The Max-Min Ant System Solving the Vehicle Routing Problem with Time Windows

Guotong Bi

Business School, Henan University, Kaifeng 475004,China

zjkbgt@163.com

Abstract

The Vehicle Routing Problem with Time Windows(VRPTW),which is more practical than VRP, is based on VRP and adds constraints of service time windows.This paper studys VRPTW. Aiming at the weaknesses of ant colony algorithm, including slow convergence and easy to get into local optimal solutions, adopts Max-Min Ant System(MMAS) to solve the problem. The pheromone is limited within a certain range. The experimental comparison shows that the MMAS algorithm is effective for solving VRPTW, can improve the ability of optimization and avoid the local optimal solutions.

Keywords

Time Windows; Vehicle Routing Problem;VRPTW; Max-Min Ant System.

1. Introduction

With the improvement of social consumption ability and the rapid development of e-commerce, logistics distribution problems have gained more and more attentions by logistics and express enterprises, and this kind of problem belongs to the Vehicle Routing Problem(VRP) in operational research. VRP refers to arrange the route reasonably for a series of delivery point and/or receiving point,making vehicles through them in an orderly way,with satisfying certain constraints (such as demand, volume, shipment delivery time, vehicle capacity limits, mileage restrictions and time limits, etc.), achieve a certain goal (such as shortest distance, minimum cost,less time,less vehicle, etc.) [1].Since this problem proposed by Dantzig and Ramser in 1959 for the first time, it is widely studied by experts in the fields of Operations Research, Combinatorial Mathematics, Graph Theory, Computer Application, and Logistics[2]. And scholars have obtained plenty of research achievements and practical applications. It is a classic combination optimization problem, having been proved to be a NP hard problem. VRPTW is based on the VRP problem and adds the constraints of customers'service windows, which is more complex than the VRP. In real life, customers often have certain requirements for service time, so VRPTW is more practical than VRP, and it is more realistic to study VRPTW.

In recent years, the ant colony algorithm is a bionic optimization algorithm which is widely used in solving VRP and VRPTW heuristic. But the ant colony algorithm has the weaknesses of slow convergence rate and easy to get into local optimal solution[3].But Max-Min Ant System limits pheromones to a certain range[4]. When the path pheromone amount is less than lower limit value or higher than the upper limit, it will be forced to set for the lower or upper limit.So this algorithm improves the convergence speed, effectively reduce the defects easily plunged into local optimal solution. In this paper, the VRPTW is studied by the MMAS and the optimal goal of the problem is the shortest length and the lowest cost of penalty. Through the calculation of standard test set of Solomon. By comparing the optimal solution with known best results, verifys the model and algorithm is feasibility and effectiveness.

2. Mathematical Model of VRPTW

2.1 Problem Description

VRPTW usually can be described as: known each customer's location and demand (no more than the maximum load), and each customer has appointed time window, having capacity limits of vehicle starts from the distribution center, by meeting capacity constraints,service the customer within the time limit specified by the customer,after complete customers' demands, return distribution center, each customer can only be serviced by one car, the optimization goal is to choose the appropriate distribution scheme,which can minimize total cost or delivery of vehicles.

2.2 Variable Definitions

The variables are defined as follows:

Assume G=(V,E) is empowerment, V={ $v_0,v_1,v_2,\ldots v_n$} stands for vertex set, E stands for edge set, $v_1,v_2,\ldots v_n$ stands for customer demand points, v_0 stands for distribution center;

K: distribution center v_0 has K vehicles of the same type truck;

Q:the maximum load of per vehicle;

q_i:The quantity demanded of customer i, i=0,1,2,…N,i=0 means distribution center,max(q_i) ≤Q;

d_{ij}: distance between customer i and customer j, $d_{ij}\geq 0$,i,j=0,1,2,…N;

