Optimum Non-equidistant Multi-variable New Information Model MGM (1, n) and Its Application

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Abstract

Grey system theory is a scientific theory to study poor information, with strong adaptability. The parameters in multivariable grey model MGM (1,n) are mutually influenced and restricted. In this work, an optimum non-equidistant multi-variable new information model MGM (1,n) was built according to the time response and reduction formula of non-equidistant new information model MGM(1,n). Such model was built using the minimum average relative error between restored value of the model and real value as objective function. Besides, the m-th component of model MGM (1, n) was taken as the initial value of solution of grey differential equations, and matrixes A and B as design variables. This optimum new information model MGM (1,n) was suitable for both equidistant and non-equidistant model, with high accuracy and strong stability. Furthermore, examples showed that this model was also practical and reliable.

Keywords

Multivariable; non-equidistant sequence; optimum new information model MGM (1, n); average relative error; nonlinear programming.

1. Introduction

Grey system theory is a scientific theory to study poor information, with strong adaptability. As an important part of grey system theory, grey model has found broad application since the grey system theory first proposed by Professor Deng Julong in 1982[1-3]. Although model MGM (1, n) was the extension of model GM (1, 1) in the case of n variables, it was not the simple combination of model GM (1, 1). Besides, it was also different with model GM (1, n) which had n first order differential equations. In model MGM (1, n), there were n-th order differential equations. Through simultaneous solution, parameters in MGM model reflected the relationship of interaction and interaction between variables [4]. In Reference [2], the optimum MGM (1, n) model was built and the coefficient q of background value was optimized by using the genetic algorithm. Based on the problem of prediction of law data serizes for the multivariable with interaction, exploring the modelling mechanism of original MGM(1,m) and discrete grey model, a type of optimization model of the discrete MGM(1,m) was constructed in Reference [5]. This model was built using the nth component as the initial condition of grey differential equation, as well as optimized initial value and coefficient of background value q . The background value was $z_i^{(1)} = qx_i^{(1)}(k+1) + (1-q)x_i^{(1)}(k)$ $(q \in [0,1])$. In Reference [6], another multivariable new information model MGM (1, n) was built by taking the nth component of $\mathbf{x}^{(1)}$ as the initial condition of grey differential equation. These models above were all equidistant models. In fact, most of the original data obtained in work are non-equidistant sequences, so the establishment of non-equidistant model is of practical and theoretical significance. Reference [7] proposed the non-equidistant multivariable model MGM (1, n) based on the fitting background value of homogeneous exponential function. However, there was still inherent defect in the modelling mechanism of this model for the prevalence of inhomogeneous exponential function. In Reference [8], another multivariable non-equidistant model MGM (1, n) was built. As the background value was generated by the average in such model, the model precision remained to be improved. Reference [9] put forward the non-equidistant multivariable model GM (1, N) based on the fitting background value of inhomogeneous exponential function, improving the model precision. Reference [10] analyzed the constructing method of background value in the multivariable grey model MGM (1, m). With the theory of vector value continued factions, background value in such model could be constructed by rational interpolation, trapezoid formula and extrapolation method of numerical integration. Although the simulation precision and prediction accuracy have been significantly improved, this model was still multivariable equidistant model MGM (1, m). Reference [11] built unequal-interval multivariate MGM (1, n) model based on improved background value. For above models, they were all built using the residual between practical value and simulation value. Therefore, there were some differences in modeling methods and evaluation standards. Namely, not all models met the conclusion that the minimum residual sum of squares achieved the highest precision. Based on this idea, the equidistance optimum grey model GM (1, 1) was established in Reference [12]. According to the modelling method in Reference [12], the optimum non-equidistant multi-variable new information model MGM (1, n) was built. Such model was built using the minimum average relative error between restored value of the model and real value as objective function. Besides, the m-th component of model MGM (1,n) was taken as the initial value of solution of grey differential equations, and matrixes A and B as design variables. This optimum new information model MGM (1,n) was suitable for both equidistance and non-equidistance model, with high accuracy and strong stability. Furthermore, examples showed that this model was of great theoretical value and application value.

2. Optimum non-equidistant multivariable grey new information model MGM(1,n)

Definition 1: Supposing $\mathbf{X}_i^{(0)}=[x_i^{(0)}(t_1),x_i^{(0)}(t_2),\cdots,x_i^{(0)}(t_j),\cdots,x_i^{(0)}(t_m)]$, then $\mathbf{X}_i^{(0)}$ was the non-equidistant sequence if $\Delta t_j=t_j-t_{j-1}\neq const$. ($i=1,2,\cdots,n,\ j=2,\cdots,m$, n was the number of variables and m the sequence number of each variable.)

Definition 2: Supposing $\mathbf{X}_i^{(1)}=\{x_i^{(1)}(t_1),x_i^{(1)}(t_2),\cdots,x_i^{(1)}(t_j),\cdots,x_{t_j}^{(1)}(t_m)\}$, then $\mathbf{X}_i^{(1)}$ was the first-order accumulated generation (1-AG0) of $\mathbf{X}_i^{(0)}$ if $x^{(1)}(t_1)=x^{(0)}(t_1)$ and $x_i^{(1)}(t_j)=x_i^{(1)}(t_{j-1})+x_i^{(0)}(t_j)\cdot\Delta t_j,\ j=2,\cdots,m,$ $i=1,2,\cdots,n,\Delta t_j=t_j-t_{j-1}$.

