Model for Neutrosophic Multiple Attribute Decision Making and Their Application to Credit Risk Evaluation of Small New Venture' Indirect Financing

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Abstract

With respect to single-valued neutrosophic multiple attribute decision making problems with completely unknown weight information, some operational laws of single-valued neutrosophic numbers, score function and accuracy function of single-valued neutrosophic numbers are introduced. To determine the attribute weights, a model based on the information entropy, by which the attribute weights can be determined, is established. We utilize the single-valued neutrosophic weighted averaging (SVNWA) operator to aggregate the single-valued neutrosophic information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example for evaluating the credit risk of small new venture' indirect financing is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

Multiple Attribute Decision Making; Single-Valued Neutrosophic Number; Single-Valued Neutrosophic Weighted Averaging (SVNWA) Operator; Entropy; Higher School Class Teaching Quality.

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [3]. The intuitionistic fuzzy set has received more and more attention since its appearance. Gau and Buehrer [4] introduced the concept of vague set. But Bustince and Burillo [5] showed that vague sets are intuitionistic fuzzy sets. In [6], Xu developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information. In [7], Xu developed some arithmetic aggregation operators, such as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid aggregation (IFHA) operator.

Using the degree of indeterminacy/neutrality as independent component in 1995, Smarandache[8] initiated the neutrosophic set theory. But, a neutrosophic set will be difficult to apply in real scientific and engineering fields. Therefore, Wang et al. [9-10] proposed the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) which are an instance of a neutrosophic set, and provided set-theoretic operators and various properties of SVNSs and INSs. Recently, the theory of neutrosophic set has received more and more attentions. Zhang et al. [11] proposed some neutrosophic aggregation operators, such as the interval neutrosophic weighted averaging (INWA) operator and the interval neutrosophic weighted geometric (INWG) operator, and applied the operators to solve the multiple attribute group decision-making problems with interval neutrosophic information.

In the process of MADM with single-valued neutrosophic information, sometimes, the attribute values take the form of single-valued neutrosophic numbers, and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. All of the above methods, however, will be unsuitable for dealing with such situations. Therefore, it is necessary to pay attention to this issue. The aim of this paper is to develop another method, based on the information entropy method, to overcome this limitation. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to single-valued neutrosophic sets. In Section 3 we introduce the MADM problem with single-valued neutrosophic information, in which the information about attribute weights is completely unknown, and the attribute values take the form of single-valued neutrosophic numbers. To determine the attribute weights, a model based on the information entropy method, by which the attribute weights can be determined, is established. We utilize the single-valued neutrosophic weighted averaging (SVNWA) operator to aggregate the i single-valued neutrosophic information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. In Section 4, an illustrative example for evaluating the credit risk of small new venture' indirect financing is pointed out. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

In the subsection, we give some concepts related to neutrosophic sets and single-valued neutrosophic sets.

Definition 1[8]. Let X be a universe of discourse, then a neutrosophic sets is defined as:

$$A=\left\{\left\langle x,T_A(x),\ I_A(x),\ F_A(x)\right\rangle \mid x\in X\right\} \tag{1}$$

which is characterized by a truth-membership function $T_A : X \to \left]0^-,1^+\right[$, an indeterminacy-membership function $I_A : X \to \left]0^-,1^+\right[$, a falsity-membership function $F_A : X \to \left]0^-,1^+\right[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le T_A(x)+I_A(x)+F_A(x)\le 3^+$.

In the following, we adopt the representations $\mu_A(x)$, $\rho_A(x)$ and $\nu_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively.

Wang et al. [9] defined the single-valued neutrosophic set which is an instance of neutrosophic set as follows:

Definition 2[9]. Let X be a universe of discourse, then a single-valued neutrosophic set is defined as:

$$A=\left\{\left\langle x,\mu_A(x),\ \rho_A(x),\ \nu_A(x)\right\rangle \mid x\in X\right\} \tag{2}$$

where $\mu_A : X \to [0,1]$, $\rho_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with $0 \le \mu_A(x)+\rho_A(x)+\nu_A(x) \le 3$ for all $x\in X$. The values $\mu_A(x)$, $\rho_A(x)$ and $\nu_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of x to A, respectively.

We will denote the set of all the SVNSs in X by Q. a single-valued neutrosophic number(SVNN) is denoted by $\tilde{a}=(\mu,\rho,\nu)$ for convenience.

