Research on Application of Computer Mathematics Software in Ordinary Differential Equations

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Abstract

Ordinary differential equation is a foundation course of mathematics, and its complex derivation and calculation can hardly be implemented by traditional manual computation which is tedious and error-prone. Under the support of modern technology, mathematical software assisted ordinary differential equation algorithm can effectively reduce the complexity of the derivation process and improve the accuracy and speed of computation. This paper discusses the application of mathematical software such as Maple, Mathematica and XPPAUT in ordinary differential equations.

Keywords

Computer; mathematical software; differential equation; application.

1. Introduction

The application of computer-aided software provides powerful support for the development of education and scientific research. Particularly, computer-aided software shows high application values in the subject field with large difficulty of manual calculation like ordinary differential equation. With the rapid development of computer software technology, all kinds of new mathematical software emerges constantly, and the function and convenient operation degree have been substantially improved. This paper analyzes the software commonly used in ordinary differential equation calculus, hoping to better play the role of mathematical software.

2. Overview of computer mathematics software

As a mathematical calculation auxiliary tool based on computer technology, computer mathematical software provides a variety of functions and drawing tools, which can effectively reduce the difficulty of mathematical calculation and derivation process.

Table 1 Application scope and advantages of computer mathematical software

Application scope	Application subject	Advantages
Function calculation, mathematical formula calculation, and data calculation	Mathematics, physics, biology, social science, and engineering technology, etc	High accuracy, fast calculation speed, and excellent expression ability

3. Characteristics of commonly used mathematical software

With the improvement of computer software development, a variety of new mathematical software emerges constantly. For instance, Maple, Mathematica, MatLab and XPPAUT have been widely used in ordinary differential equation. The characteristics of the software are as shown in Table 2.

Table 2 Characteristics of commonly used mathematical software.

Software name	Application range	Characteristics
Maple	Mathematics	Symbolic operation, and convenient drawing
Mathematica	Physics	Sign language with features is not easy for beginners to master
MatLab	Engineering, and scientific computing	Convenient data input and numerical calculation, but symbolic operation needs the assistance of Maple language
XPPAUT	Special software for differential equations	Excellent function in differential equation calculation

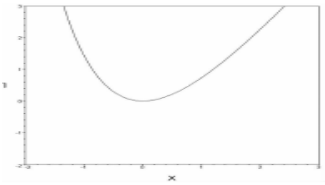
4. Practical application of mathematical software in ordinary differential equations

Application of Maple software

Example 1: Solve first-order ordinary differential equation $\frac{dy}{dx} = 2x - y$

The dsolve command, unapply command, plot command and dfieldplot command provided by Maple software are utilized in the process of solving example 1, and the solution process is as shown in Table 3.

Table 3 Solution process

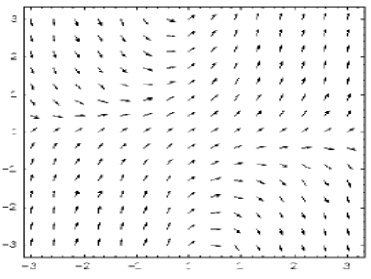
Step	Operation	Result	Specification
1	Solution:=dsolve(diff(y(x),x)=2*x-y(x),y(x))	Solution:=y(x)=-2+2x+e^{-x}C1	Solve the general solution
2	specify a specific value for C1, and suppose y(0)=0	Solution：=y(x)=-2+2x+e^{-x}	Solve the particular solution
3	Y1:=unapply(rhs(solution1),x)	Y1:=x→-2+2x+2e^{-x}	Convert the function
4	Plot(y1(x), x=-2..3, y=-2..3, color=black, axes=BOXED		Get the solution graph

Application of Mathematica software

Example 2: Solve the vector field of ordinary differential equation $\frac{dy}{dx} = 1 + xy$

Mathematica software is utilized to get the solution. Input the order in Notebook window and complete the graph drawing. The order and the result are shown in Table 4.

Table 4 Order and graph drawing result

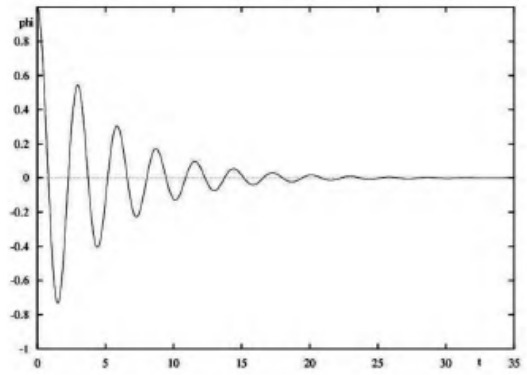
Input order	Graph drawing result
G1=PlotVectorField[{1,1+x*y},{x,-3,3},{y,-3,3}, Frame→True, ScaleFunction→(1&)， ScaleFactor→0.16， HeadLength→0.01]	

Application of XPPAUT software

Example 3: Solve the integral curve of the damp free vibration equation. It is known that $\varphi(0)=1$, $\varphi'(0)=0$, m=1, g=9.8, l=2

XPPAUT software is adopted to assist the drawing. ODE file is compiled, and then click the running button to obtain the integral curve of the equation. The contents of the ODE file and the drawing result are as shown in Table 5.

