Risk Assessment of Property Insurance Company with 2-tuple Linguistic Information

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Abstract

In this paper, we investigate the linguistic multiple attribute group decision making problems for evaluating the residential building energy-saving with 2-tuple linguistic information. We utilize the 2-tuple power weighted average (2TPWA)operator and 2-tuple weighted average (2TWA) operator to aggregate the 2-tuple linguistic information corresponding to each alternative and get the overall value of the alternatives, then rank the and select the most desirable one(s). Finally, an illustrative example for risk assessment of property insurance company is given.

Keywords

Multiple Attribute Group Decision Making (MAGDM); 2-Tuple Linguistic Information; 2-Tuple Power Weighted Average (2TPWA) Operator; Property Insurance Company.

1. Introduction

Asset liability management is one of the core activities of insurance company, whose main function is to coordinate the relationship between the assets and liabilities, to reduce risk and improve enterprise value effectively. The theory of asset liability management has been relatively well developed overseas, and the techniques have been continuously improved and widely used in operation and management practice in insurance companies, and have got good effect. In China, research on asset liability management began late, but the output of the research has shown a vigorous development momentum. The research scope keeps expanding and the methodology innovates constantly through the communication with international peers and in combination with thorough studies in relation to China insurance industry characteristics. However, from the perspective of the research object, most of the available research focuses on asset liability management of long term business such as life insurance and pensions, whereas the research of asset liability management on general insurance is relatively rare. In recent years, general insurance industry in China has developed rapidly, and the size of assets and liabilities has expanded constantly. But the uncertainty and the risk resulting from insurance business and investment increases due to the international and domestic economic status, changes of investment environment and marketization of insurances rates, which makes general insurance companies take asset liability management seriously, so as to ease the company's business risk effectively. At the same time, China Insurance Regulatory Commission agrees that investment oriented insurance business can be conditionally operated by general insurance company to some extent, which increases the difficulty and complexity of general insurance company's asset liability management.

The evaluating problems of the residential building energy-saving with 2-tuple linguistic information is classical multiple attribute group decision making problems[1-15]. In this paper, we investigate the linguistic multiple attribute group decision making problems for evaluating the residential building energy-saving with 2-tuple linguistic information. We utilize the 2-tuple power weighted average (2TPWA) operator and 2-tuple weighted average (2TWA) operator to aggregate the 2-tuple linguistic information corresponding to each alternative and get the overall value of the alternatives,

then rank the and select the most desirable one(s). Finally, an illustrative example for risk assessment of property insurance company is given.

2. Preliminaries

2.1 Power aggregation operator

Yager[16] developed a nonlinear weighted average aggregation operator called power average (PA) operator, which can be defined as follows:

$$PA(a_1, a_2, \cdots, a_n) = \frac{\sum_{i=1}^{n} (1 + T(a_i))a_i}{\sum_{i=1}^{n} (1 + T(a_i))}$$
(1)

where $T(a_i) = \sum_{j=1, j \neq i}^{n} Sup(a_i, a_j)$, and Sup(a, b) is the support for *a* from *b*, which satisfies the

following three properties:

$$(1) Sup(a,b) \in [0,1];$$

$$(2) Sup(a,b) = Sup(b,a);$$

(3) $Sup(a,b) \ge Sup(x,y)$, if |a-b| < |x-y|.

Obvoiusly, the support (Sup) measure is essentially a similarity index. The more similar, the closer two values, and the more they support each other. The PA operator is the nonlinear weighted aggregation tools, whose weighting vectors depend upon the input values and allow values being aggregated to support and reinforce each other, that's to say, the closer a_i and a_j , the more similar they are, and the more they support each other.

