The group decision making process based on probabilistic linguistic preference relations

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Abstract

Since the concept of linguistic variables was proposed, various kinds of linguistic terms have been used to express decision makers' preference information in order to solve group decision making (GDM) problems. Probabilistic linguistic term set (PLTS) is a novel extension form of the existing linguistic variables. Based on it, the probabilistic linguistic preference relation (PLPR) has been proposed. In the GDM problem, we must make sure of the consistency level of preference relations at first. In this paper, we propose the expected consistency of PLPR, develop a modeling method to check the expected consistency level of PLPR and to obtain the priority weights of alternatives. Then, we put forward the probabilistic linguistic power average (PLPA) operator to aggregate decision makers' preference relations into collective one. A practical case is given to show the effectiveness of the proposed method.

Keywords

Probabilistic linguistic term set (PLTS); Probabilistic linguistic preference relation (PLPR); Consistency measure; Priority weights; Probabilistic linguistic power average (PLPA) operator; Group decision making.

1. Introduction

Decision making is a widespread process for human being's daily life or enterprises in various domains, such as selection [1], planning [2], recommendation [3], evaluation [4], and so on. Due to the intricacy of decision making problems in the real word and the limitation of single decision maker's cognition, it is necessary to achieve the solution from the opinions provided by a cluster of decision makers (DMs). Group decision making (GDM) has been generally used in all aspects in modern life. At the same time, many studies have been developed on GDM problems [5]. DMs use different forms of information to express their preferences, such as fuzzy preference relations [6], hesitation fuzzy sets [7], and hesitant fuzzy linguistic term sets (HFLTSs), which have been generally applied to decision making [8] for the reason that DMs can express their preferences by not only one linguistic term. Nevertheless, the occurring probability of each linguistic term in HFLTS is equal. It means that the importance of each linguistic term is same, which is apparently problematic. To overcome the problem, Pang et al. [9] proposed the probabilistic linguistic term sets (PLTSs), which are applied in multi-attribute GDM. In this paper, we discuss the GDM problems on the foundation of PLTSs.

To solve the GDM problems, there are three necessary steps. Firstly, make sure that preference relations provided by DMs are acceptably consistent. Then, choose the suitable aggregation operator to aggregate individual preference relations into the collective one. Last but not least, obtain the priority weights of alternatives for the ranking. In this paper, we introduce the definition of PLPRs based on the PLTSs at first. Then, we are absorbed in the researches of the consistency and the aggregation of PLPRs. We define the expected consistency of the PLPRs, based on which we can use the index to check the expected consistency level of PLPR.

As we know, aggregation operator plays a vital role in GDM. A number of aggregation operators [10–13] have already been proposed to be applied to the aggregation process in GDM. However, the shortage of the existing aggregation operator is that aggregation values are without enough consideration during the aggregation process. With regard to this, Yager [14] presented the power average (PA) operator, which makes each value be taken into consideration and allows them being aggregated to support and enhance each other. With the generation of the PA operator, a series of PA operators [15–20] have been introduced to aggregate different types of values. With the use of linguistic variables, the PA operators have been extended to linguistic environment. The existing linguistic operators are 2-tuple linguistic power average (2TLPA) operato [21], linguistic weighted power average (LWPA) operator [22], linguistic power geometric (LPG) operator [23], and so on. According to the distance between two linguistic terms, we can calculate the support, and based on the support from other values, we can obtain the weight of each linguistic value. In order to aggregate the individual PLPRs, we put forward the probabilistic hesitant fuzzy linguistic (PLPA) operator with the idea of the PA operator.

The structure of this paper is shown below. Some elementary notions of linguistic term sets (LTSs) and probabilistic linguistic term sets (PLTSs) are reviewed in Section 2. In Section 3, we come up with the probabilistic linguistic power average (PLPA) operator under the GDM environment, which is based on the existing power average (PA) operator. In Section 4, we introduce the probabilistic linguistic preference relations (PLPRs) and develop the expected consistency measure to check the expected consistency level of PLPR. In Section 5, a practical case is provided to show the effectiveness of the techniques proposed in this paper. Finally, some conclusions are made to end this paper in Section 6.

2. Preliminary

2.1 LTSs

2.1.1 Sub-section Headings

Give a finite and ordered LTS $S=\{s_\alpha \mid \alpha=-\tau,\cdots,0,\cdots,\tau\}$, which is commonly used in the linguistic decision making, where τ is a positive integer, s_α indicates a probable value for a linguistic variable and has the features as follows [24]:

1. Ordered set: $s_\alpha \le s_\beta$, if $\alpha \le \beta$;

2. Negation operator: $s_\alpha = Neg(s_{-\alpha})$.

Example 2.1. Give a set S, which includes seven subscript-symmetric linguistic terms as follows:

$$S=\{\ s_{-3}=extremely\ poor,\ s_{-2}=very\ poor, s_{-1}=poor,$$

$$s_0=fair, s_1=good, s_2=very\ good, s_3=extremely\ good\}.$$

Further, Xu and Wang [24, 25] has expanded the discrete LTS to a continuous modality, which is more convenient for calculation and analysis, and can describe the given linguistic assessments without loss of information [26].

$$\bar{S}=\{s_\alpha | \alpha\in[-\tau,\tau]\},$$

where τ is a sufficient large positive integer. The linguistic term $s_\alpha \in S$ which is called the original linguistic term is used to estimate alternatives by DMs, and the expanded linguistic term $\bar{s}_\alpha \in \bar{S}$ which is called the virtual linguistic term can only be seen in operation.

