

## Analysis of thermal error signal and mathematical modeling for machine tool spindle

Jiang Wang <sup>a</sup>, Kang Liu <sup>b</sup> and Yuankai Meng <sup>c</sup>

<sup>1</sup>Faculty of Mechanical Engineering, Sichuan University of science & engineering, Zigong 643000, Sichuan, China.

<sup>a</sup>javonwang@hotmail.com, <sup>b</sup>315381847@qq.com, <sup>c</sup>Bryant413@163.com

### Abstract

In recent years, many studies have been conducted to develop the thermal displacement errors compensation. However, the compensation models have poor robustness, because the models can not reflect the essential characteristics of thermally induced deformation and they are just formed by the comprehensive error data not the pure thermal error data. From a novel perspective of signal, the thermally induced error signal (TIES) is a part of comprehensive error signal, and the characteristics of thermal displacement errors are the characteristics of TIES. When seeking the compensation models, the exploration of the characteristics of TIES should be put on the first place. In this paper, the characteristics of TIES of the machine tool spindle was analyzed theoretically. In order to gain the comprehensive error signal, experiments were taken on different machine tool spindles of same series. A novel algorithm of envelop attenuation was proposed for separating the TIES from the comprehensive error signal. The results of separation verify the rightness of the theoretic analysis of the characteristics of TIES and reveal the specific characteristics. Two characteristics are concluded from the experiments, one is that the variation trend of TIE signal accords with quadratic curve, another is that the curve is proportional to the change of ambient temperature. This study will increase the understanding of the law of thermally induced error. Based on the two characteristics of TIES, the robustness of the thermally induced error compensation model can be fundamentally improved.

### Keywords

thermally induced error characteristics; Robustness; Envelop attenuation; Machine tool spindle; Signal separatio.

### 1. Introduction:

With the development of society, the modern life has a strong requirement for the high precision of products. However, the key source of high precision is the precision of machine tool. In recent years, TIE are becoming more and more important in the precision engineering field where the specifications are ever increasing. The TIE has already accounted for 60%~70% of the total error [1]. Many experts and scholars have been making unremitting exploration in this field. Their works mainly concentrates on the optimization of machine tool thermal structure and thermal error compensation, and it also applies some auxiliary methods like applying and controlling additional heat loads [2], cooling from the outside and so on.

For developing the precision of a completed machine tool, the compensation method is a very economical and simple choice, it can develop the precision without changing the local structure of the machine tool. There are two kinds of compensation models, one is empirical model and another is theoretical model. However, most of the compensation models have the same serious shortcoming in robustness, the poor robustness is the essential reason why the compensation model can be hardly applied in engineering practice [3]. Based on the experimental data, a mapping relation between the temperatures of heating elements and errors has been found form the data by utilizing the pure mathematical method. Due to the neural networks has powerful processing ability in nonlinear

mapping, it is usually used to train the compensation model, and the compensation model works well just in the condition which is similar to the experiment environment. The grey theory, genetic algorithm and particle swarm optimization algorithm are added to the neural network, in order to improve the prediction effect of the compensation model [4-6]. In references [7-12], support vector machine, support vector regression, independent component analysis, projection pursuit regression and Bayesian network have made some progress for precision and less robustness. For the sake of improving the robustness, other methods like ARMA model, Elman neural network, even the way of considering different speeds in a certain time as the independent variables have their own advantages [13-15]. In terms of robustness, no matter what kind of mathematic approaches are taken, these improved models have little effect. These different modeling methods completely depends on the experimental condition, all of the researches jumped across the first step work. The study of the characteristics of TIE should be put on the first place. It is known that, theoretical compensation model has the strong robustness. The process of establishing the compensation models is described in the literatures [16-18]. According to the principle of heat generation and transfer and the simplification of the structure of the machine tool spindle, a equation of heat and deformation is built and solved. However, although the theoretical model in terms of robustness was very excellent, there are still some defects in solving the equation. Such as, the structure of the machine was too simplified to provide enough accuracy of the model in solving, the thermal boundary conditions are so complex that it will cost too much time to solve the equation by finite-element method, thus the theoretical model is unsuitable for real-time compensation. It is very suitable for the optimization of thermal structures in off-line analysis .

In this paper, according to the above analysis of the shortage of empirical models and theoretical models, a novel perspective of signal was introduced. Firstly, it needs to analyze the characteristics of comprehensive error signal components. The next is to propose a efficient algorithm to separate the TIES. The novel algorithm is the envelop attenuation method and it can successfully extract the TIES from the comprehensive error signal. Besides, the specific characteristics of the TIES is explored by using different fitting methods. The fitting curve is the real thermally induced error curve, and the robustness of the compensation model will be foundationally improved by using the real thermal error data, which will make up the deficiency between empirical models and theoretical models.

