A Study on Classical and Quantum Cosmology

Fengwei Liu

St. Mary's Ryken High School, Leonardtown, MD, United States

fengwei.liu@smrhs.org

Abstract

The classical theory of homogeneous and isotropic cosmology model based on GR, and the corresponding quantum model from loop quantization are reviewed. We will conclude how the classical singularity is solved by a bounce from the quantum effective dynamics.

Keywords

Classical cosmology,quantum cosmology,loop quantum cosmology.

1. Introduction

As many people know that today's physics has two main fundamental theories—general relativity (GR) and quantum physics, but they are not coincide together.

In GR, Albert Einstein redefines the gravity. By general relativity, gravity is not a force anymore but a representation of the space-time geometry. The geometry is described by metrics

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

Quantum physics is used to describe physics in small scale. A typical result is the uncertainty principle, which tells us that we cannot observe variables like position and momentum of a particle at the same time because the operator corresponding to the two observables do not commute with each other. Now the quantum physics, especially the quantum field theory, is very successful to construct the standard model to explain particles of our universe.

Gravity is negligible in many areas of particle physics, so that unification between general relativity and quantum mechanics is not an urgent issue in those particular applications. However, the lack of a correct theory of quantum gravity is an important issue in physical cosmology and the search by physicists for an elegant "Theory of Everything".

In those theories of quantum gravity, loop quantum gravity (LQG) is one of the promising one. It comes from quantization of generality relativity. In LQG, background independence and diffeomorphism invariance, which are the basic ideas of GR, are implemented very well [1]. Although there are a lot of efforts on the theory, the full theory is still too complicated, especially the dynamics. Hence, one used the idea of loop quantization to some reduced model to obtain some solvable theory. One of those theories is loop quantum cosmology (LQC) [2].

This paper is organized as following. In section 2, we review the classical theory reduced from general relativity. In section 3, the effective dynamic obtained from loop quantum cosmology theory is presented, where we show how the big bang is replaced by a big bounce. We summarize the paper in section 4.

2. Classical cosmology

Let us consider the homogeneous and isotropic flat spacetime. Because of the symmetry, the manifold is R^4. The matric is

$$ds^2 = -dt^2 + a(t)(dx^2 + dy^2 + dz^2)$$

Let $\phi: R^4 \to R$ be a massless scalar field coupled with gravity. Because of the symmetry, it only depends on t.

Denote π_ϕ as the conjugate momentum of ϕ, that is

$$\{\phi, \pi_\phi\} = 1 \tag{1}$$

Let V be a fiducial spatial cube in R^3. Denote v as the volume of V. Let b be its conjugate momentum. The dynamics is given by [3]

$$\dot{\phi} = \frac{\pi_\phi}{v} \tag{2}$$

$$\dot{\pi_\phi} = 0 \tag{3}$$

$$\dot{v} = \frac{3vb}{\gamma} \tag{4}$$

$$\dot{b} = -\frac{3b^2}{2\gamma} - \frac{2\pi G\gamma\pi_\phi^2}{v^2} \tag{5}$$

The Hamiltonian constraint is reduced as

$$H = -\frac{3vb^2}{8\pi G\,\gamma^2} + \frac{\pi_\phi^2}{2v} \tag{6}$$

According to equations (3), π_ϕ is a constant. Using the constraint equation $H = 0$, we will have

$$6b^2v^2 = 8\pi G\gamma^2\pi_\phi^2 \tag{7}$$

with which one will have

$$bv = \pm\sqrt{\frac{8\pi G\gamma^2\pi_\phi^2}{6}} \tag{8}$$

We can substitute equation (8) into (4) to obtain

$$\frac{dv}{dt} = \pm 3\pi_\phi\sqrt{\frac{8\pi G}{6}} \tag{9}$$

It gives us that

$$v = \pm 3\pi_\phi\sqrt{\frac{8\pi G}{6}} \cdot t \tag{10}$$

From these equations, we can conclude that when $t \to 0$, we have $v \to 0$, which leads to $b \to \infty$. Hence, singularity appears in the universe.

3. 3.Quantum cosmology

According to the above section, we see a singularity from the classical cosmology dynamic. In order to solve the problem, one develop the loop quantum cosmology. In the LQC theory, one quantized the classical cosmology model using the polymer quantization method [2]. In that theory, the quantum state is represented in the Hilbert space $\mathcal{H} = L^2(R_B, d\mu)$ consisting of square integrable functions on Bohr compactification of the real line. The dynamics is given by the Hamiltonian constraint operator $\widehat{H}$, with which a Klein-Gordon-like equation is obtained. By using the Hamiltonian constraint operator, one calculation its expected value under some coherent state, which gives us an effective Hamiltonian constraint. The effective Hamiltonian constraint reads [3]

$$H = -\frac{3v\sin^2(\lambda b)}{8\pi G\,\gamma^2\lambda^2} + \frac{\pi_\phi^2}{2v} \tag{11}$$

with which one has the following equations

$$\dot{\phi} = \frac{\pi_\phi}{v} \tag{12}$$

$$\dot{\pi_\phi} = 0 \tag{13}$$

$$\dot{v} = \frac{3v\sin(2\lambda b)}{2\lambda\gamma} \tag{14}$$

$$\dot{b} = -\frac{3\sin^2(\lambda b)}{2\gamma\lambda^2} - \frac{2\pi G\gamma\pi_\phi^2}{v^2} \tag{15}$$

Solving those equations one has

$$v^2\sin^2(\lambda b) = \frac{8\pi G\gamma^2\lambda^2}{6}\pi_\phi^2 \tag{16}$$

and

$$\frac{db}{dt} = -\frac{3}{\gamma\lambda^2}\sin^2(\lambda b) \tag{17}$$

Therefore, we have

$$\cot(\lambda\, b) = -\frac{3}{\lambda\,\gamma}t \tag{18}$$

Then we have

$$\sin^2(\lambda\, b) = \frac{1}{1 + \frac{9}{\gamma^2\,\lambda^2}t^2}$$

$$v^2 = \frac{8\pi G\gamma^2\lambda^2}{6}\left(1 + \frac{3}{\lambda^2\gamma^2}t^2\right)\pi_\phi^2 \tag{19}$$

According to (19), when $t = 0$, $v = \sqrt{\frac{8\pi G\gamma^2\lambda^2}{6}\pi_\phi^2} = v_{\min}$ and $b < \infty$. The theory gives us a bounce rather than big bang. Singularity is solved by the theory. For large t, one can find that $b \ll 1$, the trajectory comes back to the classical theory.

4. Conclusion

In the present work, we studied the homogeneous and isotropic cosmological model based on general relativity, where a singularity appears in the classical theory. This is what we called big bang theory. While, considering the quantum modification given by LQC, big bounce rather than big bang appears in the quantum region where the university is tiny. The singularity problem is then solved.

References

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