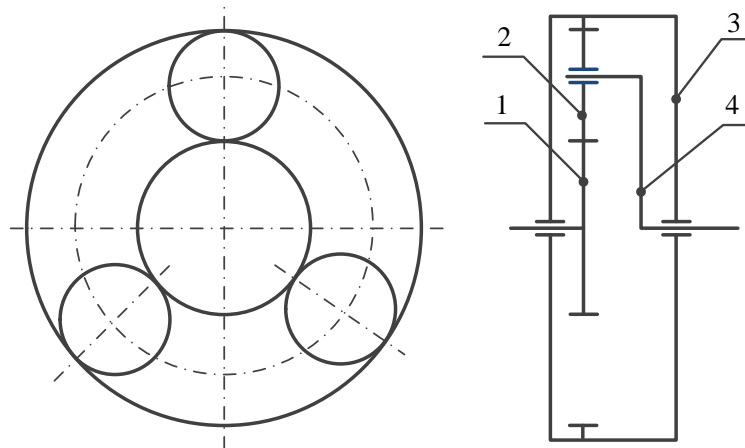


Fig. 1 Mechanism sketch of the NGW planetary gear transmission

The mechanism sketch of the NGW planetary gear transmission showed in Figure 1. As shown in Fig.1, 1 denotes the Sun, and it is the input shaft; 2 denotes the planet, and there are  $c$  ( $c$  is an integer) planets; 3 denotes the ring; 4 denotes the carrier, and it is the output shaft. The shield cutter drive system of three stage planetary gear reducer is made of three NGW planetary gears which are connected in turn. As it is convenient to study the single planetary gear system, in this paper an optimization model of a single-stage planet gear transmission will be firstly established, and then an optimization model of multi-stage planet gear transmission will be built. For a single-stage NGW planet gear transmission, more number of planets, more significant advantages. But the increase in the number of planets will lead to uneven load, and its adjacent limitations will reduce the transmission range, so the number of planets for a single stage planetary gears usually takes from 3 to 5. When the material of the gears is selected, light-weight of the planet gear transmission is similarly to minimize its volume when their bearing capacity is the same. And the weight of a profile shifted spur gear is approximately replaced by that of a standard spur gear.

**2.1 Objective function**

The volume of the sun gear is given as the equ. (1).

$$V_A = \frac{\pi}{4} D_A^2 1b = \frac{\pi}{4} z_A^2 m^2 b \tag{1}$$

Where,  $D_A$  represents the pitch diameter of the sun gear, mm;  $z_A$  represents gear number of the sun gear;  $b$  represents the gear width of the sun gear and planet gear, mm;  $m$  represents the modulus of the sun gear and planet gear, mm.

The total volume of planet gears is given as the equ. (2).

$$V_C = \frac{\pi}{4} D_C^2 bC = \frac{\pi}{4} z_C^2 m^2 bC \tag{2}$$

Where,  $D_C$  represents the pitch diameter of the planet gear, mm;  $z_C$  represents the gear number of the planet gear;  $C$  represents the number of planet gears.

the total volume of the single-stage planet gear transmission is regarded as the objective fuction, i. e.

$$F(m, b, z_A, z_C, C) = \frac{\pi}{4} m^2 b (z_A^2 + Cz_C^2) \tag{3}$$

because  $z_B - z_A = 2z_C$  and  $z_B = (i - 1)z_A$ ,

$$z_C = \frac{i - 2}{2} z_A \tag{4}$$

When equ. (4) is substituted into equ. (3), the objective function for the single planetary gear train will be given as the equ. (5).

$$F(m, b, z_A, i, C) = \frac{\pi m^2 z_A^2 b}{16} [4 + (i - 2)^2 C] \tag{5}$$

And then the objective function of the three-stage NGW planet gear transmission is given as the equ. (6).

$$F(\mathbf{X}') = \frac{\pi m_1^2 z_1^2 b}{16} [4 + (i_1 - 2)^2 C] + \frac{\pi m_2^2 z_2^2 b}{16} + \frac{\pi m_3^2 z_3^2 b}{16} [4 + (i_3 - 2)^2 C] \tag{6}$$

