An improved new delay-dependent criteria for robust stability of Networked Control systems

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Abstract

This paper provides a new stability criterion for time delay-dependent systems based on free weighting matrices and Lyapunov-Krasovskii methods. Free weighting matrices are adopted to express the relationship between the terms in the Leibniz-Newton formula. Existing methods are conservative because of adopting the upper bound approach to estimate the derivative of the Lyapunov-Krasovskii function, using some changes of variables and Schur complements, the sufficient condition for the systems is obtained, which is less conservative and more effective. Numerical examples are given to show the effectiveness of this method.

Keywords

Networked control system, upper bound,time-delay, Leibniz-Newton, stability.

1. Introduction

Time-delay phenomena extensively exist in many kinds of Industry systems, thence, the analysis and synthesis of the systems become more complicated. Meanwhile the time-delay affects the stability and performance of the systems. So time-delay analysis is one of the most hottest issues in Control theory and Control engineering. Due to the length of time-delays, there are time-delay dependent stability and independent stability. The time-delay independent stability is more conservative than dependent stability , in a general way ,due to it can be applied to delays with arbitrary size , and it is well known by [1,2]. Delay-dependent results are less conservative than the delay-independent ones, so more and more studies focus on the delay-dependent stability in recent years. An additional assumption should be required in an inequality , which is known to be conservative,used to determine the stability of the system is $-2a^{T}b < a^{T}Xa + b^{T}X^{-1}b, a, b \in R^{n}, X > 0$. An improved version was presented by Park in [3], introducing a free matrix M between a and b, which was shown, $-2a^{T}b < (a + Mb)^{T}X(a + Mb) + b^{T}X^{-1}b + 2b^{T}Mb$, and obtained some better delay-dependent criteria for linear time-delay systems than the previous ones. Moon used this bounding method to a more general form for uncertain systems with time-invariant delays in [4]. A descriptor system approach applied to obtain delay-dependent stability in terms of linear matrix inequalities, whichwas proposed in [5], Fridman & Shaked improved these stability results by incorporating the bounding method and the descriptor system approach in [6], by introducing a similar type of Lyapunov-Krasovskiifunctionals which is based on a descriptor form representation of the system, andobtained more efficient criteria for systems with or without polytopic-type uncertainties.

[7] Concluded the essentially relationships among the systems variables, and among the terms in the Leibniz-Newton formula.However, all these methods mentioned above based on the Leibniz formula and Park or Moon’s inequality cannot entirely overcome the conservative of bounding method. In the derivative of the Lyapunov functional, the term $\dot{x}$ is retained, the relationship among the terms in the systems equation is expressed by some free weighting matrices[10], as well as the relationship

between$x(t)$, $x(t-d(t)$, and $\int_{t-d(t)}^{t}\dot{x}(s)ds$ then the parameter-dependent method is used together with a relaxed LMI technique, andvarious robust stability tests based on parameter dependent method have a progress in recent years[10]. Recently, [12]summarized great efforts in time delay systems. It is found that there are redundant variables in these equivalent stability results in[6], and proved that lemma1-lemam7 is equivalent to each other respectively by Schur Completment[14] and LMI[21] method.In [13], It was noted that the method introducing more variables in the derivation of delay-dependent stability conditions is efficient and less conservative. It is well known that there are two ways to introduce more variables: one is adding slack matrix variables to certain terms in the derivative of a chosen Lyapunov-Krasovskiifunctional[5,7,17]by applying the Newton-Leibniz formula; and the other is constructing new Lyapunov-Krasovskiifunctionals with more matrix variables[9,10,11].However， these papers adopted the method of different amplifying the derivative of Lyapunov to estimate the upper bound, and got different stability criteria but conservative.

This paper aims to use the Lyapunov-Krasovskiiand free weighting matrix method to study and analysis the stability of NCSs. Due to dealing withthe networked control system model directly and not employ an system transformation, the results would be less conservative from such a transformation. some free weighting matrices but not fixed matrices are employed to express the influence of the terms of the Leibniz-Newton formula, and in the process of proving the inequality method are not employedto estimate the upper bound, so less conservative results are derived. And at last these matrices are determined by solving linear matrix inequalities.

Notation: Throughout this paper, the superscripts”-1” and “T” denote the inverse and transpose of a matrix, respectively; “P>0” represents that P is an positive and definite matrix; “I” is an appropriately dimensionidentity matrix; diag$\{\ldots\}$ denotes a block-diagonal matrix ; and the symbol “*” in a matrix stands for the symmetric terms.