## 2. Analysis of the components of comprehensive error signal of machine tool spindle

The comprehensive error of machine tool spindle is formed by many different errors, it includes geometric error of the components and structure, assembly error, transmission error, thermally induced deformation error, deformation error caused by self weight and other errors like vibration and so on. Among all of the error sources, the thermally induced deformation error has its special property, the mathematical relation between temperature and TIE error can be shown as:

$$E=F(T) \quad (1)$$

Where E is the variable of thermal error, T is the variable of temperature.

As we all known, according to the principle of heat generation and superposition, T is a continuous changing quantity, the independent variable E is also an increasing quantity caused by the continuous increasing dependent variable T. In the condition of the continuous rotation of the machine tool spindle, the heat generation is consecutive, so we can reasonably assume that the signal of TIE of machine tool spindle is a continuous monotonous curve. Because of the other error signals of the machine tool spindle such as assembly error, rotation error and transmission error do not have this characteristic, and the change trend of those errors are not have the characteristic of continuous increase in a very long time. So if a monotonous curve can be separated from the comprehensive error data, then the curve is extremely related to the thermal error in great extent. The error signal is not static error but dynamic error, and what we looking for is the monotone thermal error curve, so to

filter the other high frequency signals from the comprehensive signal is a certain goal. The comprehensive error signal of machine tool spindle can be described in mathematics as:

$$X(t) = \sum_{i=1}^n H_i(t) + L(t) \tag{2}$$

Where  $X(t)$  is the comprehensive error signal of the Z-axis of the machine tool spindle,  $H_i(t)$  is the high frequency signals, namely non monotonic dynamic error,  $L(t)$  is a low frequency signal, namely monotonic dynamic error, the change range is from the beginning of zero to the end of maximum in thermal balance.

### 3. Separation algorithm

#### 3.1 Envelope attenuation method

As it is shown the figure 1, there is a comprehensive signal and a monotonous signal, the comprehensive signal is formed by the unique monotonous signal and some other signals with high frequency. Usually, when talking about the signal separation, Fourier transform must be the first choice in head. However, the comprehensive error signal of machine tool spindle is not the regular stationary signal but the non-stationary periodic signal, so the Fourier transform are not able to deal with it effectively. Then, a new mathematical algorithm is put forward to separate the only monotonous signal from the comprehensive, called envelop attenuation method. For a certain comprehensive signal in figure 1, the monotonous signal can be extracted by attenuating the peak value along the direction of arrow, and the way of attenuation is ruled with envelop lines which are divided into up and down envelop lines in figure 2. If the high frequency signal are stationary periodic signals, then the up and down envelop lines can extract the needed signal in one step. As what mentioned before, the signal is not the stationary periodic signal. In fig.4, there is a gap between extracted signal and original signal, which reveals that the monotonous signal have not been completely extracted from the comprehensive signal by using the envelop attenuation in one time. Thus, it is necessary to deal with the signal in fig .5 till the average of its envelop lines equals zero. Then, the extracted signal can be same with the original signal before eliminated the high frequency signals.