2.2 The linguistic 2-tuple representation model

Let $S = \{s_i | i = 1, 2, \dots, t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics[17-21]:

(1) The set is ordered: $s_i > s_j$, if i > j; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \ge s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \le s_j$. For example, S can be defined as

$$S = \{s_1 = extremely \ poor(EP), s_2 = very \ poor(VP), \\ s_3 = poor(P), s_4 = medium(M), s_5 = good(G), \end{cases}$$

$$s_6 = very \ good(VG), s_7 = extremely \ good(EG)$$

Herrera and Martinez[17] developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, α_i) , where s_i is a linguistic label from predefined linguistic term set *S* and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5, 0.5)$.

Definition 1. Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set *S*, i.e., the result of a symbolic aggregation operation, $\beta \in [1,t]$, being *t* the cardinality of *S*. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [1,t]$ and $\alpha \in [-0.5, 0.5)$ then α is called a symbolic translation [17-21].

Definition 2. Let $S = \{s_1, s_2, \dots, s_t\}$ be a linguistic term set and $\beta \in [1, t]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta: [1,t] \to S \times [-0.5, 0.5) \tag{2}$$

$$\Delta(\beta) = \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5] \end{cases}$$
(3)

where round(.) is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation[17-21].

Definition 3. Let $S = \{s_1, s_2, \dots, s_t\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a function Δ^{-1} can be defined, such that, from a 2-tuple (s_i, α_i) it return its equivalent numerical value $\beta \in [1, t] \subset R$, which is [17-21].

$$\Delta^{-1}: S \times \left[-0.5, 0.5\right) \rightarrow \left[1, t\right] \tag{4}$$

$$\Delta^{-1}(s_i,\alpha) = i + \alpha = \beta \tag{5}$$

From Definitions 1 and 2, we can conclude that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$\Delta(s_i) = (s_i, 0) \tag{6}$$

Definition 4. Let (s_k, a_k) and (s_l, a_l) be two 2-tuple, they should have the following properties [17-21].

(1) If k < l then (s_k, a_k) is smaller than (s_l, a_l)

(2) If k = l then, (a) if $a_k = a_l$, then (s_k, a_k) , (s_l, a_l) represents the same information; (b) if $a_k < a_l$ then (s_k, a_k) is smaller than (s_l, a_l) ; (c) if $a_k > a_l$ then (s_k, a_k) is bigger than (s_l, a_l) .

Definition 5[17-21]. A 2-tuple negation operator.

$$neg(s_i,\alpha) = \Delta(t+1-(\Delta^{-1}(s_i,\alpha)))$$
(7)

where *t* is the cardinality of *S*, $S = \{s_1, s_2, \dots, s_t\}$.

In the following, Wei[22] developed some 2-tuple power aggregation operators, which allow the input data to support each other in the aggregating process.

Definition 6[22]. Let $x = \{(r_1, a_1), (r_2, a_2), \dots, (r_n, a_n)\}$ be a set of 2-tuple and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weighting vector of 2-tuple (r_i, a_i) $(i = 1, 2, \dots, n)$ and $\omega_i \in [0, 1]$, $\sum_{i=1}^n \omega_i = 1$, then we define the 2-top $(i = 1, 2, \dots, n)$ and $\omega_i \in [0, 1]$.

2-tuple power weighted average (2TPWA) operator as follows: $(\tilde{z}, \tilde{z}) = 2TPWA ((z, z)) (z, z) = 0$

$$(r,a) = 2TPWA_{\omega}((r_{1},a_{1}),(r_{2},a_{2}),...,(r_{n},a_{n}))$$

$$= \Delta \left(\frac{\sum_{i=1}^{n} \omega_{i}(1 + \Delta^{-1}(T(r_{i},a_{i})))\Delta^{-1}(r_{i},a_{i})}{\sum_{i=1}^{n} \omega_{i}(1 + \Delta^{-1}(T(r_{i},a_{i})))}\right)$$

$$\tilde{r} \in S, \tilde{a} \in [-0.5, 0.5)$$
(8)

where

$$T(r_i, a_i) = \Delta \left(\sum_{\substack{j=1\\j \neq i}}^n \omega_j Sup((r_i, a_i), (r_j, a_j)) \right)$$
(9)

