Make the Zambezi River Better

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Abstract

This paper establishes the mathematical model to determine the optimal number and location of small dams by considering the security of dam, locations, cost and etc. We also use the new multiple dam system to discuss the management of the Zambezi River. Since the factor of river volumes is the critical one for dam security, we establish a single factor model. Then, we introduce the dam insecurity index and give the calculation formula of the index. Based on this formula, we extend and apply the TSP model to decide the reasonable numbers and locations of dams, and then enable the insecurity index perfect. Furthermore, by improving the simulated annealing algorithm to solve the best solution of this single factor model, we obtain the results of numbers and locations.

Keywords

Multiple Dam System, Single Factor Model, Improved Simulated Annealing Algorithm, Extended-TSP Model.

1. Introduction

The Kariba Dam locates in the capital of Zambia, southeast about 300 kilometers. It stretches over the borders of two countries, Zambia and Zimbabwe. After constructing the Kariba Dam, it accomplished storing water from 1958 to 1963. Nowadays, it's also one of the most famous tourist spots in Zambia. However, it faces problems such as aging and decline of water storage. If we don't provide reasonable dams processing schemes as soon as possible, it will bring about disasters, endangering at least 3.5 million people's life and property living along the Zambezi River.

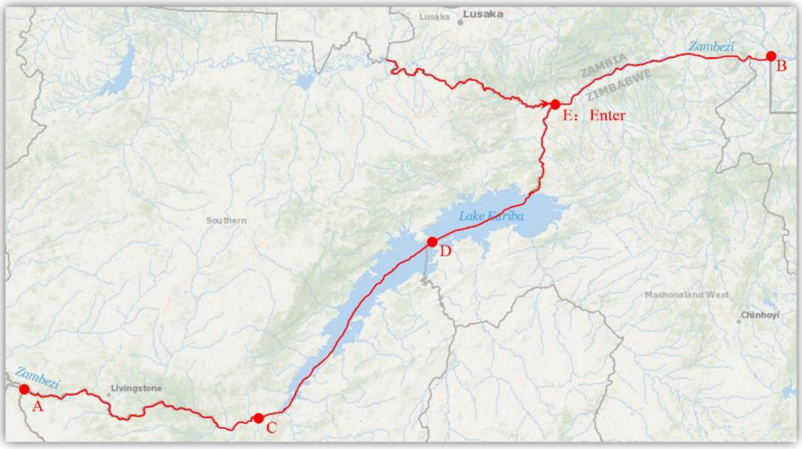

Fig1. Border between the two countries

2. Single Factor Model

From the perspective of building dam safely, we build our model under the circumstances that only water flow is considered[1]. Given that Lake Kariba is along the border between Zambia and Zimbabwe and serve for these two countries. Therefore, the location of small dams should be located in the waters near the border of these two countries. As is shown in Fig1, the border begin in Point A(25.621191,-17.792115) and end in Point B(30.421825,-15.621191). The water flow in Point A is $Q_A = 1749$ billion cubic meters. Since the tributary affect the water flow much more, we consider Kafue River, the main tributary of A and B.

We introduce ρ as insecurity degree of index to describe the safety degree of dams. The less ρ is, the safer the dam is. ρ is mainly decided by the water storage of the dams and the level of threat to the downstream. Here is how we conclude ρ .

Step1: Water storage of dam

When the dam building is of good quality, whether the dam is safe or not is decided by the amount of water it reserves. The less the amount of water is , the more dangerous the dam is. When the total amount is fixed, the further the distance is, the more water the next dam reserves. As is shown in Fig1, without thinking about the water flow of tributary, when the water flow is fixed in river entrance A, the reservoir storage q of dams which built in front of E is calculate as follows:

$$q_n = Q_A \times \frac{l_n}{L} \qquad \sum_{i=1}^{n} l_i \le L_1 \tag{1}$$

Where:

l_n represents the distance of the No.i dam and the No.i-1 dam.

L is the distance between A and B. $L = 797km$.

L_1 represents the distance between E and A. $L_1 = 557km$.