Definition 3[12]. Let $\tilde{a}=(\mu,\rho,\nu)$ be a single-valued neutrosophic number, a score function S of a single-valued neutrosophic number can be represented as follows:

$$S(\tilde{a})=\frac{1+\mu-2\rho-\nu}{2},\ S(\tilde{a})\in[-1,1]. \tag{3}$$

Definition 4. Let $\tilde{a}=(\mu,\rho,\nu)$ be a single-valued neutrosophic number, an accuracy function H of a single-valued neutrosophic number can be represented as follows:

$$H(\tilde{a})=\frac{1+\mu-\rho(1-\mu)-\nu(1-\rho)}{2},\ H(\tilde{a})\in[0,1]\ . \tag{4}$$

to evaluate the degree of accuracy of the single-valued neutrosophic number $\tilde{a}=(\mu,\rho,\nu)$, where $H(\tilde{a})\in[0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the single-valued neutrosophic number $\tilde{a}$.

Sahin & Liu [12] showed that the relation between the score function S and the accuracy function H is similar to the relation between mean and variance in statistics. Based on the score function S and the accuracy function H, in the following, Sahin & Liu [12] gave an order relation between two single-valued neutrosophic numbers, which is defined as follows:

Definition 5[12]. Let $\tilde{a}_1=(\mu_1,\rho_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\rho_2,\nu_2)$ be two single-valued neutrosophic numbers, $S(\tilde{a}_1)=\frac{1+\mu_1-2\rho_1-\nu_1}{2}$ and $S(\tilde{a}_2)=\frac{1+\mu_2-2\rho_2-\nu_2}{2}$ be the scores of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, and let $H(\tilde{a}_1)=\frac{1+\mu_1-\rho_1(1-\mu_1)-\nu_1(1-\rho_1)}{2}$ and $H(\tilde{a}_2)=\frac{1+\mu_2-\rho_2(1-\mu_2)-\nu_2(1-\rho_2)}{2}$ be the accuracy degrees of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, then if $S(\tilde{a}_1)<S(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1<\tilde{a}_2$; if $S(\tilde{a}_1)=S(\tilde{a}_2)$, then

if $H(\tilde{a}_1)=H(\tilde{a}_2)$, then $\tilde{a}_1$ and $\tilde{a}_2$ represent the same information, denoted by $\tilde{a}_1=\tilde{a}_2$; (2) if $H(\tilde{a}_1)<H(\tilde{a}_2)$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}_1<\tilde{a}_2$.

Definition 6[11]. Let $\tilde{a}_j=(\mu_j,\rho_j,\nu_j)(j=1,2,\cdots,n)$ be a collection of single-valued neutrosophic numbers, and let SVNWA: $Q^n\to Q$, if

$$\text{SVNWA}_{\omega}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\sum_{j=1}^{n}\omega_j\tilde{a}_j=\left(1-\prod_{j=1}^{n}(1-\mu_j)^{\omega_j},\prod_{j=1}^{n}\rho_j^{\omega_j},\prod_{j=1}^{n}\nu_j^{\omega_j}\right) \tag{5}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then SVNWA is called the single-valued neutrosophic weighted averaging (SVNWA) operator.

3. Model for Single-Valued Neutrosophic Multiple Attribute Decision Making Problems

The following assumptions or notations are used to represent the single-valued neutrosophic MADM problems with entropy weight information:

(1) The alternatives are known. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives;

(2) The attributes are known. Let $G=\{G_1,G_2,\cdots,G_n\}$ be a set of attributes;

(3)The information about attribute weights is incompletely known. Let $w=(w_1,w_2,\cdots,w_n)\in H$ be the weight vector of attributes, where $w_j\geq 0$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}w_j=1$.

Suppose that $\tilde{R}=(\tilde{r}_{ij})_{m\times n}=(\mu_{ij},\rho_{ij},\nu_{ij})_{m\times n}$ is the single-valued neutrosophic decision matrix, where μ_{ij} indicates the truth-membership that the alternative A_i satisfies the attribute G_j given by the decision maker, ρ_{ij} indicates the indeterminacy-membership degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, ν_{ij} indicates the falsity-membership degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij} \in [0,1]$, $\rho_{ij} \in [0,1]$, $\nu_{ij} \in [0,1]$, $0 \le \mu_{ij} + \rho_{ij} + \nu_{ij} \le 3$, $i = 1,2,\cdots,m$, $j = 1,2,\cdots,n$.