Given two linguistic terms $s_\alpha, s_\beta \in \bar{S}$, we have the combined operational law [25] as follows:

$$\lambda_1 s_\alpha \oplus \lambda_2 s_\beta = s_{\lambda_1\alpha+\lambda_2\beta}.$$

2.2 PLTSs

As we know, HFLTSs have been widely applied in decision making because DMs can express their preferences by several possible linguistic terms. While we can find that each linguistic term in HFLTS has the equal occurring probability. In other words, all linguistic terms have the same importance, which is obviously problematic. Hence, Pang et al. [9] extended HFLTSs, then propose PLTSs by the following definition:

Definition 2.1. [9]. Given a linguistic term set $S=\{s_{-\tau},\cdots,s_0,\cdots,s_\tau\}$, then a PLTS can be denoted as:

$$L(p)=\left\{L^{(k)}\left(p^{(k)}\right)\mid L^{(k)}\in S, p^{(k)}\geq 0, k=1,2,\cdots,\#L(p), \sum_{k=1}^{\#L(p)} p^{(k)}\leq 1\right\}, \quad (2.1)$$

where $L^{(k)}\left(p^{(k)}\right)$ represents a linguistic term $L^{(k)}$ and its corresponding probability $p^{(k)}$, $\#L(p)$ indicates the number of linguistic terms in $L(p)$.

There are two situations about the results of $\sum_{k=1}^{\#L(p)} p^{(k)}$ shown as below:

1. $\sum_{k=1}^{\#L(p)} p^{(k)}<1$ indicates that there exists partial probability lost because of DMs' limited cognition level;

2. $\sum_{k=1}^{\#L(p)} p^{(k)}=1$ shows the whole information of probabilities of all probable linguistic terms.

Especially, when the occurring probability of each linguistic term is equal, it means that the importance of each linguistic term is same, then PLTS is reduced to HFLTS. It is noted that if $\sum_{k=1}^{\#L(p)} p^{(k)}<1$, there always needs a normalization step in order to obtain complete assessment information. Pang at el. [9] gave the solution as follows:

Definition 2.2. [9]. Let $L(p)$ be a PLTS and $\sum_{k=1}^{\#L(p)} p^{(k)}<1$, then the associated PLTS $\dot{L}(p)$ is defined by

$$\dot{L}(p)=\left\{L^{(k)}\left(p^{(k)}/\sum_{k=1}^{\#L(p)} p^{(k)}\right)\mid k=1,2,\ldots,\#L(p)\right\}, \quad (2.2)$$

For a PLTS, there may exist two problems about the normalization of PLTSs. The first one is to assign the lost probabilistic information averagely, which is solved by the above definition. The other is to normalize the length of PLTSs for the convenience of calculation.

In order to make the linguistic term sets have the same number of terms, the proposed method in Ref. [9] is to increase the numbers of linguistic terms for the PLTSs in which the numbers of linguistic terms are relatively small. The added linguistic terms are the smallest ones, and the associated probabilities are zero. For convenience, the normalized PLTS is still denoted by $L(p)$ in this paper.

We assume that all PLTSs have already been normalized. In other word, for any $L(p)_1$ and $L(p)_2$, we have $\sum_{k=1}^{\#L(p)_1} p^{(k)}=\sum_{k=1}^{\#L(p)_2} p^{(k)}=1$ and $\#L(p)_1=\#L(p)_2$. Based on the assumption, Pang et al. [9] put forward some basic operations as follows:

Definition 2.3. [9]. Given two ordered PLTSs $L(p)_1$ and $L(p)_2$,

$L(p)_1=\left\{L^{(k)}\left(p^{(k)}\right)_1\mid k=1,2,\cdots,\#L(p)_1\right\}$ and $L(p)_2=\left\{L^{(k)}\left(p^{(k)}\right)_2\mid k=1,2,\cdots,\#L(p)_2\right\}$.

Then

$$L(p)_1\oplus L(p)_2=\bigcup_{L_1^{(k)}\in L(p)_1, L_2^{(k)}\in L(p)_2}\left\{p_1^{(k)}L_1^{(k)}\oplus p_2^{(k)}L_2^{(k)}\right\}, \quad (2.3)$$

Where $L_1^{(k)}$ and $L_2^{(k)}$ are the kth linguistic terms in $L(p)_1$ and $L(p)_2$ severally, $p_1^{(k)}$ and $p_2^{(k)}$ are the probabilities of the kth linguistic terms in $L(p)_1$ and $L(p)_2$ severally.

By means of the above operational law, the calculation results may be beyond the bounds of PLTSs, and the corresponding probability information may be lost after operation. To avoid the situation and keep the probability information, Gou and Xu [27] proposed the novel operational laws for PLTSs based on the equivalent transformation functions g and g^{-1}.