and $Sup((r_i, a_i), (r_j, a_j))$ is the support for (r_i, a_i) from (r_j, a_j) , with the conditions:

$$Sup((r_i, a_i), (r_j, a_j)) \in [0,1];$$

$$Sup((r_i, a_i), (r_j, a_j)) = Sup((r_j, a_j), (r_i, a_i));$$

$$Sup((r_i, a_i), (r_j, a_j)) \ge Sup((r_s, a_s), (r_t, a_t)), \text{ if } d((r_i, a_i), (r_j, a_j)) \le d((r_s, a_s), (r_t, a_t)), \text{ where } d \text{ is a a distance measure.}$$

Especially, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$, then the 2TPWA operator reduces to a 2-tuple power average (2TPA) operator:

$$(\tilde{r}, \tilde{a}) = 2TPA_{\omega}((r_{1}, a_{1}), (r_{2}, a_{2}), \dots, (r_{n}, a_{n}))$$

$$= \Delta \left(\frac{\sum_{i=1}^{n} (1 + \Delta^{-1}(T(r_{i}, a_{i}))) \Delta^{-1}(r_{i}, a_{i}))}{\sum_{i=1}^{n} (1 + \Delta^{-1}(T(r_{i}, a_{i})))} \right)$$

$$\tilde{r} \in S, \tilde{a} \in [-0.5, 0.5)$$

$$(10)$$

where

$$T(r_i, a_i) = \Delta \left(\frac{1}{n} \sum_{\substack{j=1\\j \neq i}}^n Sup((r_i, a_i), (r_j, a_j)) \right)$$
(11)

It can be easily proved that the 2TPWA operator has the following properties[22]. Theorem 1 . (Commutativity).

$$2TPWA_{\omega}((r_{1}, a_{1}), (r_{2}, a_{2}), \dots, (r_{n}, a_{n}))$$

= 2TPWA_{\omega}((r_{1}', a_{1}'), (r_{2}', a_{2}'), \dots, (r_{n}', a_{n}'))

where $((r_1', a_1'), (r_2', a_2'), \dots, (r_n', a_n'))$ is any permutation of $((r_1, a_1), (r_2, a_2), \dots, (r_n, a_n))$.

Theorem 2. (Idempotency) If $((r_1, a_1), (r_2, a_2), \dots, (r_n, a_n)) = (r, a)$, then

$$TPWA_{\omega}((r_1, a_1), (r_2, a_2), \dots, (r_n, a_n)) = (r, a)$$

Theorem 3. (Boundedness).

 $\min_{i} (r_{i}, a_{i}) \leq 2TPWA_{\omega}((r_{1}, a_{1}), (r_{2}, a_{2}), \dots, (r_{n}, a_{n})) \leq \max_{i} (r_{i}, a_{i})$

3. An approach to multiple attribute group decision making with 2-tuple linguistic Information

In this section, we shall utilize the power aggregation operators to multiple attribute group decision making. For a multiple attribute group decision making problems with linguistic information, let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, $G = \{G_1, G_2, \dots, G_n\}$ be the set of attributes, whose weight vector is $\omega = (\omega_1, \omega_2, \dots, \omega_n)$, with $\omega_j \ge 0$, $j = 1, 2, \dots, n$, $\sum_{i=1}^n \omega_j = 1$, and let $D = \{D_1, D_2, \dots, D_t\}$

be the set of decision makers, whose weight vector is $v = (v_1, v_2, \dots, v_s) \in H$, with $v_k \ge 0, k = 1, 2, \dots, s$, $\sum_{k=1}^{s} \lambda_k = 1$. Suppose that $R_k = (r_{ij}^{(k)})_{m \times n}$ is the multiple attribute group decision making matrix, where $r_{ij}^{(k)} \in S$ is an attribute values, which take the form of linguistic variable, given by the decision maker $D_k \in D$, for the alternative $A_i \in A$ with respect to the attribute $G_i \in G$.