Where,  $\mathbf{X}' = [m_1, b_1, z_1, i_1, m_2, b_2, z_2, i_2, m_3, b_3, z_3, i_3, C]^T$ , and when  $C=3$ , the vector  $\mathbf{X}$  of design variables is given as the equ. (7).

$$\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T = [m_1, b_1, z_1, i_1, m_2, b_2, z_2, i_2, m_3, b_3, z_3, i_3]^T \tag{7}$$

Where, the subscripts 1, 2 and 3 of  $m, z$  and  $i$  respectively denote the high-speed stage, intermediate stage and low-speed stage.

And then the objective function of three-stage NGW planet gear transmission is given as the equ. (8).

$$F(\mathbf{X}) = \frac{\pi x_2^2 x_1^2 x_3}{16} [4 + 3(x_4 - 2)^2] + \frac{\pi x_6^2 x_5^2 x_7}{16} [4 + 3(x_8 - 2)^2] + \frac{\pi x_{10}^2 x_9^2 x_{11}}{16} [4 + 3(x_{12} - 2)^2] \tag{8}$$

**2.2 Constraint conditions**

**2.2.1 Gear number and number of planet gears**

Since the objective function was derived in which the equal distance condition has been used therefore considers only the assembly and the adjacent constraints

(1) Assemblin Requirements

$$\frac{z_B + z_A}{C} = N \tag{9}$$

Where,  $z_b$  denotes the number of turns of the outer ring gear,  $N$  is a positive integer.

$$h_1(\mathbf{X}) \frac{x_1 x_4}{C} - N = 0 \tag{10}$$

(2) Adjacent conditions

$$D_{ca} \leq 2A \sin \frac{\pi}{C} \tag{11}$$

Where,  $D_{ca}$  denotes the addendum circle diameter of the planet gear,  $A$  denotes the center distance between the sun gear and the planet gear.

$$x_1 \sin \frac{\pi}{C} - \frac{1}{2} x_1 (x_4 - 2) \left( 1 - \sin \frac{\pi}{C} \right) \geq 0 \tag{12}$$

**2.2.2 Geometric constraints and strength conditions**

(1) Constraint conditions of the contact fatigue strength

The contact strength can be calculated by equ. (13).

$$S_H \leq \frac{\sigma_{Hlim} Z_{NT} Z_L Z_v Z_R Z_W Z_X}{Z_B \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{F_t}{d_1 b} \times \frac{u+1}{u}}} \tag{13}$$

Where,  $u$  is a ratio of the gear number between the gear and pinion,  $S_H$  is the contact stress safety factor and  $S_H=1.3$ . The meaning of the other parameters is shown in the literature [4], and the value of parameters is replaced by that of the traditional design.

Whether the pinion is the sun gear or the planet gear, and it is simplified as the equ. (14).

$$S_H \leq A / \frac{1000Ti}{3z_A^2 m^2 b (i - 2)} \tag{14}$$

Where,  $A$  is a constant,  $T$  is the total input torque, and  $T=1333.8N$ .

The constraint condition of the contact fatigue strength is given as the equ. (15).

$$g_1(X)=1.3 - A/\frac{1000x_4 1333.8}{3x_1^2 x_2^2 x_3 (x_4 - 2)} \geq 0 \tag{15}$$

(2) Constraint conditions of the bending fatigue strength

$$S_F \leq \frac{\sigma_{Flim} Y_{ST} Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X}{\frac{F_t}{b m} Y_{Fa} Y_{Sa} Y_{\epsilon} Y_{\beta} K_A K_V K_{F\beta} K_{F\alpha}} \tag{16}$$

Where,  $S_F$  is the safety factor of the contact stress, and  $S_F=1.6$ . The meaning of the other parameters in equ. (16) is shown in the literature [4], and initial value of thses parameters is obtained by the traditional look-up table method.