And then, we think about that the first dam should be build behind E, and the q, the reservoir storage of the dam consists of two parts. Q_B represents the water flow of tributary, and $Q_B = 74$ billion cubic meters. Here is how we calculate it:

$$q_n = Q_A \times \frac{\left(L_1 - \sum_{i=1}^{n-1} l_i\right)}{L} + \left(\sum_{i=1}^{n} l_i - L_1\right) \times \left(Q_A + Q_B\right) \tag{2}$$

As for the second dam and others built behind point E, the reservoir storage q is calculated as follows:

$$q_n = \left(Q_A + Q_B\right) \times \frac{l_n}{L} \qquad \sum_{i=1}^{n-1} l_i > L_1 \tag{3}$$

Therefore, the reservoir storage q_n is calculated as follows:

$$q_n = \begin{cases} Q_A \times \dfrac{l_n}{L} & \sum_{i=1}^{n} l_i \le L_1 \\ Q_A \times \dfrac{\left(L_1 - \sum_{i=1}^{n-1} l_i\right)}{L} + \left(\sum_{i=1}^{n} l_i - L_1\right) \times \left(Q_A + Q_B\right) & \sum_{i=1}^{n-1} l_i < L_1 \le \sum_{i=1}^{n} l_i \\ \left(Q_A + Q_B\right) \times \dfrac{l_n}{L} & \sum_{i=1}^{n-1} l_i > L_1 \end{cases} \tag{4}$$

Where:

l_n represents the distance of the ith dam and the i-1th dam.

L represents the distances of A and B.

L_1 represents the distances of E and A.

Q_B represents the water flow of the tributary.

Step2: The level of threat to the downstream

The level of threat to the downstream C mainly decided by the dam's location. As is shown in Fig1, suppose that there are two dams in C and D respectively, if the dam in D damaged, not only will it threat the dams behind D, but also it will threat the dam between C and D. Therefore, the closer the dam is to A, the much it threats and vice versa. So the level of threat C is positively related to the distance between B and the dam. We can represent it approximately as follows:

$$c_n = \mathrm{L} - \sum_{i=1}^{n} l_i \tag{5}$$

Step3： The calculation of insecurity degree ρ

Given that the dam's reservoir storage q has a different unit with the level of threat to the downstream c, we can't compute them directly. So non dimensional treatment are produced in analysis process:

$$a_n = \frac{q_n - min(q_n)}{max(q_n) - min(q_n)}$$

$$b_n = \frac{c_n - \min(c_n)}{\max(c_n) - \min(c_n)}$$

The two dimensional variable has nearly the same influence to ρ, and they are both in proportion to ρ .So we can calculate ρ :

$$\rho_n = a_n + b_n \tag{6}$$

2.1 Extended-TSP Model

We reform and extend the original TSP Model, to cope with the number and location of dams preliminary. The TSP Model is the Travelling salesman problem. The TSP Model is a basically line problem, as well as one of the most famous questions of Mathematics. The question is: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.The goal of selecting the route is to find the shortest possible route.

K dams are considered as k cities; Every ρ relating to its dam is considered as the distance between cities. If the ρ of one dam is higher, the distance of it should be further. Therefore, finding the minimum ρ of every dams equals to finding the shortest possible route[2].

We consider finding the locations as TSP problem:

$$\rho = min\sum \rho_n = min\sum (a_n + b_n) \tag{7}$$

We can find the locations of little dams through solving the TSP problem.

2.2 Improved Simulated Annealing Algorithm

Simulated annealing (SA) is a probabilistic technique for approximating the global optimum of a given function. Specifically, it is a metaheuristic to approximate global optimization in a large search space. It is often used when the search space is discrete (e.g., all tours that visit a given set of cities). For problems where finding an approximate global optimum is more important than finding a precise local optimum in a fixed amount of time, simulated annealing may be preferable to alternatives such as gradient descent[3].

Metropolis and his colleagues simulated the process of a material reaching thermal equilibrium at a in the constant problem

$$\Delta f = f\left(x\left(t+\Delta t\right)\right) - f\left(x\right) \le 0\left(\Delta t \ge 0\right) \quad (8)$$

We accept the new condition. Otherwise, we accept the condition at the probability $\mathrm{p}\left(\Delta f\right) = e^{-\Delta f/\mathrm{T}}$. $\mathrm{T} = \mathrm{T}\left(t\right)$ is a parametric variable and it descend as t increases, which equals to the temperature of annealing process [4]. The solution of this simulation question is likely to an physical annealing process, so we choose Metropolis algorithm. Controlling the cooling process appropriately to reach simulated annealing and get the optimized results, which is called simulated annealing [5]

We optimize that method and attain the result [6]. Here are what we operate:

Initial solution emerges:

$$x_n = \mathrm{L} \times \mathrm{rand}() \quad (9)$$

We use this formula to create a series of initial solutions.

In this formula rand () represents random numbers between 0-1 created by computer.

Improvement of the formula

In Simulated Annealing, a normal method is selecting one coordinate x_i randomly in a matrix x firstly, then creating new solutions by operating x_i. But that method is not suitable for our model. So we improve it.