Definition 2.4. [27]. Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,0,\cdots,\tau\}$ be a LTS, and $h_\gamma=\{\gamma \mid \gamma\in[0,1]\}$ be a HFE, $L(p)_1$ and $L(p)_2$ be two PLTSs, then:

$$g:[-\tau,\tau]\to[0,1], g(s_\alpha)=\frac{\alpha+\tau}{2\tau}=\gamma,$$

$$g^{-1}:[0,1]\to[-\tau,\tau], g^{-1}(\gamma)=s_{(2\gamma-1)\tau}=s_\alpha,$$

$$L(p)_1\oplus L(p)_2=g^{-1}\left(\bigcup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\left\{\left(\eta_1^{(i)}+\eta_2^{(j)}-\eta_1^{(i)}\eta_2^{(j)}\right)\left(p_1^{(i)}p_2^{(j)}\right)\right\}\right), \tag{2.4}$$
$$i=1,2,\cdots,\#L(p)_1,\ j=1,2,\cdots,\#L(p)_2$$

By Eq. (2.4), we can retain the probability information in the calculation results and in the final result, we rank all the linguistic terms in increasing order.

According to the concept of PLTS, the score of one PLTS has been put forward in the next definition.

Definition 2.5. [9, 28]. Let $L(p)=\left\{L^{(k)}\left(p^{(k)}\right) \mid L^{(k)}\in S, p^{(k)}\geq 0, k=1,2,\cdots,\#L(p)\right\}$ be a PLTS, and $I\left(L^{(k)}\right)$ be the function to obtain the subscript of $L^{(k)}$. Then the score of $L(p)$ can be denoted as:

$$E(L(p))=s_{\bar{\alpha}}, \tag{2.5}$$

where $\bar{\alpha}=\sum_{k=1}^{\#L(p)}I\left(L^{(k)}\right)\cdot p^{(k)}\Big/\sum_{k=1}^{\#L(p)}p^{(k)}$.

3. The probabilistic linguistic power average operator

3.1 The power average operator

The PA operator makes each value be taken into consideration during the aggregation process and use them to obtain the aggregated weights. Yager [14] gave the following definition.

Definition 3.1. [14, 29]. Let $a_1,a_2,\cdots,a_m$ be the aggregated values. The PA operator is denoted as

$$PA(a_1,a_2,\cdots,a_m)=\frac{\sum_{k=1}^{m}\left(1+T(a_k)\right)a_k}{\sum_{k=1}^{m}\left(1+T(a_k)\right)}, \tag{3.1}$$

where $T(a_k)=\sum_{l=1,l\neq k}^{m}Sup(a_k,a_l), k=1,2,\cdots,m$.

The $Sup(a,b)$ represents the support for a from b, which meets the conditions:

1. $Sup(a,b)\in[0,1]$;
2. $Sup(a,b)=Sup(b,a)$;
3. $Sup(a,b)\geq Sup(c,d)$ if $|a-b|\leq|c-d|$.

The PA operator reflects that if aggregated values are more similar, then the weight is bigger.

3.2 The probabilistic linguistic power average operator

Under the linguistic decision making environment, a series of linguistic PA operators were introduced, such as the 2TLPA operator in Ref. [21]nand LP2TPA operator in Ref. [29]. Here we put forward the definition of the PLPA operator. First, we propose the distance measure between two PLTSs.

Definition 3.2. Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a LTS, $L(p)_1$ and $L(p)_2$ be two PLTSs, $L(p)_1=\left\{L^{(k)}\left(p^{(k)}\right)_1 \mid k=1,2,\cdots,\#L(p)_1\right\}$ and $L(p)_2=\left\{L^{(k)}\left(p^{(k)}\right)_2 \mid k=1,2,\cdots,\#L(p)_2\right\}$, then the distance measure between $L(p)_1$ and $L(p)_2$ is defined as $d\left(L(p)_1,L(p)_2\right)$, which satisfies:

1. $0\le d\left(L(p)_1,L(p)_2\right)\le 1$;
2. $d\left(L(p)_1,L(p)_2\right)=0$ if and only if $L(p)_1=L(p)_2$;
3. $d\left(L(p)_1,L(p)_2\right)=d\left(L(p)_2,L(p)_1\right)$.

Xu [24] has defined the distance between two linguistic variables $s_\alpha,s_\beta\in S$, which is shown by the next formula:

$$d\left(s_\alpha,s_\beta\right)=\frac{|\alpha-\beta|}{2\tau},$$

where 2τ is the number of the linguistic terms in the set S.

According to the score function, we can obtain the expectation linguistic term of PLTS. Then the definition of the distance measure between two PLTSs is given as follows:

Definition 3.3. Let $L(p)_1=\left\{L^{(k)}\left(p^{(k)}\right)_1 \mid k=1,2,\cdots,\#L(p)_1\right\}$ and $L(p)_2=\left\{L^{(k)}\left(p^{(k)}\right)_2 \mid k=1,2,\cdots,\#L(p)_2\right\}$ be two PLTSs, then we have

$$d\left(L(p)_1,L(p)_2\right)=\frac{\left|I\left(E\left(L(p)_1\right)\right)-I\left(E\left(L(p)_2\right)\right)\right|}{2\tau}. \tag{3.2}$$

Similar to the method proposed by Yager [14], we propose the support function of PLTSs.