Definition 7. Let $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = (\mu_{ij}, \rho_{ij}, \nu_{ij})_{m\times n}$ be a single-valued neutrosophic decision matrix, $\tilde{r}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{in})$ be the vector of attribute values corresponding to the alternative A_i, $i = 1,2,\cdots,m$, then we call

$$\tilde{r}_i = (\mu_i, \rho_i, \nu_i) = \mathrm{SVNWA}_w(\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{in})$$

$$= \left(1 - \prod_{j=1}^{n} (1-\mu_{ij})^{w_j}, \prod_{j=1}^{n} \rho_{ij}^{\ w_j}, \prod_{j=1}^{n} \nu_{ij}^{\ w_j}\right), \quad i = 1,2,\cdots,m. \tag{6}$$

the overall value of the alternative A_i, where $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of attributes.

In the situation where the information about attribute weights is completely known, i.e., each attribute weight can be provided by the expert with crisp numerical value, we can weight each attribute value and aggregate all the weighted attribute values corresponding to each alternative into an overall one by using Eq. (6). Based on the overall attribute values $\tilde{r}_i$ of the alternatives A_i $(i = 1,2,\cdots,m)$, we can rank all these alternatives and then select the most desirable one(s). The greater $\tilde{r}_i$, the better the alternative A_i will be.

Entropy[13] was one of the concepts in thermodynamics originally and then Shannon first introduced the concept of information entropy in connection with communication theory. He considered entropy was an equivalent to uncertainty. It made a pervasive impact to many other disciplines in extending his work to other fields, ranging from management science, engineering technology and the sociological economic field. In these disciplines entropy is applied as a measure of disorder, unevenness of distribution and the degree of dependency or complexity of a system. Information entropy is an ideal measure of uncertainty and it can measure the quality of effective information. In the single-valued neutrosophic MADM problems which have m alternatives and n attributes, the jth attribute's entropy is defined as follows:

$$H_j = -k \sum_{i=1}^{m} f_{ij} \ln f_{ij},\ i = 1,2,\cdots,m,\ j = 1,2,\cdots,n.$$

where

$$f_{ij} = \frac{H(\tilde{r}_{ij})}{\sum_{i=1}^{m} H(\tilde{r}_{ij})},\ k = \frac{1}{\ln m}. \tag{7}$$

Assume that if $f_{ij} = 0$, then $f_{ij} \ln f_{ij} = 0$.

Then, the jth attribute's entropy is defined as follows:

$$w_j = \frac{1 - H_j}{n - \sum_{j=1}^{n} H_j}. \tag{8}$$

Based on the above models, we develop a practical method for solving the MADM problems, in which the information about attribute weights is completely unknown, and the attribute values take the form of single-valued neutrosophic numbers. The method involves the following steps:

Step 1. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$ be the single-valued neutrosophic decision matrix, where $\tilde{r}_{ij}=\left(\mu_{ij},\rho_{ij},\nu_{ij}\right)$, which is an attribute value, given by an expert, for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$, $w=\left(w_1,w_2,\cdots,w_n\right)$ be the weight vector of attributes, where $w_j\in\left[0,1\right]$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}w_j=1$.

Step 2. Determine the entropy weight of each attribute according to equations (7) and (8).

Step 3. Utilize the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)$ and by Eq. (6), we obtain the overall values $\tilde{r}_i$ of the alternative $A_i\left(i=1,2,\cdots,m\right)$.

Step 4. calculate the scores $S\left(\tilde{r}_i\right)$ of the overall single-valued neutrosophic numbers $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s) (if there is no difference between two scores $S\left(\tilde{r}_i\right)$ and $S\left(\tilde{r}_j\right)$, then we need to calculate the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$ of the overall single-valued neutrosophic numbers $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$.

Step 5. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

Step 6. End.