2. Preliminaries

A typical structure of singe loop NCS is shown in Fig.1, where control components, sensor, controllers, actuators, and physical plant, are connected through a network which could be a wired or wireless. In the NCS, the sensor is time-triggered and the controller and actuator is event-triggered. Due to the transmission of data in the network, the Network-induced delays, which can be formulated as : $\tau(t_k) = \tau_{SC}(t_k) + \tau_{ca}(t_k)$, where $\tau(t_k)$denotes the total transmission delays, and the$\tau_{SC}(t_k)$ and $\tau_{ca}(t_k)$ represent the network delays from the sensor to the controller and from the controller to the actuators, respectively, are invertible in the NCS and degrade the stability performance of the system. In the normal situation, network-induced delays could be assumed as stochastic time-varying and bounded.

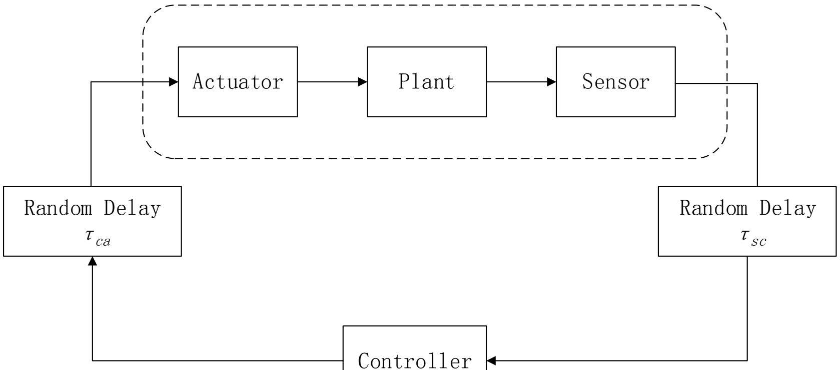

Fig .1 Structure of singe loop NCS

The system in Fig.1 can be described as :

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) \\ x(t) = \varphi(t) \end{cases} \tag{1}$$

In which, $x(t) \in R^n$, $y(t) \in R^n$, $u(t) \in R^m$ are the systems state vector, output vector and the control input vector, respectively. n, m is the dimensions of state vector; A, B are known real constant matrices with compatible dimensions. $\varphi(t)$ is the initial condition.

And the state feedback controller of the object can be described as :

$$u(t) = u_c(t - \tau_\infty) = Kx\big(t - (\tau_{sc} + \tau_{ca})\big) = Kx(t - d(t)) \tag{2}$$

Where

$$d(t) = \tau_{sc} + \tau_{ca}$$

Assume the state vector of system is complete detected， and the time delay induced by the controller is so small that can be ignored. Then the input state vector could be seen as delay state vector , which is similar to the time-delay system:

$$u(t) = A_d x(t - d(t)) \tag{3}$$

Actually, there are several issues induced from an NCS and the packet dropouts could be seen as delays as well. Then $d(t) = \tau_{sc} + \tau_{ca} + d_{sa}h$, in which dsc is an constant number means the numbers of the packet dropouts, satisfy $0 \le d_{sc} < \infty$, Apparently, when $d_{sc} = 0$ and $d_{sc} \to \infty$ means that there is no packet dropouts and no connection, respectively. Consider the τ_{sc} and τ_{ca} is stochastic and the total delay time is bounded, which satisfy for: $0 < d(t) \le \tau$, which tao is constant. Then combine the formula (1) and(3), the whole system $\sum_0$ can be described as:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d(t)) \\ x(t) = \varphi(t), t \in [-\tau, 0] \end{cases} \tag{4}$$

Where $A_d = BK$, denotes the delay state or also can be seen as state feedback control, $\varphi(t)$ is the initial condition denoting an continuous initial vector function.