[ET_i,LT_i]: the time window of each customer specified, ET_i means the earliest service start time of the customer i,LT_i means the latest service start time of the customer i, the vehicle must service all customers within the time limits set by the customers. If the vehicle arrives early, it have to wait for the customer's appointed time window. If the vehicle arrives late, it will have to pay a certain amount of punishment;

p_1: the cost of waiting per hour if early;

p_2: the cost of punishment per hour if late;

T_i: the time of the vehicle arrives customer i;

T_{ij}: the time that the vehicle takes from customer i to customer j.

decision variables:

$$x_{ijk}=\begin{cases}1, \text{vehicle k runs from customer i to customer j}\\ 0, \text{others}\end{cases}$$

$$y_{ik}=\begin{cases}1, \text{the demend of customer i is delivered by vehicle k}\\ 0, \text{others}\end{cases}$$

2.3 Mathematical Model

The Mathematical model of VRPTW can be described:

Among them, formula (1) represents the objective function of total cost,including transportation cost, waiting cost and punishment cost. Formula (2) represents the total amount of cargo per car is not exceeding the maximum capacity of the vehicle. Formula (3) represents each customer is serviced by and only by one truck. Formula (4) ensures that neighboring nodes is only one before customer j. Formula (5) ensure customer i has only one adjacent node. Formula (6) represents the vehicle from the distribution center will return to the distribution center after completing the delivery.Formula (7) represents eliminateing child loop. Formula (8), Formula (9) represents the range of the decision variables.

$$Min\ Z = \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K} \mathrm{d}_{ij}\mathrm{x}_{ijk} + p_1\sum_{i=1}^{N}\max(\mathrm{ETi}\text{-}\mathrm{Ti},0) + p_2\sum_{i=1}^{N}\max(\mathrm{Ti}\text{-}\mathrm{LTi},0) \quad (1)$$

$$\text{s.t.:} \quad \sum_{i=1}^{N} q_i \mathrm{y}_{ik} \le Q, \forall k \in \{1,2,\ldots,K\} \quad (2)$$

$$\sum_{k=1}^{K} \mathrm{y}_{ik} = 1, \forall i \in \{1,2,\ldots,N\} \quad (3)$$

$$\sum_{i=0}^{N} \mathrm{x}_{ijk} = \mathrm{y}_{jk}, \forall j \in \{1,2,\ldots,N\}, \forall k \in \{1,2,\ldots,K\} \quad (4)$$

$$\sum_{j=0}^{N} \mathrm{x}_{ijk} = \mathrm{y}_{ik}, \forall i \in \{1,2,\ldots,N\}, \forall k \in \{1,2,\ldots,K\} \quad (5)$$

$$\sum_{j=1}^{N} x_{0jk} = \sum_{i=1}^{N} x_{i0k} = K \quad (6)$$

$$\sum_{i\in S}\sum_{j\in S} \mathrm{x}_{ijk} \le |S|-1, S \subseteq \{1,2,\ldots,N\}, 2 \le |S| \le n-1 \quad (7)$$

$$\mathrm{x}_{ijk} \in \{0,1\}, i, j \in \{1,2,\ldots,N\}, \forall k \quad (8)$$

$$\mathrm{y}_{ki} \in \{0,1\}, i \in \{1,2,\ldots,N\}, \forall k \quad (9)$$

3. MMAS Solving VRPTW

3.1 Algorithm Principle

Comparing with the basic ant colony algorithm, the improvements of the MMAS are as follows:

(1) Forcing every path pheromone amount in [τ_{min}, τ_{max}].When the pheromone values of one path greater than τ_{max}, the pheromone values will be set τ_{max}. When the pheromone values of one path smaller than τ_{min}, the pheromone values will be set τ_{min}. This will increases the speed of searching the optimal solution speed, effectively avoid the concentration of information on a particular path that causes all the ants to focus on the same path.

(2) The value of the pheromone is initialized to τ_{max}, and the pheromone volatilization coefficient is set to a smaller value, making the algorithm search more feasible solutions at the beginning.

