Two and Three Dimensional Temperature Coupling Model based on Bathtub Design

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Abstract

In the ordinary bathtub, people will face water gradually cool . Therefore, it is proposed to continue to add water to the bath. We propose a model in order to maintain the water temperature .For the design of the shape and volume of the bathtub, the mathematical model of heat transfer is simulated, obtaining the distribution of temperature field.According to the analysis, when the water inlet Ann near the position of the foot at the bottom of the bathtub, an outlet in the head position, make the heat to maximize the use of. Then the water saving problem is converted into the least amount of heat dissipation problem. When the bathtub is a cylinder and the volume of 650 liters, using the least water. Finally, the use of fuzzy control algorithm to control the flow of hot water, making the water temperature control in the most appropriate range $(37^{\circ}\mathrm{C}-42^{\circ}\mathrm{C})$ **.**

Keywords

Two and three dimensional temperature coupling model; Linear programming ; Fuzzy control algorithm.

1. Introduction

Taking a shower is a part of people's daily life, but ordinary bathtub does not have secondary heating system and circular jet. With time going by, water gradually becomes cold. We must continuously put heating water into bathtub in order to adapt to temperature the body need. How to control water flow and choose the shape of bathtub is the problem we need to urgently resolve.

The process of constant-temperature's heat transferring is widely used in daily life, Zhang Huixin has sufficiently explained for a three-dimensional heat transfer model in the research of Mariculture temperature, 3D model and the study of control algorithm[1]. One model gives full consideration to the exchange of water heat, inflow, thermal diffusion, the influence of factors such as heat convection[2]. Feng Zhijie once used porous brick as the object of research, analyzing the similar bubble insulation heat preservation process[3]. This research involves various fields, and we will study the problem on the basis of predecessors[4].

2. Assumptions

Convective heat transfer between water surface and air, that is the third kinds of boundary conditions.

Water is a stable, incompressible viscous fluid.

Except for the density, the other physical properties are constants. And the Prandl number is 0.71.

The density of the momentum equation is only considered in terms of the volume force, and the density of the rest is considered as constant.

Regard the cross section of the bath as regular equilateral graphics.

3. Two or three dimensional temperature's coupling model based on bathtub design

3.1 Analysis of water temperature and heating system in bathtub

The Cause of flow regime and flow

The flow of fluid along the solid wall is two kinds of flow regimes, such as laminar flow and turbulent flow. When the turbulence is turbulent, the effect of convection is enhanced, and the heat transfer is better due to the eddy current disturbance. The heat transfer process of fluid can be divided into forced convection heat transfer and natural convection heat transfer.

Thermal physical properties of fluids

Because fluids have different types, their thermal properties are also different. Even if it is the same kind of fluid, the thermal physical properties will vary with the change of temperature and pressure. Physical property parameters which effect convective heat's transferring such as thermal conductivity λ, specific heat c, density ρ, internal friction μ and coefficient of cubical expansion α.

Geometrical factors of heat transfer surface

Geometric factors are involved in the wall size, roughness, shape and relative position of the fluid. They influence the heat exchange by effecting fluid flow on the wall , the velocity and temperature distribution.

In summary, for non-phase change heat transfer, factors that affect the h of the convective heat transfer coefficient can be expressed as:

$$h = f\left(u, t_w, t_f, \lambda, \rho, c_p, \mu, \alpha, l\right) \tag{1}$$

3.2 he solution of natural convection heat transfer coefficient

Heat transfer coefficient of water surface and air h_1

When the system is in a stable state, the water temperature of the bathtub remains constant. The temperature of water's surface is constant at $t_1 = 40^{\circ}C$;The temperature of air is t_2 .The bath water by natural convection heat dissipation of the Q_1 is equal to the hot water faucet to bath water heat exchange Q_2.

$$Q_2 = Q_1 = c\rho q\left(t_3 - t_4\right) \tag{2}$$

t_3 is the temperature that hot water get into the bath; t_4 is the temperature of bath water which overflow from bathtub; q is the flow rate of hot water in the unit time

Air quality temperature is $t_n = \left(t_1 + t_2\right)/2$, λ_2, v, P_r is obtained from the physical properties of air.

Grashof number:

$$Gr = \frac{g\alpha\Delta t l_0^3}{v^2} \tag{3}$$

Nusselt number:

$$Nu = 0.54Ra^{\frac{1}{4}} \tag{4}$$

$$h_1 = Nu\frac{\lambda_2}{l_0} \tag{5}$$

According to the following formula, the natural convection heat transfer coefficient of the surface air of the water can be obtained $h_1 = 2239W/(m^2 ℃)$

Test water surface temperature hypothesis: $t_1^{'} = t_2 + Q_2 / A_0 h_1 = 39.87^{\circ}C$ .The deviation between $t_1^{'}$ and t_1 is $0.325\% < 1\%$, So Hypothesis values. $h_1 = 2239W/(m^2 ℃)$ is acceptable.