Whether the pinion is a sun wheel or a planetary wheel, and it is simplified as the equ. (17).

$$S_F \leq B \frac{3m^2 b z_A}{1000T} \tag{17}$$

Where,  $B$  denotes a constant which is determined by the relevant parameters.

The constraint conditions of the bending fatigue strength is given as the equ. (18).

$$g_2(X)=1.6 - B \frac{3x_1 x_2^2 x_3}{1000 \times 1333.8} \geq 0 \tag{18}$$

(3) Constraint conditions of undercutting phenomenon

For spur gear without undercut, the minimum number of teeth is usually equal to 17, and constraint condition of undercutting phenomenon is given as the equ. (19).

$$g_3(X)=x_1 - 17 \geq 0 \tag{19}$$

(4) Constraint conditions of modulus

In order to ensure the strength, the modulus of the gears is not less than 2. So, the modulus constraint condition is given as the equ. (20).

$$g_4(X)=x_2 - 2 \geq 0 \tag{20}$$

(5) Constraint conditions of tooth widths

As  $0.3 \leq z_1 m/b \leq 0.8$ , the constraint conditions of the tooth width coefficient are given as the equ. (21).

$$\begin{cases} g_5(X)=0.3x_1 x_2/x_3 - 1 \geq 0 \\ g_6(X)=x_3/0.8x_1 x_2 - 1 \geq 0 \end{cases} \tag{21}$$

Except for the above constraints conditions, there are the other ones, such as the tooth number is a positive integer and the modulus of a gear is and a series value. Because the bearing capacity of the internal meshing gear pair is high and it easy to satisfy the design requirment, it is not included in the constraint conditions. Especially, the above constraint conditions are only involved in a single stage planet gear transmission, however, For a multi-stage planet gear transmission, the constraints conditions should satisfy the design requirement for every stage.

### 3. Design example

For a three-stage planet gear reducer for the cutterhead drive system of shield tunneling machine, its total gear ratio is 51.4, the rated power is 1600KW, the rated rotation is 1145.6r/min, there is a medium impact, external gear ring is fixed. The optimized optimization function is solved by the fmincon toolbox under the MATLAB environment. Firstly, the objective function, a M\_file myfun.m, is constructed. Secondly the nonlinear constraint function, a M\_file mycon.m, is programmed. Finally, the main optimization fucitons is programmed by called the M\_file myfun.m and mycon.m. The input initial value  $x_0$  is  $[24; 4; 50; 4; 25; 6; 70; 3.6; 27; 9; 90; 3.57]^T$ , the optimizaiton results  $x$  is

[23.9990; 4.01; 48.8572; 4.01; 24.9990; 5.9915; 69.3393; 3.6004; 26.9028; 9.0100; 79.6716; 3.5686]. The value of the final objective function  $fx_{al}=1.2342 \times 10^7$  and it is round of results shown in Table 1.

Table 1. Optimization result of the planetary gear reducer of the cutterhead drive system of shield

High speed stage				Intermediate stage				Low speed stage			
$z_A$	$m/mm$	$b/mm$	$i$	$z_A$	$m/mm$	$b/mm$	$i$	$z_A$	$m/mm$	$b/mm$	$i$
24	4	48	4	25	6	70	3.6	27	9	80	3.57

The volume of the reducer before optimization is  $1.5452 \times 10^7$ , and that after optimization is  $1.2342 \times 10^7$ , the volume decreased by 25.2%. On the premise that the performance is not reduced, the weight of the reducer is significantly reduced.

#### 4. Conclusion

In this paper the light-weight optimization model of the three stage planetary gear transmission system for the cutter-head of shield tunneling machine are proposed, and the column vector is used to denote the design variables and constraint conditions in the proposed. Especially, several single-stage planet gear transmission are connected by the gear ratio to construct the optimization model for multi-stage planet gear transmission. The simulation results show that the optimization model of the multi stage planetary gear transmission can be quickly solved by the optimization toolbox of MATLAB and obtain the precise results. The proposed method also has reference value for the optimization design of the other multi-stage gear transmission.

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