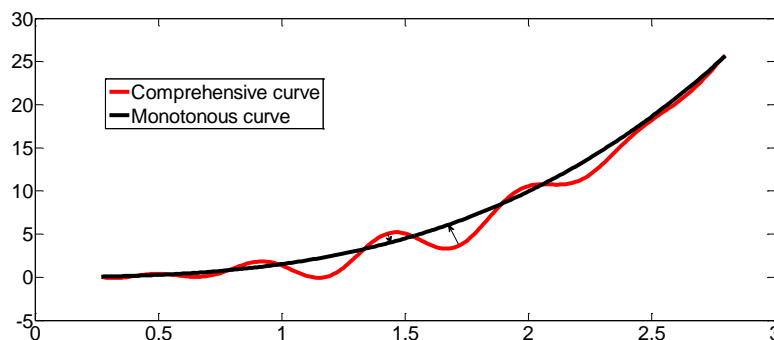


Fig. 1 Monotonous signal hiding in comprehensive signal

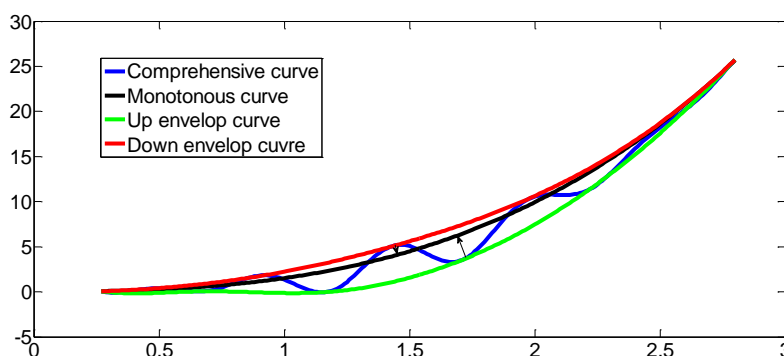


Fig.2 Up and down envelop lines of the comprehensive signal

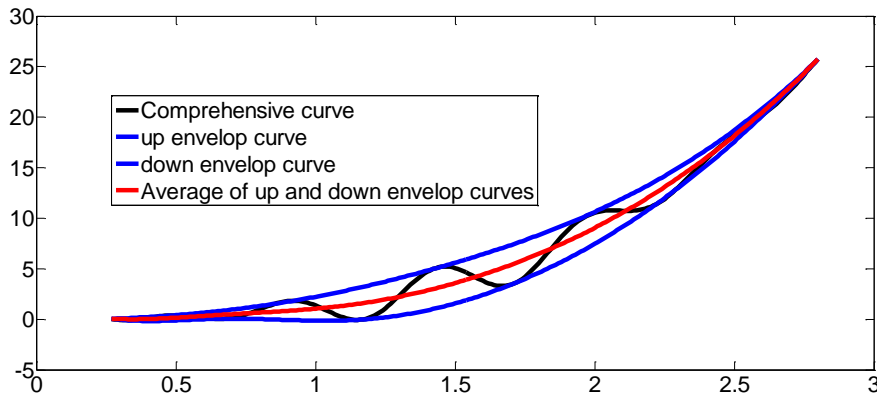


Fig.3 Average of up and down envelop lines

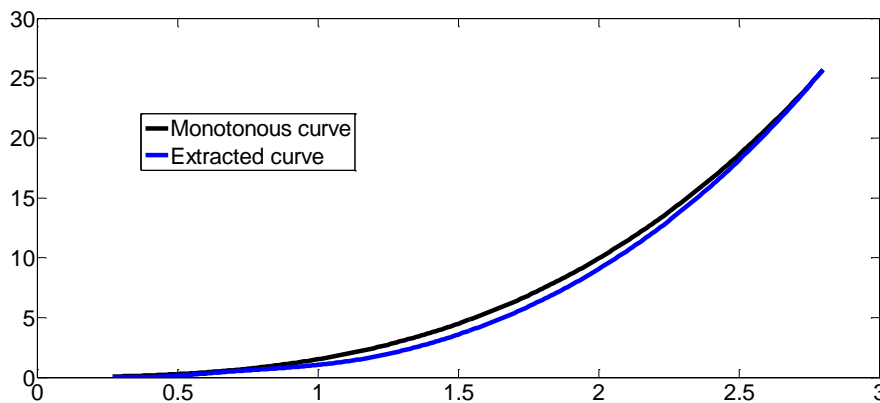


Fig.4 Gap between original signal and extracted signal

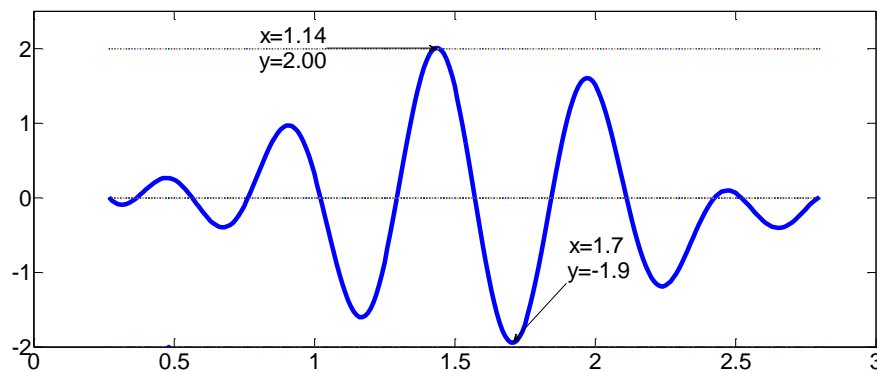


Fig.5 High frequency signal separated by one step separation

As formula (1-2) shows that the comprehensive error signal of machine tool spindle is composed of non monotonic dynamic signal and monotonic inherent thermal error signal, in order to filter out  $H_i(T)$  through the attenuation of  $X(t)$ , we use the envelope attenuation method to down the extreme points. The envelope attenuation method includes the following steps:

Step 1: In order to obtain comprehensive error signal  $X(T)$ 's local maximum value and local minimum values, and due to the combined error signal at the endpoint at both ends with the non malleability, we set the both endpoints of error signal as that both of them have the special characteristics of maximum value and minimum value to ensure the integrity of the amplitude of thermal error signal. Three times spline interpolation method is used to fit the maximum and minimum values to generate respectively the upper and lower envelope of  $S_1$  and  $S_2$  of  $X(t)$ .

Step 2: The average value of the envelope line, as shown in equation (3):

$$A = (S_1 + S_2)/2 \tag{3}$$

Utilizing the original signal and the average value to build a high frequency and low amplitude signal, as shown in equation (4):

$$X_1(t) = X(t) - A \tag{4}$$

Then turn to the first step to fit the upper and lower envelop of  $X_1(t)$ .

Step 3: If the average value of new envelopes is close to zero to some extent, then we set  $X_1(t)$  as the first high frequency signal which needs to be removed immediately, otherwise, continuing the above steps for  $X_1(t)$  until the mean of envelopes is close to zero. The judgment condition is:

$$|\max(A)| \leq 0.01 \tag{5}$$

Step 4: After eliminate  $X_1(t)$ , the source signal is attenuated into a new  $X(t)$ , then repeat those steps above for the new  $X(t)$ , until the next new  $X(t)$  only has no extreme point except for the endpoint we set before. However, there is still a problem that, when the signal of  $H_i(t)$  are mixed by a member of single peak signal, we can hardly fit the envelop by utilizing three times spline interpolation curve.