Definition 3.4. Let $L(p)_1,L(p)_2,\cdots,L(p)_m$ be a set of PLTSs. The support function between two PLTSs is defined as the function $Sup\left(L(p)_k,L(p)_l\right)$, which meets the conditions:

1. $Sup\left(L(p)_k,L(p)_l\right)\in[0,1]$;
2. $Sup\left(L(p)_k,L(p)_l\right)=Sup\left(L(p)_l,L(p)_k\right)$;
3. $Sup\left(L(p)_k,L(p)_l\right)\ge Sup\left(L(p)_i,L(p)_j\right)$ if $d\left(L(p)_k,L(p)_l\right)\le d\left(L(p)_i,L(p)_j\right)$, where is defined by Eq. (3.2);

Definition 3.5. Let $L(p)_1,L(p)_2,\cdots,L(p)_m$ be the aggregated PLTSs. A PLPA operator is defined as

$$PLPA\left(L(p)_1,L(p)_2,\cdots,L(p)_m\right)=\frac{\sum_{k=1}^{m}\left(1+T\left(L(p)_k\right)\right)L(p)_k}{\sum_{k=1}^{m}\left(1+T\left(L(p)_k\right)\right)}, \tag{3.3}$$

where $T\left(L(p)_k\right)=\sum_{l=1,l\ne k}^{m}Sup\left(L(p)_k,L(p)_l\right),k=1,2,\cdots,m$.

4. The probabilistic linguistic preference relations

4.1 PLPRs

Similar to linguistic preference relations (LPRs) and hesitant fuzzy linguistic preference relations (HFLPRs). Zhang et al. [28] proposed probabilistic linguistic preference relations (PLPRs) on the basis of PLTSs.

Definition 4.1. [28] If $X=(x_1,x_2,\cdots,x_n)$ is a fixed set, then the PLPR H^P on X is defined as a matrix $H^P=\left(L_{ij}\left(p_{ij}\right)\right)_{n\times n}$, where $L_{ij}\left(p_{ij}\right)=\left\{L_{ij}^{(1)}\left(p_{ij}^{(1)}\right), L_{ij}^{(2)}\left(p_{ij}^{(2)}\right),\cdots, L_{ij}^{\left(\#L_{ij}\left(p_{ij}\right)\right)}\left(p_{ij}^{\left(\#L_{ij}\left(p_{ij}\right)\right)}\right)\right\}$ is a FLPR, which indicates the preferred degree or intensity of the alternative x_i over x_j, and $\sum_{q=1}^{\#L_{ij}\left(p_{ij}\right)} p_{ij}^{(q)}=1$. $L_{ij}\left(p_{ij}\right)$ is a PLTS, and must satisfy the following conditions as well:

1. $L_{ij}^{(k)}\oplus L_{ji}^{(k)}=s_0, p_{ij}^{(k)}=p_{ji}^{(k)}$;
2. $L_{ii}^{(k)}=s_0, p_{ii}^{(k)}=1$;
3. $L_{ij}^{(k)}<L_{ij}^{(k+1)}$ and $L_{ji}^{(k+1)}<L_{ji}^{(k)}$, where $j>i$, $k\in\left\{1,2,\cdots,\#L_{ij}\left(p_{ij}\right)-1\right\}$, $i=1,2,\cdots,n-1$, $j=2,3,\cdots,n$, $\#L_{ij}\left(p_{ij}\right)$ is the number of possible elements in $L_{ij}\left(p_{ij}\right)$, and $\#L_{ij}\left(p_{ij}\right)=\#L_{ji}\left(p_{ji}\right)$, $p_{ij}^{q}\in[0,1]$ is the probability of $L_{ij}^{(k)}$ in $L_{ij}\left(p_{ij}\right)$.

4.2 Consistency measures

Definition 4.2. [30]. For a set of alternatives $X=\{x_1,x_2,\cdots,x_n\}$, there is a fuzzy preference relation $R=\left(r_{ij}\right)_{n\times n}$, where $r_{ij}\in[0,1]$, reflects the preference degree of x_i over x_j, $r_{ii}=0.5, r_{ij}+r_{ji}=1$, R is said to be multiplicatively consistent if $r_{ik}\cdot r_{kj}\cdot r_{ji}=r_{ki}\cdot r_{kj}\cdot r_{ij}\left(i,j,k=1,2,\cdots,n\right)$, which can be denoted as follows:

$$r_{ij}=\frac{\omega_i}{\omega_i+\omega_j},\forall i,j=1,2,\cdots,n, \tag{4.1}$$

where $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)$ is the priority vector of R. This vector must satisfy $\sum_{i=1}^{n}\omega_i=1$, and $\omega_i>0, i\in N$.

According to the above definition of consistency, we propose the definition of expected consistency for PLPRs, which is multiplicatively consistent as well.