4. Numerical example

From a global perspective, it is a common phenomenon of the existence of a largenumber of new small ventures, regardless of countries, regions and stage of development. But,new small ventures have, for a long term,been facing with the difficulty of micro-financingwhile they are creating significant macro social and economic benefits. This is mainlyreflected in: on one hand, new small ventures have played an important role in promotingeconomic development, increasing employment, creating innovation, maintaining marketcompetitive vigor, and protecting price mechanism, etc; on the other hand, as the micro mainbody,new small ventures have constantly fallen into the plight of survival and developmentdue to difficulties in financing.Three reasons result in the financing difficulty of new small ventures. First, the inherentdeficiencies of new small ventures,such as: smaller scale of assets in the early days; greatuncertainty of future development; lack of credit, high risk,etc; second, for a long term, creditrisk evaluation models of commercial banks in China have mainly been focused on large andmedium enterprises,resulting in its failure to build effective credit risk evaluation models fornew small ventures,and to make scientific and objective evaluation of credit risk and creditdecision for new small ventures; third, lack of theoretical research. This is mainly reflected in: The academic community, when building credit risk evaluation models, tends to focus onlarge ventures and ignore small ventures; focus on financial indicators and ignorenon-financial indicators, focus on tangible assets and ignore intangible assets, focus on theinformation in financial statements and ignore the information out of financial statements,focus on the normative of information disclosure and ignore cooperative enterprises, transaction process and financing products' function in mitigating information asymmetry. These perspective limitations on the theoretical research not only restrict the innovations andpractices of financial bodies in finance services to new small ventures, but also result in asituation that theoretical research of credit risk measurement, evaluation and management fornew small ventures lags far behind practice needs.Therefore, it is undoubtedly of great practical and theoretical significance to solving thefinancing difficulties of new small ventures through a correct understanding of the operationcharacteristics of and financing rules new small ventures,and a scientific command of riskcharacteristics,so as to further effectively identify the credit risk situation and key influence factors in various stages in their growth process, build a scientific and rational credit riskevaluation model for small new venture, and to guide and optimize banks'financing policies, credit ideas and product design for new small ventures, and to enrich the theory of credit riskmanagement. This section presents a numerical example to illustrate the method proposed in this paper. Suppose a company plans to evaluate the credit risk of small new venture' indirect financing. There is a panel with five possible enterprises $T_i\left(i=1,2,3,4,5\right)$ to select. The company selects four attribute to evaluate the five possible enterprises: ①G_1 is the solvency ability; ②G_2 is the viability ability; ③G_3 is the corporate profitability; ④G_4 is the development capacity. The five possible enterprises $A_i\left(i=1,2,3,4,5\right)$ are to be evaluated using the single-valued neutrosophic numbers by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix} (0.2,0.4,0.7) & (0.1,0.3,0.8) & (0.4,0.3,0.5) & (0.5,0.4,0.4) \\ (0.6,0.3,0.3) & (0.3,0.5,0.6) & (0.6,0.5,0.4) & (0.6,0.6,0.3) \\ (0.4,0.5,0.4) & (0.5,0.6,0.4) & (0.5,0.6,0.5) & (0.4,0.5,0.5) \\ (0.2,0.4,0.5) & (0.1,0.5,0.7) & (0.7,0.4,0.2) & (0.5,0.4,0.4) \\ (0.6,0.4,0.2) & (0.4,0.6,0.4) & (0.5,0.6,0.3) & (0.3,0.6,0.4) \end{bmatrix}$$

Procedure for evaluating the credit risk of small new venture' indirect financing contains the following steps.

Step 1 According to equations (7) and (8), we get the weight vector of attributes:

$$w=\left(0.3068\ \ 0.1190\ \ 0.2433\ \ 0.3309\right)^T$$

Step 2 Utilize the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)$ and by Eq. (6), we obtain the overall values $\tilde{r}_i$ of the alternative $A_i\left(i=1,2,\cdots,m\right)$.

$$\tilde{r}_1=\left(0.3525,0.2333,0.5445\right),\tilde{r}_2=\left(0.5725,0.3255,0.3494\right)$$
$$\tilde{r}_3=\left(0.4384,0.4366,0.4547\right),r_4=\left(0.4530,0.2354,0.3868\right)$$
$$\tilde{r}_5=\left(0.4666,0.3287,0.3015\right)$$

Step 3 Calculate the scores $S\left(\tilde{r}_i\right)$ of the overall single-valued neutrosophic numbers $\tilde{r}_i\left(i=1,2,\cdots,m\right)$

$$S\left(\tilde{r}_1\right)=-0.1932,S\left(\tilde{r}_2\right)=0.2247,S\left(\tilde{r}_3\right)=-0.0132$$
$$S\left(\tilde{r}_4\right)=0.0676,S\left(\tilde{r}_5\right)=0.1684$$

Step 4 Rank all the enterprises $A_i\left(i=1,2,3,4,5\right)$ in accordance with the scores $S\left(\tilde{r}_i\right)\ \ \left(i=1,2,\cdots,5\right)$ of the overall single-valued neutrosophic numbers $\tilde{r}_i\left(i=1,2,\cdots,m\right)$: $A_2\succ A_5\succ A_3\succ A_4\succ A_1$, and thus the most desirable enterprises is A_5.

5. Conclusion

In this paper, we have investigated the problem of MADM with completely unknown information on attribute weights to which the attribute values are given in terms of single-valued neutrosophic numbers. To determine the attribute weights, a model based on the information entropy, by which the attribute weights can be determined, is established. We utilize the single-valued neutrosophic weighted averaging (SVNWA) operator to aggregate the single-valued neutrosophic numbers corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example for evaluating the credit risk of small new venture' indirect financing is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, we shall continue working in the application of the single-valued neutrosophic multiple attribute decision-making to other domains [14-22].

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