Step5: If we want to continuously apply the envelop attenuation method, there must exist two envelopes. We already have two extreme points, added with the single peak point, we obtained one signal curve by three times spline interpolation curve, another signal curve is made by a formed straight line of linking the two endpoint, then we got the two needed envelopes as the upper and lower envelop. Go on these steps described above for the new formed envelopes, we surprisingly found that, along with the peak value attenuated, the fluctuation of the new signal is increased, and the new signal has the enough extreme points again. Until the amplitude are attenuated in an acceptable arrangement, the final stop condition is:

$$\max|A_{i+1} - A_i| \leq 0.01 \tag{6}$$

The last  $X(t)$  approximate to an error signal curve without extreme point  $L(t)$ , namely thermal error signal.

### 3.2 Envelope attenuation method flow chart

As is shown in figure (1), the algorithm flow of envelop attenuation method to separate the thermal error signal is very simple and brief.

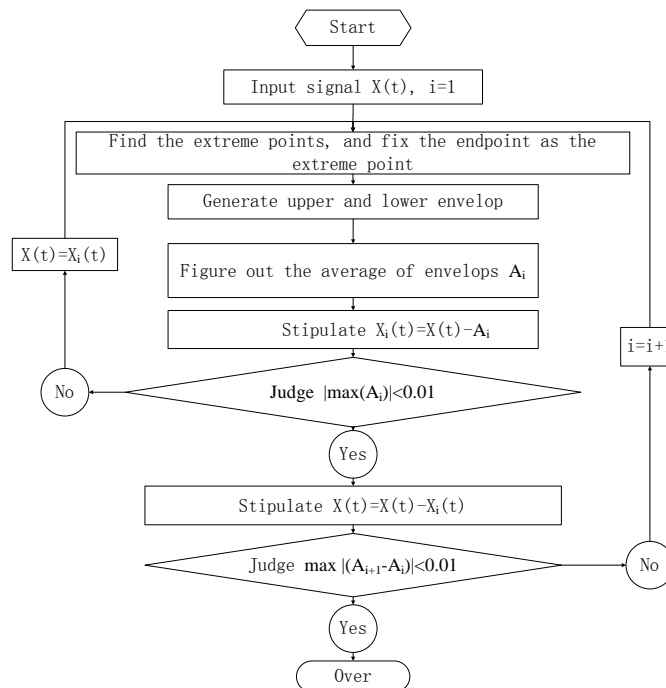


Fig.6 Flow chat of Envelope attenuation method

The model of the inherent thermal error signal is established based on the extracted inherent thermal error signal, according to the previous analysis, the inherent thermal error signal has characteristics

of monotonicity, so the mathematic method of curve fitting can be used to establish the corresponding inherent thermal error model. The way of curve fitting including linear, quadratic curve and cubic curve.

## 4. Experiments

### 4.1 Experimental scheme

(1) If there is a TIE signal of machine tool spindle, then the ITE signal coming from two spindle error of the same type should comply with the same rules. However, because the machine assembly is different, the machine TIE should not be completely same. So we can conclude that the TIE must have a same trend for a certain type of machine tool spindle, however, it must have differences in specific parameters. In order to explore the TIE signal from the machine tool spindle, we simultaneously measure the Z-axis error of two different machine tools spindle which are of the same series under the same external temperature.

(2) The lack of robustness of the thermal error compensation which mentioned in the introduction is largely caused by the change of the ambient temperature. Because of the influence of ambient temperature, there will be different TIE compensation models in different external environments. In order to make up for this deficiency, the inner relation among different models with different ambient temperature must be found. Therefore, we measure the Z-axis error of the same machine tool spindle in different environmental temperatures.

### 4.2 Experimental setup

The KVC series of Changzheng machine tools, KVC-640 and KVC850, which have the same similar structure, are taken to be the experimental machine tools. The Eddy current sensor DT3005 is adopted to measure the displacement of the Z direction of machine tool spindles. It's a non-contact linear measuring instrument about displacement, which can measure the distance between the measured metal conductor and the probe surface in a static and dynamic way. As a non contact displacement sensor, it has a lot of characteristics which make it become the best choice in this experiment. The characteristics of high precision, high linearity, high resolution and powerful ability of measuring the static and dynamic displacement between the measured object (metal conductor) and the probe end surface.



Fig.7 Installation of DT3005 displacement sensor

### 4.3 Visualization of experimental data

The Z-axis error data of the machine tool spindle of two different machine tools measured at ambient temperature of 9.8 degree Celsius are shown in figure (2,3) respectively. The same test scheme was

carried out at ambient temperature of 20.1 degree Celsius, and the recorded experimental data are shown in figure (4, 5).