Definition 4.3. If $X=\{x_1,x_2,\cdots,x_n\}$ is a set of alternatives, and $H^P=\left(L_{ij}\left(p_{ij}\right)\right)_{n\times n}$ is a PLPR, where $L_{ij}\left(p_{ij}\right)=\left\{L_{ij}^{(1)}\left(p_{ij}^{(1)}\right), L_{ij}^{(2)}\left(p_{ij}^{(2)}\right),\cdots, L_{ij}^{\left(\#L_{ij}\left(p_{ij}\right)\right)}\left(p_{ij}^{\left(\#L_{ij}\left(p_{ij}\right)\right)}\right)\right\}$ is a PLTS, indicating the possible priority intensity for $x_i\succ x_j$, then H^P is expectedly consistent if $I\left(\bar{e}_{ik}\right)\cdot I\left(\bar{e}_{kj}\right)\cdot I\left(\bar{e}_{ji}\right)=I\left(\bar{e}_{ki}\right)\cdot I\left(\bar{e}_{jk}\right)\cdot I\left(\bar{e}_{ij}\right)$, $\left(i,j,k=1,2,\cdots,n\right)$, which can be presented by the following formula:

$$I\left(\bar{e}_{ij}\right)=\sum_{q=1}^{\#L_{ij}\left(p_{ij}\right)}\frac{I\left(L_{ij}^{(q)}\right)+\tau}{2\tau}\cdot p_{ij}^{(q)}=\frac{\omega_i}{\omega_i+\omega_j},\forall i,j=1,2,\cdots,n \tag{4.2}$$

where $\bar{e}_{ij}=s_{\bar{\alpha}}$ is the expected linguistic term of the PLTS,

then $I\left(\bar{e}_{ij}\right)=I\left(s_{\bar{\alpha}}\right)=\bar{\alpha}=\sum_{q=1}^{\#L_{ij}\left(p_{ij}\right)}\frac{I\left(L_{ij}^{(q)}\right)+\tau}{2\tau}\cdot p_{ij}^{(q)}$, and $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)$ is the priority vector of H^P meeting $\sum_{i=1}^{n}\omega_i=1$, and $\omega_i>0, i\in N$.

Based on the above definition, we develop a modeling method inspired by the method proposed in Ref. [31] to check whether the PLPR is expectedly consistent or not and to obtain the priority weights of alternatives.

By Eq. (4.2), we have $\sum_{q=1}^{\#L_{ij}(p_{ij})} \frac{I(L_{ij}^{(q)})+\tau}{2\tau} \cdot p_{ij}^{(q)} = \frac{\omega_i}{\omega_i+\omega_j}, \forall i, j = 1, 2, \cdots, n$.

Let $\varepsilon_{ij} = (\omega_i+\omega_j) \cdot \sum_{q=1}^{\#L_{ij}(p_{ij})} \frac{I(L_{ij}^{(q)})+\tau}{2\tau} \cdot p_{ij}^{(q)} - \omega_i$, we must find the minimization of ε_{ij} for all $i, j \in N$ to make sure of more consistent preferences by the following model.

$$(\text{MOD 1}) \quad \begin{aligned} &\min \quad \varepsilon_{ij} = (\omega_i+\omega_j) \cdot \sum_{q=1}^{\#L_{ij}(p_{ij})} \frac{I(L_{ij}^{(q)})+\tau}{2\tau} \cdot p_{ij}^{(q)} - \omega_i \\ &\text{s.t.} \quad \begin{cases} \sum_{i=1}^{n} \omega_i = 1, \omega_i > 0 \\ i, j = 1, 2, \ldots, n, j > i \end{cases} \end{aligned} \tag{4.3}$$

Without loss of generality, we consider that all the goal functions $\varepsilon_{ij}(i, j = 1, 2, \ldots, n, j > i)$ are fair, and d_{ij}^{+}, d_{ij}^{-} represent the positive and negative deviation to the objective ε_{ij} respectively. Then we can transform the model into the following optimization model.

$$(\text{MOD 2}) \quad \begin{aligned} &\min \quad D = \sum_{i=1}^{n} \sum_{j>i}^{n} d_{ij}^{+} + d_{ij}^{-} \\ &\text{s.t.} \quad \begin{cases} (\omega_i+\omega_j) \cdot \sum_{q=1}^{\#L_{ij}(p_{ij})} \frac{I(L_{ij}^{(q)})+\tau}{2\tau} \cdot p_{ij}^{(q)} - \omega_i - d_{ij}^{+} + d_{ij}^{-} = 0, \\ \sum_{i=1}^{n} \omega_i = 1, \omega_i > 0, i \in 1, 2, \ldots, n, \\ d_{ij}^{+}, d_{ij}^{-} \geq 0, i, j = 1, 2, \ldots, n, j > i. \end{cases} \end{aligned} \tag{4.4}$$

In order to check the expected consistency of PLPR, ECI_{H^P} of PLPR is proposed, which is based upon the d_{ij}^{+} and d_{ij}^{-}.

$$ECI_{H^P} = \frac{2\sum_{j=i+1}^{n} \sum_{i=1}^{n-1} (d_{ij}^{+} + d_{ij}^{-})}{n(n-1)} \tag{4.5}$$

In general, if $ECI_{H^P} \leq 0.02$, then the expected consistency of the PLPRs is acceptable. If $ECI_{H^P} > 0.02$, then the ECI_{H^P} needs to be improved. In chapter 5, we give an illustration to explain the feasibility of this modeling method.