(3) After complete one iteration, only update the information on the path of the best ant in every generation, the optimal solution of historical information will be fully retained , so as to make better use of historical information to speed up the convergence speed.

3.2 Algorithm Steps

Step1：Colony initialization. Assume there are N customers point, pheromones on each side of the initial value is τ_{max}, place m ants on the node of the distribution center. The load capacity of each ant represents the payload of the vehicle Q. Set up an empty tabu table $tabu_k$(k=1,2, ··;m). Initialize parameter.

Step2：Determine to terminate Conditions. When the iteration reaches the specified number of times,return step7;else return step3.

Step3: Build path. Choose the next target location according to the transfer probability, which is the complement of the forbidden watch, and then add the selected city to the tabu table.

Step4: Update the optimal solution. Records the length of the ant path, the optimal solution of the iteration and the optimal path, the optimal path, the optimal solution and the optimal path. Update the optimal solution

Step5: Update pheromone. According to the information element, increase the concentration of pheromones on the path of the best performing ant in the iteration. For τij(t) of each path,

$\tau_{min}<\tau_{ij}(t)<\tau_{max}$.After each iteration, the value of the pheromone level is still within this range. If $\tau_{ij}(t)>\tau_{max}$,set $\tau_{ij}(t)=\tau_{max}$,if $\tau_{ij}(t)<\tau_{min}$,set $\tau_{ij}(t)=\tau_{min}$.

Step6：Number of iterations plus one and return step2;

Step7：Output result and end the algorithm

4. Simulation Experiment

4.1 Parameters Setting

To verify the validity of the algorithm, the algorithm based on Matlab software platform simulation experiments.The algorithm of each parameter is set as: pheromone stimulating factor α= 1, visibility stimulating factor β= 5, volatilization coefficients ρ =0.1, the maximum number of iterations NCmax = 3000, ant number m=50, $\tau_0=1/D_0$ (D_0 stands for the first oldest born route length produced by the nearest neighbor method), $\tau_{max}=\tau_0$, $\tau_{min}=0.005\tau_0$.

4.2 Experiment and Result Analysis

We adopt the VRPTW test set raised by Solomon as test data[5]. The test set consists of different vehicle numbers, vehicle capacities, customer locations, demands, time windows and service time. The data set is divided into six data sets,including C1,C2,R1,R2,RC1,RC2. Each contains 100 customer points. The experiment takes six data sets in R2, calculate 10 times for each data set and adopt the optimal solution.Then compare with the optimal solutions of known, the results of basic ant colony algorithm. The results are shown in Table 1.

Table 1. The comparation of simulation results by different algorithms

data set	The optimal solutions of known		AS		MMAS	
	Vehicle Number	Total Length/km	Vehicle Number	Total Length/km	Vehicle Number	Total Length/km
R201	4	1252.37	6	1457	4	1421.33
R202	3	1198.45	6	1389	3	1319.11
R203	3	942.64	4	1106	4	1110.89
R204	2	854.88	3	1067	3	969.86
R205	3	1013.47	5	1196	3	1123.79
R206	3	833.00	4	1154	3	1059.75

By contrast,we can find that the MMAS algorithm has fewer vehicles than the AS algorithm, the total length is shorter than the AS algorithm,all results is closer to the optimal solution than the AS algorithm.So the MMAS algorithm can effectively solve VRPTW problem, shows that this model and the algorithm of this paper is effective and feasible.

5. Conclusion

This paper studies VRPTW problem. Aiming at the weaknesses of ant colony algorithm,including slow convergence and easy to get into local optimal solutions,adopts MMAS algorithm to solve the problem.The pheromone is limited within a certain range.The experimental comparison shows that the MMAS algorithm is effective for solving VRPTW problem, which can improve the ability of searching, avoid the local optimal solution, and expand the solution of VRPTW. But MMAS still has some drawbacks, such as pheromone volatilization coefficients ρ is fixed, the ants in the late algorithm is not sensitive to the pheromone concentration gradually become.Thus it is conducive to solving, needs further research to improve.

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