Lemma 1 (Schur completment)[14]Given a symmetric matrix S with appropriate dimensions: $S^T = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, in which $S_{11} \in R^{r\times r}$. The following three conditions is equivalent：

(1) $S < 0$;

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0, S_{11} - S_{12}^T S_{22}^{-1} S_{12} < 0$

Lemma 2 [24]Given matrices Y, D, E , Rwith appropriate dimensions, and in which the matrices Y and R are symmetric, then

$$Y + DFE + E^T F^T D^T < 0$$

For all $F^T F$ satisfy ≤R, if and only if there exists some γ such that

$$Y + \gamma F^T F + \gamma^{-1} E^T R E < 0$$

3. MAIN RESULTS

Using the Leibniz-Newton formula to obtain a delay-dependent condition, free weighting matrices N and T are added to the left side of the equation:

$$2[x^T(t)N + x^T(t - d(t)T)] \times \left[x(t) - \int_{t-d(t)}^{t} \dot{x}(s)ds - (t - d(t))\right] = 0 \tag{5}$$

Then the equation are added into the derivative of the Lyapunov functional. The matrices N and T can be determined by solving the corresponding linear matrix inequalities.

Theorem 2 Given a scalar τ>0, if there exists positive matrices, $P = P^T > 0$, $S = S^T > 0$, $Z = Z^T > 0$, M and any appropriately dimension matrices N and T, such that the following LMI holds:

$$\Gamma_1 = \begin{bmatrix} \pi & -\tau N \\ * & -\tau Z \end{bmatrix} < 0, \Gamma_2 = \begin{bmatrix} \pi & -\tau T \\ * & -\tau Z \end{bmatrix} < 0 \tag{6}$$

$\pi = \pi_1 + \pi_2 + \pi_3 < 0$

where

$$\pi_1 = \begin{bmatrix} AP + PA^T + S & A_d P & 0 \\ * & 0 & 0 \\ * & * & -S \end{bmatrix}$$

$$\pi_2 = \tau[A^T \quad PA_d{}^T \quad 0]Z[A \quad A_d P \quad 0]$$

$$\pi_3 = \begin{bmatrix} N_1 + N_1^T & T_1 - N_1 + N_2^T & N_{3-}T_1 \\ * & T_2 + T_2^T - N_1 - N_1^T & T_3^T - N_3^T - T_2 \\ * & * & -T_3 - T_3^T \end{bmatrix}$$

Then the single closed-loop system $\sum_0$ is asymptotically stable.

Proof: using the Leibniz-Newton formula

$$x(t - d(t)) = x(t) - \int_{t-d(t)}^{t} \dot{x}(s)ds$$

$$x(t-\tau) = x(t - d(t)) - \int_{t-\tau}^{t-d(t)} \dot{x}(s)ds$$

Introducing the free weighting matrices N_i, T_i to indicate the relationship between the terms in the Leibniz-Newton formula

$$2\psi^T(t)N\left[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)ds\right] = 0$$

$$2\psi^T(t)T\left[x(t-d(t)) - x(t-\tau) - \int_{t-d(t)}^{t} \dot{x}(s)ds\right] = 0$$

(7)

In which

$$\psi^T(t) = [x^T(t) \ \ x^T(t-d(t)) \ \ x^T(t-\tau)]^T$$

Choosing the following Lyapunov functional form which are given by:

$$V(t, x_t) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Sx(s)ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha d\beta \quad (8)$$

In which, $P=P^T>0$, $S=S^T>0$, $Z=Z^T>0$ are to be determined. Calculating the derivative of V(xt)for system Σ_0.

$$\begin{aligned}
\dot{V}(t, x_1) &= 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) + \tau\dot{x}^T(t)Z\dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^T(s)Z\dot{x}(s)ds \\
&= 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) + \tau\dot{x}^T(t)Z\dot{x}(t) \\
&\quad - \int_{t-d}^{t} \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-\tau}^{t-d(t)} \dot{x}^T(S)Z\dot{x}(s)ds \\
&\quad + 2\psi^T(t)N\left[x(t) - x\left(t - d(t) - \int_{t-d(t)}^{t} \dot{x}(s)ds\right)\right] \\
&\quad + 2\psi^T(t)T\left[x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)ds\right] \\
&= 2\psi^T(t)\Xi\psi(t) - 2\psi^T(t)N\int_{t-d(t)}^{t} \dot{x}(s)ds \\
&\quad - 2\Psi^T(t)T\int_{t-\tau}^{t-d(t)} \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-d(t)}^{t} \dot{x}^T(s)Z\dot{x}(s)ds - \int_{t-d}^{t-d(t)} \dot{x}^T(s)Z\dot{x}(s)ds \\
&= \frac{1}{\tau}\int_{t-d(t)}^{t} \xi(t,s)\Gamma_1\xi(t,s)ds + \frac{1}{\tau}\int_{t-\tau}^{t-d(t)} \xi^T(t,s)\Gamma_2\xi(t,s)ds
\end{aligned}$$