5. Illustrative example

ERP (enterprise resource planning) is a new generation of information system, which is based on the era of network. It is widely used to achieve the update of management, so that information technology plays an important role in the development of enterprises.

One company is prepared to introduce a suitable ERP software for the company's development. There are four candidate ERP software available, denoted as $X = \{x_1, x_2, x_3, x_4\}$. The vice-general manager in charge of information technology, group information center director, and information technology expert are responsible for this decision making problem, whose weight vector $v=(v_1,v_2,\dots,v_n)$ is unknown.

Step 1. Three decision makers give the PLPRs as shown below.

$$H_1^P=\begin{pmatrix} \{s_0(1)\} & \{s_1(0.8),s_2(0.2)\} & \{s_{-1}(0.7),s_0(0.3)\} & \{s_{-2}(0.5),s_{-1}(0.5)\} \\ \{s_{-1}(0.8),s_{-2}(0.2)\} & \{s_0(1)\} & \{s_{-2}(0.9),s_{-1}(0.1)\} & \{s_{-3}(0.7),s_{-2}(0.3)\} \\ \{s_1(0.7),s_0(0.3)\} & \{s_2(0.9),s_1(0.1)\} & \{s_0(1)\} & \{s_{-1}(0.8),s_0(0.2)\} \\ \{s_2(0.5),s_1(0.5)\} & \{s_3(0.7),s_2(0.3)\} & \{s_1(0.8),s_0(0.2)\} & \{s_0(1)\} \end{pmatrix}$$

$$H_2^P=\begin{pmatrix} \{s_0(1)\} & \{s_{-1}(0.5),s_0(0.5)\} & \{s_1(0.4),s_2(0.6)\} & \{s_0(0.6),s_1(0.4)\} \\ \{s_1(0.5),s_0(0.5)\} & \{s_0(1)\} & \{s_1(0.9),s_2(0.1)\} & \{s_0(0.1),s_1(0.9)\} \\ \{s_{-1}(0.4),s_{-2}(0.6)\} & \{s_{-1}(0.9),s_{-2}(0.1)\} & \{s_0(1)\} & \{s_{-2}(0.2),s_{-1}(0.8)\} \\ \{s_0(0.6),s_{-1}(0.4)\} & \{s_0(0.1),s_{-1}(0.9)\} & \{s_2(0.2),s_1(0.8)\} & \{s_0(1)\} \end{pmatrix}$$

$$H_3^P=\begin{pmatrix} \{s_0(1)\} & \{s_2(0.6),s_3(0.4)\} & \{s_{-2}(0.5),s_{-1}(0.5)\} & \{s_0(0.8),s_1(0.2)\} \\ \{s_{-2}(0.6),s_{-3}(0.4)\} & \{s_0(1)\} & \{s_{-3}(0.9),s_{-2}(0.1)\} & \{s_0(0.2),s_1(0.8)\} \\ \{s_2(0.5),s_1(0.5)\} & \{s_3(0.9),s_2(0.1)\} & \{s_0(1)\} & \{s_1(0.3),s_2(0.7)\} \\ \{s_0(0.8),s_{-1}(0.2)\} & \{s_0(0.2),s_{-1}(0.8)\} & \{s_{-1}(0.3),s_{-2}(0.7)\} & \{s_0(1)\} \end{pmatrix}$$

Step 2. Use MOD 2 to check whether the expected consistency of PLPR is acceptable or not. We take H_1^P as an example, then get the following optimization model.

$$\min \quad D=\sum_{i=1}^{4}\sum_{j>i}^{4}d_{ij}^{+}+d_{ij}^{-}$$

$$\text{s.t.}\quad \begin{cases} (\omega_1+\omega_2)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{12}^{(q)}\right)+3}{6}\cdot p_{12}^{(q)}-\omega_1-d_{12}^{+}+d_{12}^{-}=0, \\ (\omega_1+\omega_3)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{13}^{(q)}\right)+3}{6}\cdot p_{13}^{(q)}-\omega_1-d_{13}^{+}+d_{13}^{-}=0, \\ (\omega_1+\omega_4)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{14}^{(q)}\right)+3}{6}\cdot p_{14}^{(q)}-\omega_1-d_{14}^{+}+d_{14}^{-}=0, \\ (\omega_2+\omega_3)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{23}^{(q)}\right)+3}{2\tau}\cdot p_{23}^{(q)}-\omega_2-d_{23}^{+}+d_{23}^{-}=0, \\ (\omega_2+\omega_4)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{24}^{(q)}\right)+3}{6}\cdot p_{24}^{(q)}-\omega_2-d_{24}^{+}+d_{24}^{-}=0, \\ (\omega_3+\omega_4)\cdot\sum_{q=1}^{2}\dfrac{I\left(L_{34}^{(q)}\right)+3}{6}\cdot p_{34}^{(q)}-\omega_3-d_{34}^{+}+d_{34}^{-}=0, \\ \sum_{i=1}^{4}\omega_i=1,\omega_i>0 \\ d_{ij}^{+},d_{ij}^{-}\geq 0, i,j=1,2,3,4, j>i. \end{cases}$$

By solving this optimization model, we obtain the following results: $D=0.0484$, $\omega_1=0.1632$, $\omega_2=0.0636$, $\omega_3=0.2835$, $\omega_4=0.4897$, $d_{12}^-=0.0044$, $d_{13}^+=0.0080$, $d_{24}^-=0.0360$, $d_{12}^+=d_{13}^-=d_{14}^+=d_{14}^-=d_{23}^+=d_{23}^-=d_{24}^+=d_{34}^+=d_{34}^-=0$. According to the Eq. (4.5), we have $ECI_{H_1^P}=0.0081<0.02$, so the expected consistency of H_1^P is acceptable.