When the ambient temperature is 9.8 degree Celsius.

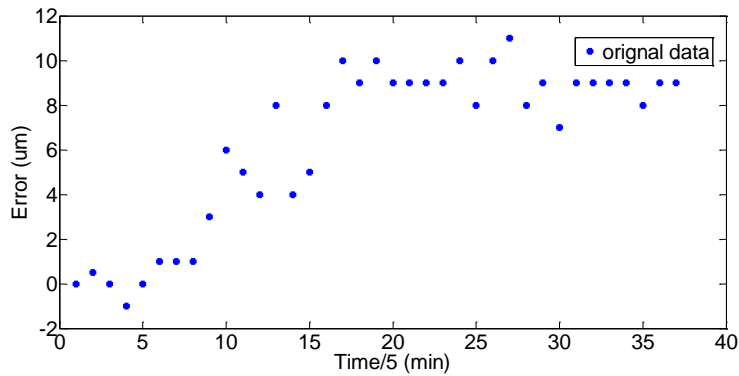


Fig.8 Z-axis comprehensive error of machine tool spindle of KVC-850

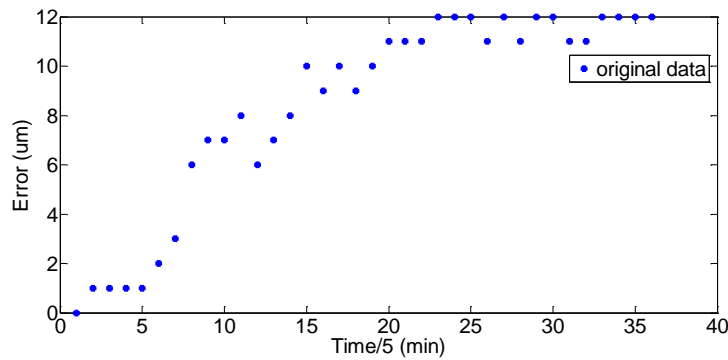


Fig.9 Z-axis comprehensive error of machine tool spindle of KVC-640

When the ambient temperature is 25.1 degree Celsius.

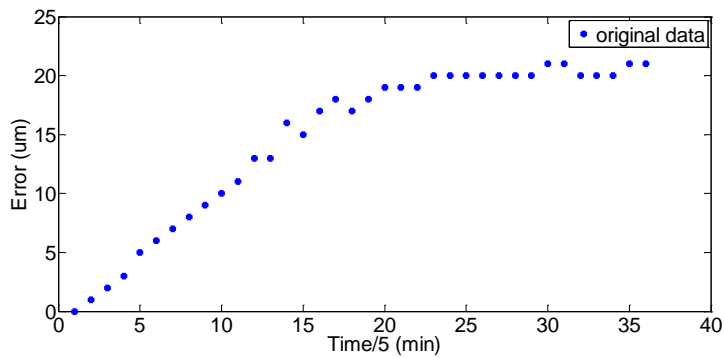


Fig.10 Z-axis comprehensive error of machine tool spindle of KVC-850

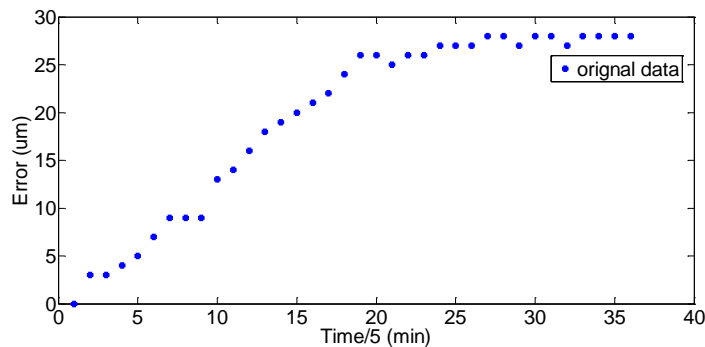


Fig.11 Z-axis comprehensive error of machine tool spindle of KVC-640



From the comprehensive original experiment data, it could be concluded that the error of the machine tool spindle would reach a steady value in about three hours, and the fluctuation of error data is larger at low temperature.

**4.4 Processing of experimental data**

Based on the MATLAB platform, using the envelope attenuation algorithm to separate the inherent thermal error signal from the total error signal of machine tool spindle. The extracted thermal error signal is fitted by linear, quadratic curve and cubic curve. the fitted equations and the residuals are respectfully showed in the figure (6,7,8,9).

When the ambient temperature is 9.8 degree Celsius.