Then, we utilize the 2TPWA operator to develop an approach to multiple attribute group decision making problems with linguistic information, which can be described as following:

Step 1. Transforming linguistic decision matrix $R_k = (r_{ij}^{(k)})_{m \times n}$ into 2-tuple linguistic decision matrix $\tilde{R}_k = (r_{ij}^{(k)}, 0)_{m \times n}$

Step 2. Calculate the support measure as follows:

$$Sup((r_{ij}^{(k)}, 0), (r_{ij}^{(l)}, 0)) = 1 - d((r_{ij}^{(k)}, 0), (r_{ij}^{(l)}, 0)), \ l = 1, 2, \cdots, s$$
(12)

which satisfy the support conditions 1)-3) in section 3. Here we calculate $d\left(\Delta^{-1}\left(r_{ij}^{(k)},0\right),\Delta^{-1}\left(r_{ij}^{(l)},0\right)\right)$ with distance as following:

$$d\left(\left(r_{ij}^{(k)},0\right),\left(r_{ij}^{(l)},0\right)\right) = \frac{\left|\Delta^{-1}\left(r_{ij}^{(k)},0\right) - \Delta^{-1}\left(r_{ij}^{(l)},0\right)\right|}{t} \ l = 1, 2, \cdots, s$$
(13)

Step 3. Utilize the weights $v = (v_1, v_2, \dots, v_s)$ of the decision maker D_k $(k = 1, 2, \dots, s)$ to calculate the weighted support $T(r_{ij}^{(k)}, 0)$ of the 2-tuple linguistic preference value $(r_{ij}^{(k)}, 0)$ by the other 2-tuple linguistic preference value $(r_{ij}^{(l)}, 0)$ for the preference value $r_{ij}^{(l)}$ $(l = 1, 2, \dots, s, and l \neq k)$:

$$T(r_{ij}^{(k)}, 0) = \sum_{\substack{l=1\\l \neq k}}^{s} \nu_l Sup((r_{ij}^{(k)}, 0), (r_{ij}^{(l)}, 0))$$
(14)

And calculate the weights $v_{ij}^{(k)}(k=1,2,\dots,s)$ of the 2-tuple linguistic preference value $(r_{ij}^{(k)},0)(k=1,2,\dots,s)$:

$$v_{ij}^{(k)} = \frac{v_k \left(1 + T\left(r_{ij}^{(k)}, 0\right) \right)}{\sum_{k=1}^{s} v_k \left(1 + T\left(r_{ij}^{(k)}, 0\right) \right)}, \ k = 1, 2, \cdots, t.$$
(15)

Where $v_{ij}^{(k)} \ge 0, k = 1, 2, \dots, s$, and $\sum_{k=1}^{s} v_{ij}^{(k)} = 1$.

Step 4. Utilize the decision information given in matrix \tilde{R}_k , and the 2TPWA operator

$$\tilde{r}_{ij} = 2TPWA((r_{ij}^{(1)}, 0), (r_{ij}^{(2)}, 0), \cdots, (r_{ij}^{(s)}, 0))$$

$$= \Delta \left(\sum_{k=1}^{s} \frac{\nu_{k} \left(1 + T(r_{ij}^{(k)}, 0) \right) \Delta^{-1}(r_{ij}^{(s)}, 0)}{\sum_{k=1}^{s} \nu_{k} \left(1 + T(r_{ij}^{(k)}, 0) \right)} \right)$$

$$i = 1, 2, \cdots, m, j = 1, 2, \cdots, n.$$
(16)

to aggregate all the individual decision matrices $\tilde{R}_k (k = 1, 2, \dots, s)$ into the collective decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m \times n} = (r_{ij}, a_{ij})_{m \times n}$, where $v = \{v_1, v_2, \dots, v_s\}$ be the weighting vector of decision makers.