By the same way, we obtain $ECI_{H_2^P}=0.0117<0.02, ECI_{H_3^P}=0.0192<0.02$. Hence, the PLPRs given by three decision makers are all of the acceptable consistency. However, the weight vector $v=(v_1,v_2,\ldots,v_n)$ of DMs is unknown. We can use the PLPA operator to obtain the weight of each linguistic value. Then use them to aggregate.

Step 3. The individual PLPRs are aggregated into one aggregated HFLPR by using the PLPA operator. Assume that the support function is given by

$$Sup\left(L_{ij}\left(p_{ij}\right)_k, L_{ij}\left(p_{ij}\right)_l\right)=1-\frac{d\left(L_{ij}\left(p_{ij}\right)_k, L_{ij}\left(p_{ij}\right)_l\right)}{\sum_{k,l=1,k<l}^{m} d\left(L_{ij}\left(p_{ij}\right)_k, L_{ij}\left(p_{ij}\right)_l\right)}, \tag{5.1}$$

where

$$d\left(L_{ij}\left(p_{ij}\right)_k, L_{ij}\left(p_{ij}\right)_l\right)=\frac{\left|I\left(E\left(L_{ij}\left(p_{ij}\right)_k\right)\right)-I\left(E\left(L_{ij}\left(p_{ij}\right)_l\right)\right)\right|}{2\tau} \tag{5.2}$$

The supports of the PLTS $L_{ij}\left(p_{ij}\right)_k$, $T\left(L_{ij}\left(p_{ij}\right)_k\right), i, j=1,2,3,4, i\neq j$, are got as

$$T_1=\begin{pmatrix} - & 1.5000 & 1.5001 & 1.0526 \\ 1.5000 & - & 1.5000 & 1.0139 \\ 1.5001 & 1.5000 & - & 1.4999 \\ 1.0526 & 1.0139 & 1.4999 & - \end{pmatrix}$$

$$T_2=\begin{pmatrix} - & 1.2069 & 1.1291 & 1.4473 \\ 1.2069 & - & 1.1250 & 1.4861 \\ 1.1291 & 1.1250 & - & 1.4311 \\ 1.4473 & 1.4861 & 1.4311 & - \end{pmatrix}$$

$$T_3=\begin{pmatrix} - & 1.2931 & 1.3710 & 1.5001 \\ 1.2931 & - & 1.3750 & 1.5000 \\ 1.3710 & 1.3750 & - & 1.0690 \\ 1.5001 & 1.5000 & 1.0690 & - \end{pmatrix}$$

We take $T\left(L_{12}\left(p_{12}\right)_k\right), k=1,2,3$ as an example to show the computational process clearly.

The distances between $L_{12}\left(p_{12}\right)_1, L_{12}\left(p_{12}\right)_2, L_{12}\left(p_{12}\right)_3$ are calculated by Eq. (5.2) as $d\left(L_{12}\left(p_{12}\right)_1, L_{12}\left(p_{12}\right)_3\right)=0.2$, and $d\left(L_{12}\left(p_{12}\right)_2, L_{12}\left(p_{12}\right)_3\right)=0.4833$. The supports between $L_{12}\left(p_{12}\right)_1, L_{12}\left(p_{12}\right)_2, L_{12}\left(p_{12}\right)_3$ are obtained by Eq. (5.1) as follows:

$Sup\left(L_{12}\left(p_{12}\right)_1, L_{12}\left(p_{12}\right)_2\right)=1-\frac{0.2833}{0.9666}=0.7069$, $Sup\left(L_{12}\left(p_{12}\right)_1, L_{12}\left(p_{12}\right)_3\right)=0.7931$, $Sup\left(L_{12}\left(p_{12}\right)_2, L_{12}\left(p_{12}\right)_3\right)=0.5$.

Hence, we can get the supports of $L_{12}(p_{12})_1, L_{12}(p_{12})_2, L_{12}(p_{12})_3$ as

$T\left(L_{12}(p_{12})_1\right) = Sup\left(L_{12}(p_{12})_1, L_{12}(p_{12})_2\right) + Sup\left(L_{12}(p_{12})_1, L_{12}(p_{12})_3\right) = 1.5$,

$T\left(L_{12}(p_{12})_2\right) = Sup\left(L_{12}(p_{12})_1, L_{12}(p_{12})_2\right) + Sup\left(L_{12}(p_{12})_2, L_{12}(p_{12})_3\right) = 1.2069$,

$T\left(L_{12}(p_{12})_3\right) = Sup\left(L_{12}(p_{12})_1, L_{12}(p_{12})_3\right) + Sup\left(L_{12}(p_{12})_2, L_{12}(p_{12})_3\right) = 1.2931$.