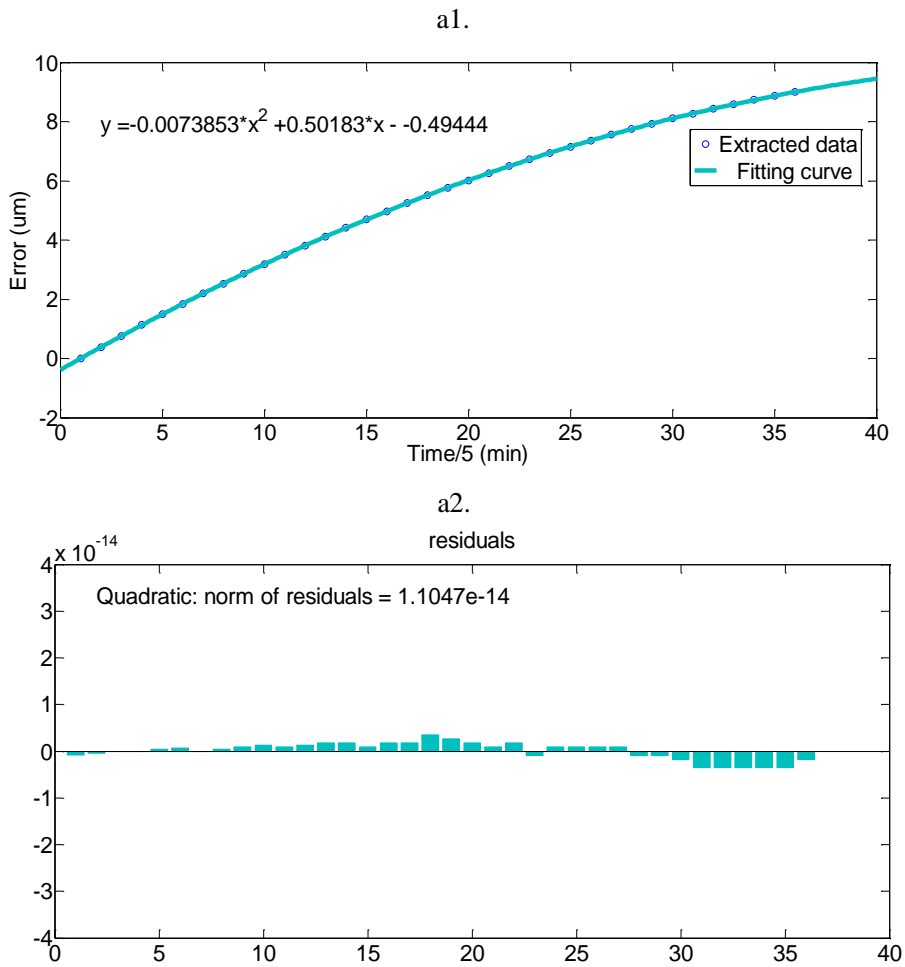
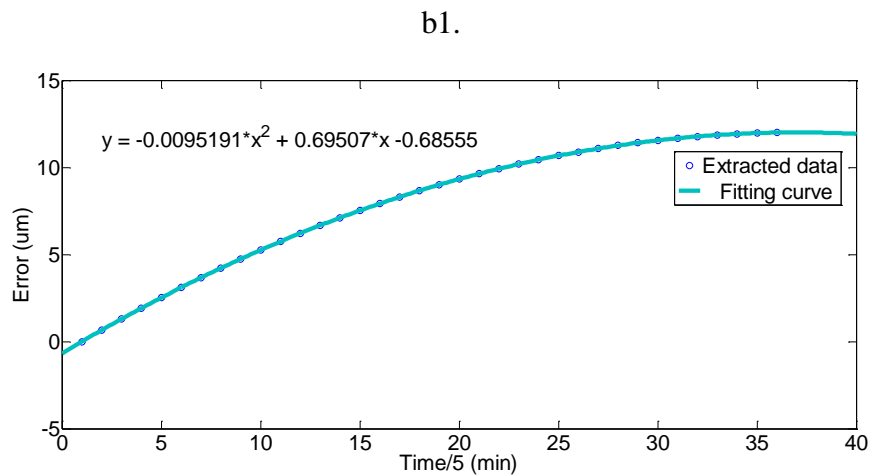


Fig.12 Fitting curve of TIE signal (a1) and the fitting residuals (a2) in KVC-850





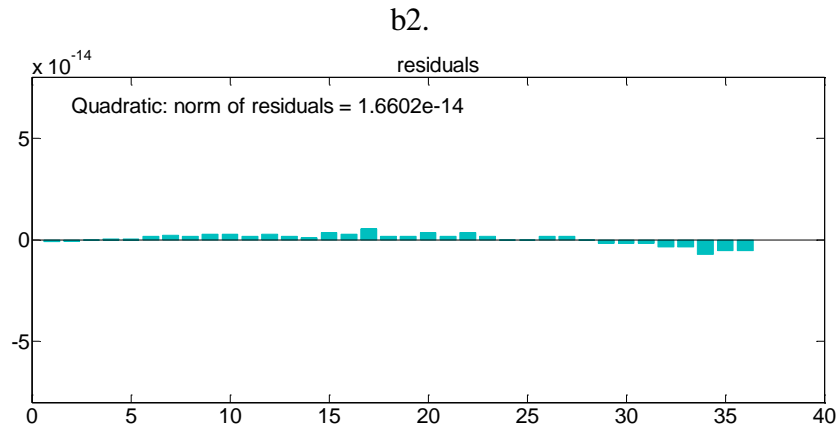


Fig.13 Fitting curve of TIE signal (b1) and the fitting residuals (b2) in KVC-640 When the ambient temperature is 20.1 degree Celsius.