Table 5 ODE file content and graph drawing result

ODE file content	Graph drawing result
#the damped pendulum Phi'=v V'=-(g/l)*sin(phi)-(mu/m)*v Par g=9.8,l=2,mu=0.4,n=1 Init phi=1,v=0 @total=35 done	

5. Comparative analysis of the commonly used mathematical software

Among the several kinds of mathematical software introducted in this paper, Maple, Mathematica, and MatLab are widely used in China, and XPPAUT is a special software for differential equations developed by American professor of mathematics, Bard Ermentrout. Based on the analysis of the above examples, it can be seen that the commonly used software is substantially different in application process and practical application process.

Maple software has universal programming language in the field of mathematics, and specializes in symbolic computation. It can directly draw the integral curve by function definition without numerical solution in advance. Meantime, it can draw multiple curves simultaneously. Therefore, the application is relatively convenient. The programming language of Mathematica software has certain particularity, and the symbols commonly used by people can be directly input and displayed in the display interface. Consequently, it has certain advantages in displaying the results. However, Mathematica software also defines some special rules and symbols with special meaning, so the operation is relatively difficult. Maple software is more convenient in the numerical input and calculation, and the graph can be displayed more flexibly. Since the symbolic operation process needs the help of Maple language, this paper does not give the numerical example.

In contrast, XPPAUT software specially developed for differential equation calculation and graph drawing has more advantages in the application in ordinary differential equation: (1) The numerical simulation and branch analysis function of equation solution; (2) Animation demonstration of the curve approach; and (3) advantages in the application of partial differential equations and differential equations.

In the process of studying and researching of ordinary differential equations, beginners can hardly understand the concepts such as orientation field and trajectory. XPPAUT software can be utilized to draw orientation field and trajectory, so that beginners can understand the key points of graph drawing in the process of compiling ODE file and operating software, and digest the relevant knowledge. XPPAUT has more advantages in the application in damp vibration equation. Take Example 3 in this paper as an example. After the integral curve of the ordinary differential equation is drawn, the animation demonstration can be implemented by compiling animation file, and the software users can see the pendulum swing from the left to the right and stop at the equilibrium position. However, XPPAUT software is seldom utilized in China. Therefore, it is necessary to strengthen the application of XPPAUT and such advanced software.

6. Application methods

In the application of computer mathematical software, the process of solving ordinary differential equations become more simple, and its application mainly includes: (1) The eigenvectors and eigenvalues of matrix, determinant and the exponential function calculation are adopted to solve the linear ordinary differential equations. Additionally, algebraic equations are needed to calculate the equilibrium point.

(2) Draw the assistant curvilinear figure of the solution of equation, including vector fields, oblique line and integral curve. A variety of mathematical software provides the corresponding mapping function library, and the graph is drawn by establishing and calling function library. Since accurate solution can hardly be obtained in ordinary differential equation, the numerical solution of the graph should be solved first, and then the graph is drawn by the approximate numerical solution. (3) The Laplasse transform method and power series method and other special functions are adopted to solve the ordinary differential equation. (4) Direct integral solution. In Maple and some other mathematical software, symbols and relevant functions can be utilized to solve the equation without initial condition. If there are initial conditions, they can be directly added to the equation. In the application process of computer mathematical software, it should be noted that mathematical software cannot guarantee that the equation can be solved, and artificial intelligence processing is needed. The matters needing attention in the application of mathematical software in ordinary differential equations are summarized in Table 6.

Table 6 Matters needing attention in the application of mathematical software

Application object	Matters needing attention
Linear differential equation with constant coefficients	Convert the equation into the form of x=Ax+f(t). Calculate the eigenvalues and the exponential function, and then solve the general solution and particular solution.
Laplace transform	Laplace transform is adopted to convert the equation into algebraic equation. Solve the equation by reverse change. For convenient operation, the variable name should be redefined.
Integral curve and vector diagram drawing	The given time interval and the numerical solution of the equation with the initial value are calculated and converted to graphics. Determine the vector size and scope of the vector diagram.

7. Conclusion

Computer mathematical software has become an important auxiliary tool to solve ordinary differential equation, and has important position in the field of education and scientific research. Despite of the substantial difference of mathematical software in calculation process, operation steps and language features, the function of auxiliary solution includes derivation calculus and auxiliary drawing. Therefore, it is necessary to strengthen the application of mathematical software and simplify the solution process of ordinary differential equations.

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