In which,

$$\Xi = \begin{vmatrix} AP + PA^T + S + N_1 + N_1^T + \tau A^T ZA & BM + T_1 - N_1 + N_2^T + \tau A^T ZBM & N_{3-}T_1 \\ * & T_2 + T_2^T - N_1 - N_1^T + \tau B^T M^T ZBM & T_3^T - N_3^T - T_2 \\ * & * & -S \pm T_3 - T_3^T \end{vmatrix}$$

$$\xi(t,s) = \left[\psi^T(t)\dot{x}^T(s)\right]^T$$

$$\Gamma_1 = \begin{bmatrix} \Xi & -\tau N \\ * & -\tau Z \end{bmatrix}, \qquad \Gamma_2 = \begin{bmatrix} \Xi & -\tau T \\ * & -\tau Z \end{bmatrix}$$

And matrix Ξ multiply both sides by $diag\{P^{-1}, P^{-1}, I\}$, set$P = P^{-1}$, $M = KP^{-1}$, A_dP=BM. then we will get π. There exist a small positive scalar ε, which satisfy $\dot{V}(t) < -\varepsilon\|x(t)\|^2$ based on the inequality (6), So the system $\sum_0$ is asymptotically stable if LMIs(6) is true, this completes the proof.

Remark 1. In the procedure of the proof of the theorem2, it is clear thatthe terms in the derivative of V(xt) is not amplified, and all of the terms are reserved, then it is more less conservative than other methods due to abandoning the term of $-\int_{t-\tau}^{t-d(t)} \dot{x}^T(s)T\dot{x}(s)ds$.

Remark 2. Set the matrices X, T, N to zero, and Z=I, then we will get the delay-independent stability condition .

$$\begin{bmatrix} PA + A^T P + S & PB \\ B^T P & -S \end{bmatrix} < 0$$

Remark 3. Without introducing new matrix variables, the formation of LMIs is more simple and the computation is low complexity and less conservative. And an interesting result is similar to the theorem 2 in[7] when the Leibniz formula $x(t) - x\big(t - d(t)\big) - \int_{t-d(t)}^{t} \dot{x}(s)ds$ is adopted.

Consider the system with uncertain state-delay, ΔA and ΔA_d, then the system $\sum_0$ can be described as $\sum_1$:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)) \\ x(t) = \varphi(t), t \in [-\tau, 0] \end{cases} \tag{9}$$

The uncertainties are assumed to be of the form

$$[\Delta A \quad \Delta A_d] = DF(t)[E_a E_b]$$

In which, D,E_a, E_b are constant matrices with appropriate dimensions, and F(t) is an unknown real time-varying matrix satisfy the condition of Lebesgue-measurable elements:

$$F^T(t)F(t) \le I$$

Base on the lemma 2, the robust stability can be deduced as follow.

Theorem 3 Given a positive scalar τ, if there existspositive and symmetric matrices P, S, and Z, and matrices N and T with appropriate dimension, and positive scalar ε_i , $i = 1,2$ then the inequality holds as follows:

$$\begin{bmatrix} \Gamma_i & \varepsilon_i L & X \\ * & -\varepsilon_i I & 0 \\ * & * & -I \end{bmatrix} < 0, i = 1,2 \tag{10}$$

In which, $\Gamma_i = \Gamma_1, \Gamma_2$ are defined in (6), and$L = [PD^T\ 0\ 0\ 0\ \tau D^T Z]$, $X = [E_a\ E_b\ 0\ 0\ 0]$.

Proof: substitute the $A + \Delta A$ and $A_d + \Delta A_d$ into the inequality (6) to replace the matrices A and B, and by lemm2, we have:

$$\Gamma_i + \varepsilon_i LL^T + \varepsilon_i{}^{-1} XX^T < 0,\ \ i = 1,2$$

Then with the help of lemma1, the formation (11) is equivalent to (10). This completes the proof.

4. Numerical examples

To show the effectiveness of our method, one simple example is used to further illustrate the less conservativeness of the present results.

Example 1

Consider the time delay system in (1) and (2), with the matrix

A=[-1.0 0.5;   0.5 -1]; Ad=[-2 2.0; -0.5 -1.0];

It is clear that when the time delay d=0, the eigenvalue of the system is -1.5 and -0.5, then the system is asymptotically stable. And when time delay exists in the system, which is shown in Fig.2, the maximum allowed delay τcould be calculated by the theorem2 with the LMI Toolbox, which is shown in table1. And the step response of NCS with time delay satisfies the dynamic and static property as shown in Fig 3. And some other theorems are applied in this system and get different results. Clearly, contrast with them, our method is less conservative than the existing methods, thus demonstrating its validity.

