A Critical Case Analysis of the Equilibrium Point of Two-Dimensional Lotka-Volterra System

Yuanyuan Gu [a], Dan Zhang [b]

Southwest Petroleum University, Chengdu 610000, China

[a]guyuanyuan1010@126.com, [b]836699985@qq.com

Abstract

In this paper, the dynamic properties of the critical case of the equilibrium point for Lotka -Volterra neural network with two-dimensional are studied. By carrying out blow-up methods and studying the network parameters, the type of equilibrium point and the trajectory of solution curve near the equilibrium point are found. Finally,simulation examples are used to illustrate the theory developed in this paper.

Keywords

Higher Oder Singular Point, Blow-Up Methods, The Critical Case.

1. Introduction

In recent years, with the development of science and technology, mathematics has been widely used in fish fishing, neural network, food chain and population dynamics[1,2] and so on. For these problems, scholars mainly use mathematical theory and mathematical model to analyze the internal characteristics of complex system[3]. For example, the existence of periodic solution of population model can be obtained by using the theory of nonlinear analysis. Using the qualitative theory of ordinary differential equations, we can study the various dynamical properties of population model [4]. Automatic control theory, radio technology, rocket flight, missile launch, operation of the missile, vibration of the valve oscillator, chemical reaction stability, neural network, biotechnology, image processing, population, infectious diseases and financial problems can often be transformed as differential equations problems. Therefore, the study of differential equations is very important and practical significance. This paper focuses on the population dynamics system, in which Lotka -Volterra system is a very important model in biological mathematics. This system reflects the interaction between species. For example, parasite and hermit, predation and prey [1], coexistence relationship, reciprocity and mutual benefit relationship and so on belong to Lotka -Volterra model.

In this paper, we use blow-up methods study the critical case of the system, then giving the types of the equilibrium point in section II. At the same time, we describe the solution curve near the equilibrium point. In section III, simulation examples are used to illustrate the theory developed in this paper. Finally, conclusions are given in sectionIV.

2. Organization of the Text

2.1 Section Headings.

This paper studies the Lotka -Volterra model [5] described by the following:

$$\begin{cases} \dfrac{dx}{dt} = x\left(r_1 + a_{11}x + a_{12}y\right) \\ \dfrac{dy}{dt} = y\left(r_2 + a_{21}x + a_{22}y\right) \end{cases} \tag{1}$$

Where $\gamma_i, a_{ij}\left(i, j = 1,2\right)$ are constants. a_{11} and a_{22} reflect the density of the two populations of factors, known as intraspecific effect coefficient; a_{12} and a_{21} reflect the interaction between two species, known as the coefficient of interaction; r_1 and r_2 respectively two species of intrinsic growth rate.

The model (1) exists equilibrium points (0, 0), it is easy to verify that the equilibrium is hyperbolic equilibrium[6]. According to The Hartman-Grobman Theorem [3], we know, the nonlinear system (1) and its corresponding linear system have the same topologically structure in a neighborhood of the hyperbolic equilibrium point. For the linear system of the system (1), we get its eigenvalues r_1 and r_2. If the equilibrium point is the origin, one of the eigenvalues of the corresponding linear system is zero, in this case, we called the equilibrium point as a high order singularity [6].

For $r_1 = 0, r_2 > 0$, the system(1) can be written as

$$\begin{cases} \dfrac{dx}{dt} = x\left(a_{11}x + a_{12}y\right) \\ \dfrac{dy}{dt} = y\left(r_2 + a_{21}x + a_{22}y\right) \end{cases} . \tag{2}$$

In this case, we brow up the higher order singularity along the x-axis and y-axis [7]. The basic idea: the higher order singularity of a planar system is decomposed into several primary singular points by the coordinate transformation. And studying the distribution of the trajectories near these primary singularities and then shrinking them to a point. Finally we can get the trajectory structure near the high-order singular point. Based on the above ideas, we have the transformation:

$$\begin{cases} x = r\bar{x} \\ y = r\bar{y} \end{cases} \qquad r \in [0, \infty) \quad . \tag{3}$$

In the (3), respectively, let $\bar{x} = \pm 1$ and $\bar{y} = \pm 1$, The transformation (3) turn into the following four transformations:

$$\begin{cases} x = r \\ y = r\bar{y} \end{cases} (I), \quad \begin{cases} x = -r \\ y = r\bar{y} \end{cases} (II), \quad \begin{cases} x = r\bar{x} \\ y = r \end{cases} (III), \quad \begin{cases} x = r\bar{x} \\ y = -r \end{cases} (IV),$$

We call them as the x- direction coordinate card and the y-direction coordinate card. This is, the points which lie the x-y plane are turned to the points which lie the $r - \bar{y}$ plane or $r - \bar{x}$ plane. After the trajectories on the four coordinate cards are clearly studied, the information of these coordinate cards is transferred to the field outside the unit circle. Finally, the unit circumference is shrinked to a

By the transformation (I), we can get $\bar{y} = \dfrac{y}{x} = \tan\theta$, when $(r,\theta) \in [0,\delta] \times (-\dfrac{\pi}{2}, \dfrac{\pi}{2})$, transformation $\phi : (r,\theta) \to (r,\bar{y})$ map the right half cylinder into its tangent rectangle neighborhood $(r,\bar{y}) \in [0,\delta] \times (-\infty, +\infty)$. So we translate the calculation on the cylinder into the rectangle plane. In the near each elementary singular point, $\dot{r}$ gives the the normal direction of the trajectory and $\dot{\theta}$ gives the tangent direction of trajectory. Thereby the trajectory structure is obtained on the outside of the unit (r> 0). Finally shrinking the unit circle to the origin, we get the trajectory structure of original system (1) near the high order singular point (0,0).

a_{11} reflect intraspecific effect coefficient. Considering the actual situation, the individual is mutually exclusive as $t \to \infty$, so $a_{11} < 0$. Based on the above idea, we get the trajectories of the system (1) in the near singular point of higher order as shown in figure.1. Obviously, the equilibrium point is a saddle-node.