A0 is contact area of water surface and air .g is acceleration of gravity;l0is bath width;v is kinematic viscosity.

Convection heat transfer coefficient of water and bathtub

According to Fourier's law:

$$q_x = -\lambda \left(\frac{\partial t}{\partial y} \right)_{y=0,x} \tag{6}$$

$\left(\frac{\partial t}{\partial y} \right)_{y=0,x}$ is temperature gradient of fluid in coordinate (1, 2).

According to Newton cooling formula:

$$q_x = h_x \left(t_w - t_\infty \right) [W/m^2] \tag{7}$$

$$Q_x = S_1 q_x \tag{8}$$

From the above, convection heat transfer differential equation:

$$h_x = -\frac{\lambda}{t_w - t_\infty} \frac{\partial t}{\partial y} |_{y=0,x} \tag{9}$$

3.3 wo or three dimensional temperature coupling model

Natural convection heat transfer of hot water and cold water Q_1

As for the heat transfer of natural convection, the basic governing equations are composed of three parts, such as continuity equation, momentum conservation equation and energy conservation equation. The model is a three-dimensional heat transfer process, the water temperature is mainly achieved by natural convection heat exchanging, and water in the bathtub without inner heat source, so under the model assumptions, three-dimensional natural convection for heat control equation is as follows:

Continuous equation:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{10}$$

Momentum conservation equation:

$$\rho\left(\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{11}$$

$$\rho\left(\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{12}$$

$$\rho\left(\frac{\partial w}{\partial \tau} + u\frac{\partial w}{\partial x} + u\frac{\partial w}{\partial y} + u\frac{\partial w}{\partial z} \right) = -\rho g - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \tag{13}$$

Energy conservation equation:

$$\frac{\partial t}{\partial \tau} + u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z} = \frac{\lambda}{\rho c}\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) \tag{14}$$

The （14）formula constitutes the control equation of cold water and hot water bath of natural convection heat transfer.

The heat transfer process of continuous hot water is a steady state process. At this time, the system reaches a steady state, and the temperature is no longer changed with time.

$$\frac{\partial t}{\partial \tau}=0$$

λ is thermal conductivity $\left[W/(m\cdot {}^{\circ}C)\right]$; g is acceleration of gravity $9.8\,m/s^2$

u Is velocity of water in X direction m/s ; v Is velocity of water in Y direction m/s ; w Is velocity of water in Z direction m/s ; μ Is dynamic viscosity coefficient $Pa\cdot s$.

Boundary conditions:

Inlet constantly put hot water into water flow as the heat source, the boundary conditions of temperature on the external wall of the bathtub are the calculation of temperature out of the bathtub wall t_0 ;The cylinder bottom can be approximately considered as adiabatic $-\lambda\frac{\partial t}{\partial z}|_{z=0}=0$;

The wall around the cylinder can also be approximately considered as adiabatic;

Water surface and indoor air convection transfer heat.

Initial condition:

Indoor air temperature is t ;

Hot water and cold water inlet and outlet temperature t_3 , t_4 ;

The initial temperature of water is t_0 ;

For the surface of the water body, the convection heat transfer and radiation heat transfer are carried out at the same time, the radiation in the form of heat dissipation, absorption and re emission of the indoor air are neglected. Convective heat transfer and radiation heat transfer can be considered to be independent of each other. Its formula is as follows:

Convective heat transfer:

$$Q_a=h_1A_0(t_0-t_1) \tag{15}$$

Radiant heat dissipation:

$$Q_b=\varepsilon\sigma A_0(t_0^4-t_1^4) \tag{16}$$

Because the water surface temperature is lower, the radiation heat radiation is very small compared with the convection heat radiation, so it can consider the heat radiation.

For the surface of water body, the boundary conditions are third kinds of boundary conditions:

$$-\lambda\frac{\partial t}{\partial \tau}|_{z=h}=h_1(t_0-t_1) \tag{17}$$

Above all, the governing equations and the definite conditions for the dynamic simulation of the three-dimensional water temperature and heat transfer system of the bathtub are formed, see Fig.1.