Step 5. Aggregate all 2-tuple linguistic preference value $(r_{ij}, a_{ij})(j = 1, 2, \dots, n)$ by using the 2-tuple weighted average (2TWA) operator:

$$\tilde{r}_{i} = 2TWA((r_{i1}, a_{i1}), (r_{i2}, a_{i2}), \cdots, (r_{in}, a_{in}))$$

$$= \Delta \left(\sum_{j=1}^{n} w_{j} \Delta^{-1}(r_{ij}, a_{ij})\right)$$

$$i = 1, 2, \cdots, m.$$
(17)

to derive the overall 2-tuple linguistic preference values $\tilde{r}_i (i = 1, 2, \dots, m)$ of the alternative A_i , where $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weighting vector of the attributes.

Step 6. Rank all the alternatives A_i ($i = 1, 2, \dots, m$) in accordance with the overall 2-tuple linguistic preference values \tilde{r}_i ($i = 1, 2, \dots, m$) of the alternative A_i . The larger the overall preference values \tilde{r}_i , the better the alternative A_i will be.

4. Numerical example

With the improvement of economic and financial environment, China's insurance industry is facing a stage of rapid growth. Owing to the specialcharacteristic of insurance and the large scale of premium, insurance industryappears a surge of capital increase, in order that the companies can have enough capital to satisfy their own development aims, the requirment of regulators and rating organizations. Under this condition, all the insurance companies muststrengthen the capital management, improve the utilization efficiency of capital, better off the capital allocation. The importmance is that, companies need to seek thefinancing channels, plan for financing in advance to prenvent themselves fromfalling into insolvency. Because capital management is a complicated systemsengineering, so insurance companies must analyse on the whole to realize the objectives which are mutual independent, incompatible, complementary. Based on he analysis of the current capital management conditions, this paper combines thecapital management theory with the multi-objective planning models to improve theability of capital management. In this section, we investigate the linguistic multiple attribute group decision making problems for evaluating the residential building energy-saving with 2-tuple linguistic information. There is a panel with five possible property insurance companies to evaluate the risk according to the following four attributes: $(1)G_1$ is the interest rate risk; $(2)G_2$ is the inflation risk; $(3)G_3$ is the catastrophe risk; $(4)G_4$ is the other business-environment risk. The five possible alternatives A_i (i = 1, 2, 3, 4, 5) are to be evaluated using the linguistic term set S by three decision makers $D_k(k=1,2,3)$ (whose weighting vector v = (0.4, 0.3, 0.3) under the above four attributes (whose weighting vector $\omega = (0.3, 0.1, 0.2, 0.4)$), and construct, respectively, the linguistic decision matrices are shown in Table 1-3:

	G_1	G_2	G ₃	G ₄
A ₁	VP	VP	Р	М
A ₂	G	G	VP	EP
A ₃	VG	EG	VG	G
A ₄	VP	Μ	EG	VP
A ₅	М	VP	Р	VP

Table 1. Decision matrix \tilde{R}_1

	1 a01	le 2. Decision matri		
	G_1	G_2	G ₃	G_4
A1	Р	Р	Μ	VP
A ₂	М	Р	Р	VP
A ₃	G	EG	G	EG
A ₄	Р	G	VG	Р
A5	VP	М	EG	EP

Table 2. Decision matrix \tilde{R}_2

Table 3. Decision matrix \tilde{R}_3

			5	
	G1	G ₂	G ₃	G4
A_1	VP	М	G	Р
A_2	Р	G	VP	G
A ₃	G	EG	VG	EG
A_4	EG	VP	G	VG
A_5	М	VP	Μ	VP

If the weights of the decision makers are known, we shall utilize the following steps to select the most desirable property insurance company:

Step 1. Utilize (12)-(14) to calculate the weight $v_{ij}^{(k)}$ (i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4, k = 1, 2, 3) associated with the attribute values $(r_{ij}^{(k)}, 0)(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4, k = 1, 2, 3)$, which are expressed in the matrices $V^{(k)} = (v_{ij}^{(k)})_{5\times4}$ (k = 1, 2, 3) which are given in Table 4-6, respectively.