Table. 1 Comparison of delay dependent stability condition of example 1

Method	Ref.[3]	Ref.[8]	Ref.[9]	Theorem1
Maximum time delay	0.3558	0.4428	0.4445	0.6841

As shown in Fig2, the time delay is stochastic variable in the NCS, with the control feedback and theorem2, it is clear that the system is asymptotically stable, which is shown in Fig2.2.

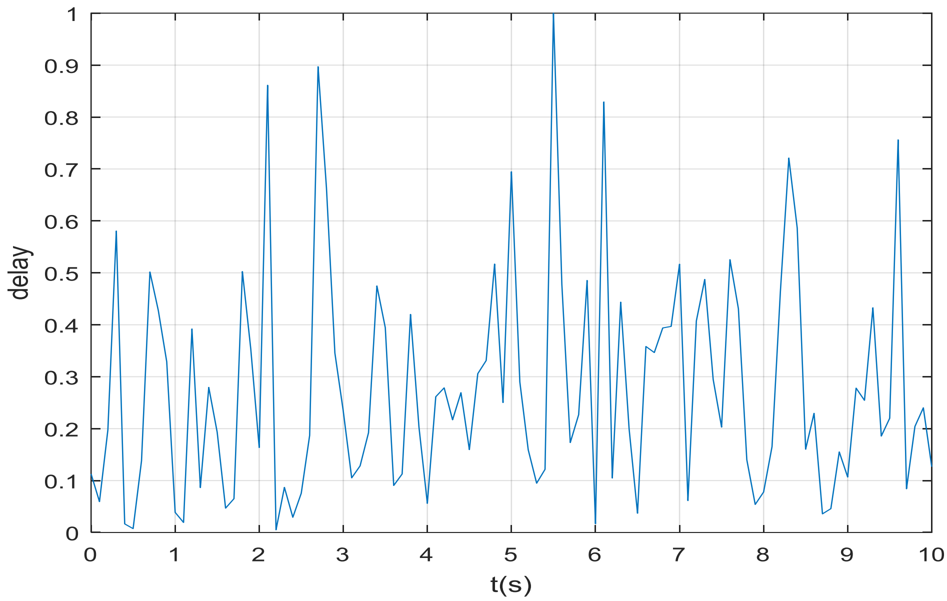

Fig.2 Stochastic time delay in NCS

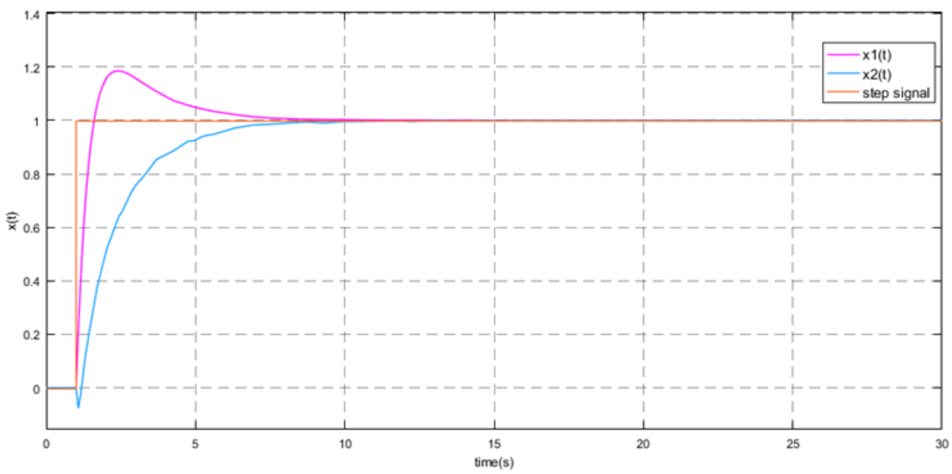

Fig.3 Step response of NCS with time delay

5. Conclusion

This paper presents a less conservative LMI method to obtain the maximum time delay in the system. Based on the free weighting matrices and Lyapunov-krasovskii approach, an improved LMI form in terms of N, T slack matrices gives the sufficient conditions. Without introducing the scaling technique, the results are less conservative than the existing methods, which are shown with numerical examples. It reveals that other existing delay-dependent results could be improved by this method.

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