By means of the PLPA operator, we can aggregate the single PLPRs into the collective PLPR H^P as follows.

$$H^P = \begin{pmatrix} \{s_0\} & \left\{\begin{matrix} s_{0.2676}(0.04), s_{0.3357}(0.04), \\ s_{0.3625}(0.06), s_{0.3905}(0.06), \\ s_{0.4381}(0.24), s_{0.4451}(0.24), \\ s_{0.4518}(0.16), s_{0.4688}(0.16) \end{matrix}\right\} & \left\{\begin{matrix} s_{0.2754}(0.14), s_{0.3225}(0.14), \\ s_{0.345}(0.06), s_{0.3707}(0.06), \\ s_{0.5057}(0.09), s_{0.54}(0.09), \\ s_{0.57}(0.21), s_{0.6329}(0.21) \end{matrix}\right\} & \left\{\begin{matrix} s_{0.1714}(0.24), s_{0.2464}(0.24), \\ s_{0.3771}(0.16), s_{0.3857}(0.06), \\ s_{0.3921}(0.16), s_{0.3982}(0.06), \\ s_{0.42}(0.04), s_{0.4225}(0.04) \end{matrix}\right\} \\ / & \{s_0\} & \left\{\begin{matrix} s_{0.1371}(0.729), s_{0.2057}(0.081), \\ s_{0.2282}(0.081), s_{0.2754}(0.009), \\ s_{0.6329}(0.009), s_{0.6957}(0.081), \\ s_{0.7257}(0.009), s_{0.8171}(0.081) \end{matrix}\right\} & \left\{\begin{matrix} s_{0.1071}(0.014), s_{0.1786}(0.006), \\ s_{0.375}(0.056), s_{0.375}(0.126), \\ s_{0.3869}(0.024), s_{0.3869}(0.054), \\ s_{0.4196}(0.504), s_{0.4216}(0.216) \end{matrix}\right\} \\ / & / & \{s_0\} & \left\{\begin{matrix} s_{0.2754}(0.048), s_{0.3225}(0.192), \\ s_{0.345}(0.012), s_{0.3707}(0.048), \\ s_{0.5057}(0.112), s_{0.54}(0.028), \\ s_{0.57}(0.448), s_{0.6329}(0.112), \end{matrix}\right\} \\ / & / & / & \{s_0\} \end{pmatrix}$$

We take $L_{12}(p_{12})$ as an example to show the aggregation process. For convenience, we keep one decimal of $T\left(L_{ij}(p_{ij})_k\right)$.

$$\begin{aligned} L_{12}(p_{12}) &= PLPA\left(L_{12}(p_{12})_1, L_{12}(p_{12})_2, L_{12}(p_{12})_3\right) \\ &= \frac{\sum_{k=1}^{3}\left(1+T\left(L_{12}(p_{12})_k\right)\right)L_{12}(p_{12})_k}{\sum_{k=1}^{3}\left(1+T\left(L_{12}(p_{12})_k\right)\right)} \\ &= \frac{2.5\{s_1(0.8), s_2(0.2)\} \oplus 2.2\{s_{-1}(0.5), s_0(0.5)\} \oplus 2.3\{s_2(0.6), s_3(0.4)\}}{2.5+2.2+2.3} \\ &= \left\{\begin{matrix} s_{0.4451}(0.24), s_{0.4688}(0.16), s_{0.4381}(0.24), s_{0.4518}(0.16), \\ s_{0.3625}(0.06), s_{0.2676}(0.04), s_{0.3905}(0.06), s_{0.3357}(0.04) \end{matrix}\right\} \end{aligned}$$

Step 4. Then still use MOD 2 to check whether the expected consistency of collective PLPR is acceptable or not. By solving this optimization model, we obtain the following results: $D = 0.0720$, $\omega_1 = 0.3279$, $\omega_2 = 0.2460$, $\omega_3 = 0.2385$, $\omega_4 = 0.1876$, $d_{14}^- = 0.0440$, $d_{23}^+ = 0.0183$, $d_{34}^+ = 0.0097$, $d_{12}^+ = d_{12}^- = d_{13}^+ = d_{13}^- = d_{14}^+ = d_{23}^- = d_{24}^+ = d_{24}^- = d_{34}^- = 0$. According to the Eq. (4.5), we have $ECI_{H_1^P} = 0.0120 < 0.02$, so the expected consistency of H^P is acceptable and we have $\omega_1 > \omega_2 > \omega_3 > \omega_4$, which implies that the priorities of the alternatives is $x_1 \succ x_2 \succ x_3 \succ x_4$.

6. Conclusion

PLTS is a novel extension form of the existing linguistic variables. Based on it and PLPRs, we develop the expected consistency of PLPRs. By the modeling method, we can check whether one PLPR is expected consistent or not. Then we put forward the PLPA operator to aggregate individual preference relations into the collective one. Finally, we can obtain the priority weights of alternatives for the ranking to solve the GDM problems. In this paper, we are absorbed in the researches of the consistency and the aggregation of PLPRs, which have never been studied.

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