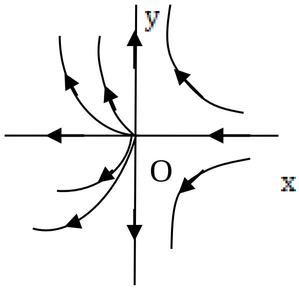

Fig.1 The trajectories of the system (1) in the near origin.

2.2 Simulation example

In this section, two simulation examples are presented to illustrate the theorems developed in the paper.

Example 1: consider a 2-D neural network, as the following:

$$\begin{cases} \dfrac{dx}{dt} = x(-x-2y) \\ \dfrac{dy}{dt} = y(2+3x+y) \end{cases} \quad (4)$$

It's solution trajectories in the near 0(0, 0) as shown in Figure. 2 by Maple.

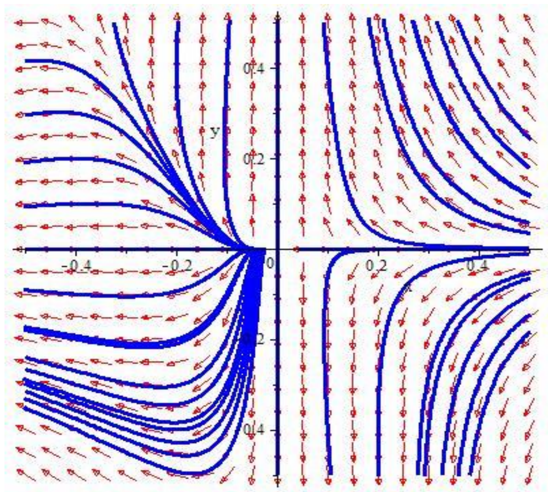

Fig.2 The trajectories of the system (4) in the near origin.

Example 2: consider a 2-D neural network, as the following:

$$\begin{cases} \dfrac{dx}{dt} = x(-2x-y) \\ \dfrac{dy}{dt} = y(1+3x+y) \end{cases} \quad (5)$$

The solution trajectories of the system (5) in the near 0(0, 0) as shown in Figure. 3.

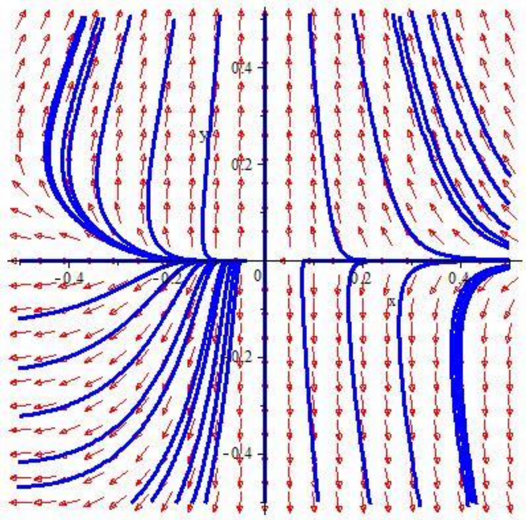

Fig.3 The trajectories of the system (3) in the near origin.

From above the two examples, we can see that the solution curve obtained by Maple is consistent with our expected theory.

3. Conclusion

In this paper, the structure of the solution curve near the equilibrium point of the L-V neural network is analyzed when the equilibrium point is the higher order singular point. Firstly, we decompose the higher order singularity into two primary singular points by blow-up technique. Studying the trajectories distribution of near these primary singular points, then they are shrinked to a point. Finally drawing the trajectory structure of higher order singular point according to the reversibility of polar coordinate transformation. However, this article only studies the situation of real number system, the complex system is not involved.

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References

[1] J. Zhen and Z. Ma, Stability for a competitive Lotka-Volterra system with delays, Nonlinear Analysis: TMA 51(2002), p. 1131-1142.

[2]Ju H. Park. Stability for a competitive Lotka-Volterra system with delays: LMI optimization approach, Appl. Math. Lett. 18(2005), p. 689-694.

[3]B. Kosko, Bi-directional associative memories, IEEE Trans. Syst. Man Cybern. 18(1988), p. 49-60.

[4] J. Hale, Theory of Functional Differential Equations, Springer, Heidelberg(1997).

[5]G. Gause, The Struggle for Existence, Dover, New York 1971.

[6]F. John, Differential Equations and Dynamical Systems: Linear Systems in R^2. New York: springer-verlag, 1991.

[7]F. Dumortir and J. Llibre, Qualitative Theory of Plannar Differential Systems. Berlin: Springer Verlag, 2006.