	G ₁	G_2	G ₃	G_4	
A1	0.4077	0.3883	0.3933	0.4024	
A ₂	0.3753	0.3755	0.3730	0.3806	
A ₃	0.3806	0.3933	0.3933	0.3843	
A4 0.3933 0.3754 0.3806 0.4024					
A ₅	0.3893	0.3855	0.3807	0.3934	
Table 5 Weight matrix $U^{(2)}$					

Table 4. Weight matrix $V^{(1)}$

Table 5. Weight matrix $V^{(2)}$					
	G_1	G_2	G ₃	G_4	
A1	0.3151	0.3175	0.3033	0.3109	
A ₂	0.3124	0.3124	0.3076	0.3097	
A ₃	0.3097	0.3033	0.3033	0.3022	
A4	0.3033	0.3124	0.3098	0.3109	
A5	0.3007	0.3072	0.3097	0.2922	

	G ₁	G_2	G ₃	G_4
A1	0.2773	0.2947	0.3033	0.2867
A ₂	0.3125	0.3123	0.3197	0.3097
A ₃	0.3097	0.3033	0.3033	0.3134
A4	0.3038	0.3128	0.3097	0.2867
A5	0.3103	0.3072	0.3097	0.3145

Table 6. Weight matrix $V^{(3)}$

Step 3. Utilizing the 2TPWA operator to aggregate all the individual decision matrices into the collective decision matrix, the aggregating results are shown in Table 7.

	G_1	G_2	G ₃	G_4		
A ₁	(M, -0.22)	(M, -0.24)	(VG, 0.07)	(M, -0.33)		
A ₂	(P, 0.26)	(P, -0.21)	(G, -0.12)	(VP, -0.24)		
A ₃	(VP, 0.34)	(P, 0.04)	(M, 0.12)	(P, -0.13)		
A ₄	(M, 0.08)	(M, 0.27)	(VP, 0.21)	(P, -0.25)		
A5	(G, 0.34)	(EG, 0.01)	(VG,-0.29)	(VG,0.49)		

Table 7. Decision matrix \tilde{R} (2TPWA)

Step 4. By utilizing the decision information given in Table 7, and the 2TWA operators, $\omega = (0.3, 0.1, 0.2, 0.4)$ is the weighting vector of the attributes, we derive the overall preference values of the attractives. The accuractions must are shown in Table 10.

of the alternatives. The aggregating results are shown in Table 10.

Table 10. The overall preference values of the property insurance companies

	A ₁	A_2	A ₃	A_4	A5
2TPWA and 2TWA	(P,0.23)	(P, 0.36)	(VG, 0.25)	(M, 0.17)	(P, 0.29)

Step 5. According to the aggregating results shown in Table 10, the ordering of the alternatives are shown in Table 11. Note that > means "preferred to". As we can see, depending on the aggregation operators used, the ordering of the alternatives is slightly different. Therefore, depending on the aggregation operators used, the results may lead to different decisions. However, the best alternative is A_3 .

Table 11. Ordering of the alternative

	Ordering
2TPWA and 2TWA	$A_3 > A_4 > A_2 > A_5 > A_1$

5. Conclusion

In this paper, based on the ideal of power aggregation, we have introduced the 2-tuple power weighted average (2TPWA) operator. The prominent characteristic of these four operators is that they take into account information about the relationship between the 2-tuple linguistic variables being aggregated. Then, we have utilized this operator to develop an approach to solve the linguistic multiple attribute group decision making problems with the known weights or completely unknown weights information of decision makers. Finally, an illustrative example for risk assessment of property insurance company is given to verify the developed approach and to demonstrate its practicality and effectiveness. The prominent characteristic of the developed approaches is that they

can take all the decision arguments and their relationships into account. Our operators could usefully be applied to many other areas such as data mining, information retrieval, and pattern recognition, which we suggest are the possible paths for future research. In the future, we shall extend the proposed modles to other domains.

Acknowledgements

The work was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People's Republic of China (15XJA630006).

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