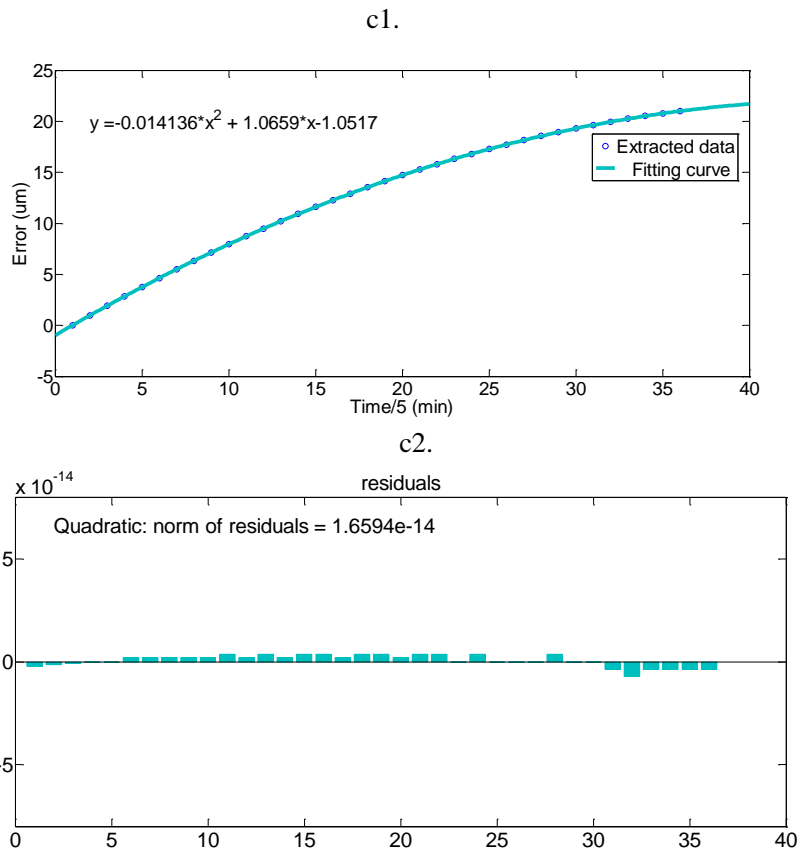
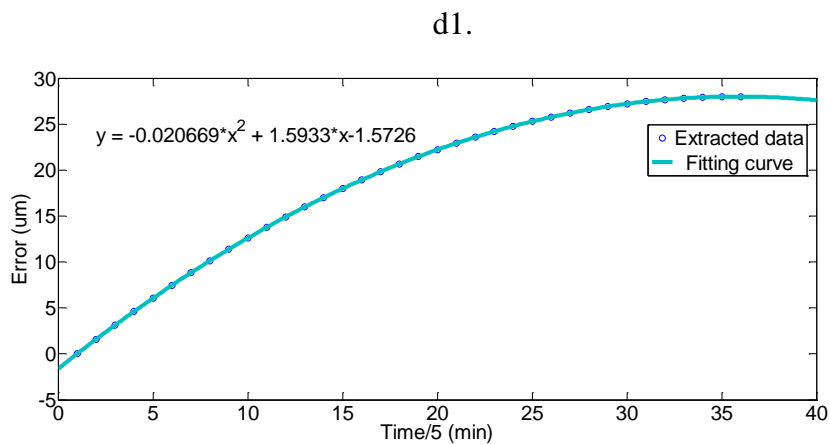


Fig.14 Fitting curve of TIE signal (c1) and the fitting residuals (c2) in KVC-850



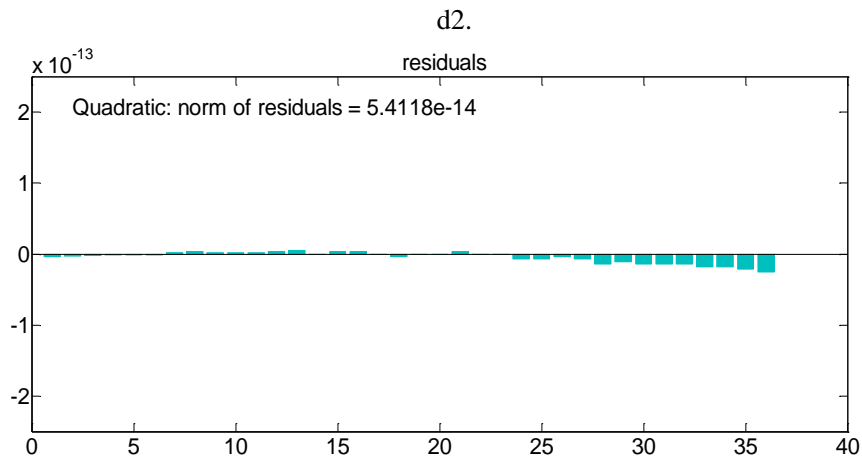


Fig.15 Fitting curve of TIE signal (d1) and the fitting residuals (d2) in KVC-640

Form the four pictures (a1, b1, c1, d1) in above, it is clear to see that, for different machine tools, there is a certain curve which can be extracted successfully from the comprehensive error signal. Because of fitting residual which is shown in the pictures (aa2,b2,c2,d2), the max fitting residual is so small that the separated TIE signal can be treated as a quadratic curve. In order to explore the relation between the TIE signal curve and ambient temperature, the coefficients of the fitting equation and ambient temperatures are put in the table (1, 2).