To explain the process of improvement better, we introduce the selected probability. The selected probability is calculated as follows:

$$Q_{nmin} = \frac{\sum_{i=1}^{n-1}\left(\rho_i\right)}{\sum\left(\rho_i\right)} \quad (10)$$

$$Q_{nmax} = \frac{\sum_{i=1}^{n}\left(\rho_i\right)}{\sum\left(\rho_i\right)} \quad (11)$$

Where：

Q_n represents the range of probability of the i th little dam

Q_{min} is the lower boundary

Q_{max} is the upper boundary.

Then , a random number between 0 and 1 ,r, is created:

If $Q_{min} < r \le Q_{max}$, x_1 is selected;

If $Q_{mmin} < r \le Q_{m\,max}$, x_m is selected[7].

Eventually, we operate x_m as follows:

$$x_m = x_m \times StepFactor \times L \times rand \quad (12)$$

In the formula, StepFactor is step length to control the changing rate of new solutions. We set StepFactor=0.2.

Judge whether new solutions are qualified

Judge whether new solutions satisfy constraint condition. If so, execute next step; if not ,repeat it.

Judge whether replace current solution

Compute the objective function value f_1 of new solution. If $f_1 < f_0$, replace current solution x_0 with new solution x_1. Otherwise decide if accept new solutions by the proportion $e^{-\Delta t'/T}$.

Compute the objective function value

Compute the objective function value of current solution x_0. If it is within the tolerance, then the formula comes to an end and output the current solution; if not, repeat it and lower the temperature until reach a suitable solution[8].

Above all, the Simulated annealing optimization algorithm flowchart is shown as follows:

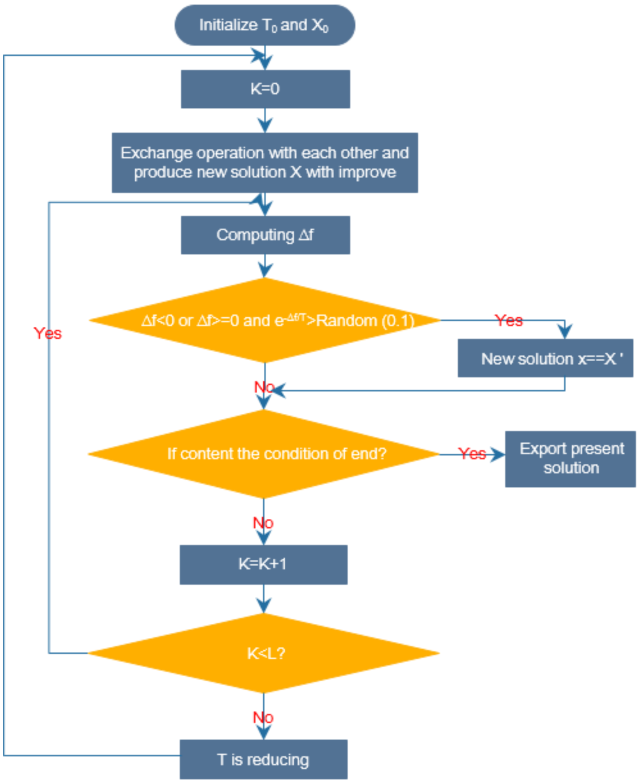

Fig2. Simulated annealing optimization algorithm flowchart

2.3 Model Results

We get the results by Matlab programing. Through analyzing the result, we found that the results remain consistent in the range that the error allows every time we run the program. Accordingly, from a side view,it proves that there is a multiple dam system that makes a minimum ρ .

Base on that, after removing the best geographical location, we get the second-best geographical location after we run the program again. And so on, we get 20 practicable geographical locations listed as followed:

Table1. Best location-related data

Number	Distance	Longitude(E)	Latitude(S)	Number	Distance	Longitude(E)	Latitude(S)
1	571.3	28.85	-16.07	11	423.71	28.11	-16.83
2	671.1	29.29	-15.74	12	459.63	28.38	-16.67
3	695.5	29.90	-15.62	13	398.17	27.91	-16.92
4	721.7	30.16	-15.61	14	334.35	27.57	-17.39
5	625.5	29.68	-15.64	15	356.71	27.65	-17.21
6	555.7	28.89	-15.64	16	276.34	27.19	-17.74
7	707.5	30.00	-15.63	17	210.61	26.74	-18.02
8	519.4	28.84	-16.39	18	122.71	26.08	-17.93
9	493.4	28.70	-16.55	19	103.91	25.98	-17.99
10	471.6	28.50	-16.62	20	76.47	25.85	-17.92

The point 6 is at the intersection of the Zambezi River and its tributary, so we can build the dam on the tributary.

3. Conclusion

We introduce the dam insecurity index and give the calculation formula of the index. Based on this formula, we extend and apply the TSP model to decide the reasonable numbers and locations of dams, and then enable the insecurity index perfect. we determine the optimal number and location of dams, and get the new multiple dam system.

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