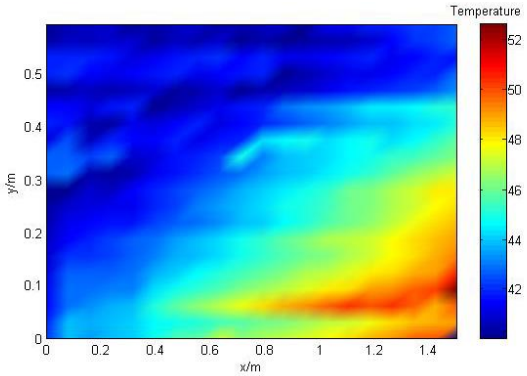

Fig.1 Distribution image of two dimensional temperature field

Convection heat transfer of water in the bathtub and the surface of the human body Q_b.

The contact area of the human body and the fluid can be approximately regarded as the surface area of the human body.

$$S_3 = 0.0061H_p + 0.012W_p - 0.0099 \tag{18}$$

According to the health department announced in 2015 the country's 18 year old and above (including the elderly) adult male and female average height were: 167.1 cm and 155.8 cm, the average weight of 66.2 kg and 57.3 kg. Therefore, according to the mass height and weight can be solved to regulate the body surface area.

The convective heat transfer between water and body surface heat dissipation, by Newton cooling formula:

$$Q_b = h_2 S_3 \left(t_p - t_l\right) \tag{19}$$

t_l is body temperature; h_2 is surface heat dissipation coefficient of the human body.

3.4 Determination of optimal bath size based on linear programming model

In real life the bathtub shape variety, in order to facilitate the study of approximation of the bathtub cross-sections as rules of equilateral graphics.

When the bath capacity of V_1 at H height of the bathtub, transverse length n and the number of transverse plane length R is constantly changing. Transverse section area:

$$S_1 = n \cdot R^2 \sin\left[\frac{(n-2)\pi}{2n}\right],\ n = 3,4,5\cdots \tag{20}$$

The area of the side wall of the bathtub is:

$$S_2 = nRH,\ n = 3,4,5\cdots \tag{21}$$

To sum up, the flow of water through various forms of heat transfer for overall heat loss:

$$Q = Q_3 S_1 + Q_1(S_1 + S_2) + Q_2 \tag{22}$$

With the conversion of minimum water into the overall heat dissipation problem of bath in which the objective function is minimum:

$$Q = Q_3 S_1 + Q_1(S_1 + S_2) + Q_2 \tag{23}$$

Constraint conditions are:

$$\begin{cases} V_1 = S_1 H \\ 0.58 \le H \le 0.9 \\ 0 \le R \le 0.9 \end{cases} \tag{24}$$

Solving model

By using the formula above, the calculation is obtained by calculating, see Table.1:

Table.1 Heat transfer calculation results chart

Q_1	Q_2	Q_3
10.725	3.569	61.8646

Through the lingo software programming to get the best size Fig.2.

Variable	Value	Reduced Cost
S1	1.0935884	0.000000
N	2106155	0.000000
R	0.0000008	0.000000
S2	2.7803695	0.000000
H	0.5963262	0.000000
V	0.6500000	0.000000

Fig.2 Results of linear programming

The lingo data were analyzed by $n \to \infty$, and the $R \to 0$ was informed that the transverse plane was approximately a circle, and its appearance was approximately a cylinder, see Fig.3.

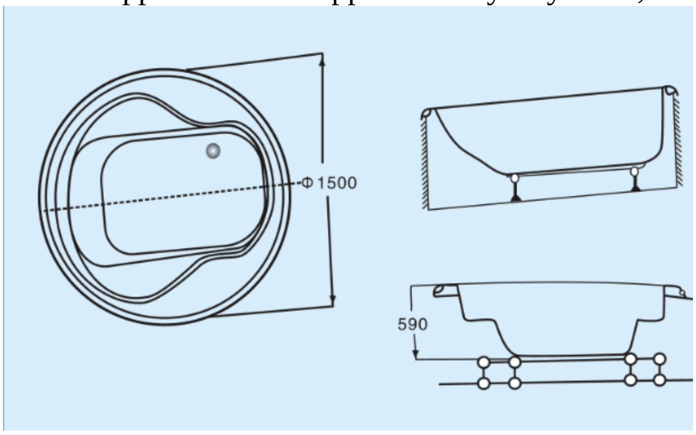

Fig.3 The sketch of the bathtub

4. Weaknesses and Strengths

The problem of how much water is converted into the problem of the size of the heat and the different shape of the bathtub to abstract into the rules of the column, so that the problem is more simple to solve.People's behavior may affect the size of the bath tub. This paper only considers the static state.

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