Table 1.Fitting equation of extracted data from KVC-850

Expression	Coefficient of quadratic equation under the environment temperature P <sub>0</sub>	Coefficient of quadratic equation under the environment temperature P <sub>0</sub> '	The corresponding coefficient ratio C <sub>i</sub> = P <sub>i</sub> ' / P <sub>i</sub>
$y = P_1 * x^2 + P_2 * x + P_3$	P <sub>0</sub> =9.8°C	P <sub>0</sub> '=20.1°C	C <sub>0</sub> = 2.0510
	P <sub>1</sub> =-0.0073853	P <sub>1</sub> '=-0.014136	C <sub>1</sub> = 1.9140
	P <sub>2</sub> =0.50183	P <sub>2</sub> '=1.0659	C <sub>2</sub> = 2.0565
	P <sub>3</sub> =-0.49444	P <sub>3</sub> '=-1.0517	C <sub>3</sub> = 2.1270

Table 2.Fitting equation of extracted data from KVC-640

Expression	Coefficient of quadratic equation under the environment temperature P <sub>0</sub>	Coefficient of quadratic equation under the environment temperature P <sub>0</sub> '	The corresponding coefficient ratio C <sub>i</sub> = P <sub>i</sub> ' / P <sub>i</sub>
$y = P_1 * x^2 + P_2 * x + P_3$	P <sub>0</sub> =9.8°C	P <sub>0</sub> '=20.1°C	C <sub>0</sub> = 2.0510
	P <sub>1</sub> =-0.0095191	P <sub>1</sub> '=-0.020669	C <sub>1</sub> = 2.1713
	P <sub>2</sub> =0.69507	P <sub>2</sub> '=1.5933	C <sub>2</sub> = 2.2922
	P <sub>3</sub> =-0.68555	P <sub>3</sub> '=-1.5726	C <sub>3</sub> = 2.2939

A certain machine tool under different environment temperatures will generate different curves which are showed in the tables above. The corresponding coefficient ratio in these tables reveals that the effect of temperature on error for a certain machine tool is proportional in some degree. It is proved that the ambient temperature is an important and relevant factor for the TIE signal curves change. Within error permissibility, the coefficient ratios may be seen as a stable value. According to the tables above, the relation between ambient temperature and TIE signal curve can be expressed in mathematics as:

$$E = \frac{T}{T_0} (A * y^2 + B * y + C) \tag{7}$$

Where A, B and C are coefficients of the inherent thermal error of the machine tool spindle, which represent the different installation structure of machine tool spindle, T is the different stage of ambient temperature, and T<sub>0</sub> is the based temperature when measuring the inherent thermal error curve of machine tool spindle, y is the variable about time.

## 5. Thermally induced error compensation

When the TIE model of the machine tool spindle have been built, it needs to be apply into the machine tool to develop the precision. In order to realize real-time compensation, the TIE model needs a mechanism of deciding when to compensate and how to feedback. The TIE model a variable about time, and the temperature of the machine tool temperature sensitive point is also a variable about time, so we can use the corresponding mathematical method to make the thermal error model associated with the temperature. In document [2-10], the mainly object of study is the relationship between the temperature and the thermal error, adding with the description about the application of error compensation method by using the dot offset method. These works will not be repeated in the article. The TIE compensation model established with the TIE signal has strong accuracy and robustness, will greatly improve the engineering application of thermal error compensation model.

## 6. Conclusion

In present study, based on the theoretical analysis of the error components of the machine tool spindle and the experiments, and from a perspective of signal, a new algorithm of signal separation were adopted for the exploration of the TIE of machine tool spindle. The main conclusions are drawn as follows:

The theoretical analysis of the components of the error signal of machine tool spindle reveals that the thermal error signal of the machine tool spindle is a continuous monotonic curve but the others are not.

A monotonic signal was successfully extracted by using the newly proposed algorithm of envelope attenuation from the comprehensive signal, which shows the validity of the algorithm.

The experimental results show that the measured errors are not regular signals, which cannot stand for the TIE signals, but the general trend is similar with the TIE signal.

The processing of experimental data reveals two conclusions. One is that the trend of thermal error characteristics which accords with the quadratic curve is same for different machine tools of the same structure. The other is that the thermal error characteristics has close relation with the ambient temperature as the formula (7) saying.

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