The original data matrix of multivariable was supposed as follows.

$$\mathbf{X}^{(0)}=\{\mathbf{X}_1^{(0)},\mathbf{X}_2^{(0)},\cdots,\mathbf{X}_n^{(0)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m)\\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m)\\ \cdots & \cdots & \cdots & \cdots\\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m)\end{bmatrix} \tag{1}$$

Where the observed value of each variable $\mathbf{X}^{(0)}(t_j)(j=1,2,\cdots m)$ at t_j was $\mathbf{X}^{(0)}(t_j)=[x_1^{(0)}(t_j),x_2^{(0)}(t_j),\cdots,x_n^{(0)}(t_j)]$; the sequence $[x_i^{(0)}(t_1),x_i^{(0)}(t_2),\cdots,x_i^{(0)}(t_j),\cdots,x_i^{(0)}(t_m)]$ $(i=1,2,\cdots,n,\ j=1,2,\cdots,m)$ was non-equidistant sequence, namely the distance t_j-t_{j-1} was constant.

To build the model, one accumulated generating operation was conducted on the original data, with the following new sequence.

$$\mathbf{X}^{(1)}=\{\mathbf{X}_1^{(1)},\mathbf{X}_2^{(1)},\cdots,\mathbf{X}_n^{(1)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m)\\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m)\\ \cdots & \cdots & \cdots & \cdots\\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m)\end{bmatrix} \tag{2}$$

Where $x_i^{(1)}(t_j)(i=1,2,\cdots,n)$ met the requirement of definition 2, namely

$$x_i^{(1)}(t_j)=\begin{cases}\sum_{j=1}^{k}x_i^{(0)}(t_j)(t_j-t_{j-1}) & (k=2,\cdots,m)\\ x_i^{(0)}(t_1) & (k=1)\end{cases} \tag{3}$$

The multivariable non-equidistant model MGM (1, n) was expressed as n -element first order differential equations.

$$\begin{cases}\frac{dx_1^{(1)}}{dt}=a_{11}x_1^{(1)}+a_{12}x_2^{(1)}+\cdots+a_{1n}x_n^{(1)}+b_1\\ \frac{dx_2^{(1)}}{dt}=a_{21}x_1^{(1)}+a_{22}x_2^{(1)}+\cdots+a_{2n}x_n^{(1)}+b_2\\ \cdots\\ \frac{dx_n^{(1)}}{dt}=a_{n1}x_1^{(1)}+a_{n2}x_2^{(1)}+\cdots+a_{nn}x_n^{(1)}+b_n\end{cases} \tag{4}$$

Noting $\mathbf{A}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}b_1\\ b_2\\ \cdots\\ b_n\end{bmatrix}$, then Equation (4) was expressed as follows.

$$\frac{dX^{(1)}(t)}{dt}=AX^{(1)}(t)+B \tag{5}$$

According to priority principle of new information in grey system theory, the new information was underutilized when the first component $\mathbf{x}_i^{(1)}(t_j)(j=1,2,\cdots,m)$ of $\mathbf{x}_i^{(1)}(t_j)(j=1,2,\cdots,m)$ was taken as initial condition of grey differential equation. However, it was possible to make full use of the new information by using the m-th component $\mathbf{x}_i^{(1)}(t_m)$ of $\mathbf{x}_i^{(1)}(t_j)(j=1,2,\cdots,m)$ as the initial condition. The continuous time response formula was as follows.

$$\mathbf{X}^{(1)}(t)=e^{At}\mathbf{X}^{(1)}(t_m)+\mathbf{A}^{-1}(e^{\mathbf{A}t}-\mathbf{I})\mathbf{B} \tag{6}$$

Where $e^{\mathbf{A}t}=\mathbf{I}+\sum_{k=1}^{\infty}\frac{\mathbf{A}^k}{k!}t^k$, $\mathbf{I}$ was unit matrix.

The calculated value of new information model MGM (1, n) as follows.

$$\hat{\mathbf{X}}_j^{(1)}(i)=e^{\mathbf{A}(t_j-t_m)}\mathbf{X}_{t_m}^{(1)}(i)+\mathbf{A}^{-1}(e^{\mathbf{A}(t_j-t_m)}-\mathbf{I})\mathbf{B}\quad (j=1,2,\cdots,m) \tag{7}$$

Using the m-th component of Equation (7) as the initial condition of grey differential equation, then new information will be fully used. The fitting value of original data was as follows.

$$\hat{\mathbf{X}}_i^{(0)}(t_j)=\begin{cases}\lim_{\Delta t\to 0}\frac{\mathbf{X}_i^{(1)}(t_1)-\mathbf{X}_i^{(1)}(t_1-\Delta t)}{\Delta t}, & j=1\\ (\hat{\mathbf{X}}_i^{(1)}(t_j)-\hat{\mathbf{X}}_i^{(1)}(t_{j-1}))/(t_j-t_{j-1}), & j=2,3,\cdots,m\end{cases} \tag{8}$$

The absolute error of the i-th variable was defined as follows.

$$q(t_k)=\hat{x}_i^{(0)}(t_j)-x_i^{(0)}(t_j) \tag{9}$$

The relative error (%) of the i-th variable was defined as follows.

$$e_i(t_j)=\frac{\hat{x}_i^{(0)}(t_j)-x_i^{(0)}(t_j)}{x_i^{(0)}(t_j)}*100 \tag{10}$$

The average relative error of the i-th variable was defined as follows.

$$\frac{1}{m}\sum_{j=1}^{m}\left|e_i(t_j)\right| \tag{11}$$

The average relative error of the i-th variable was defined as follows.

$$\frac{1}{m}\sum_{j=1}^{m}\left|e_i(t_j)\right| \tag{12}$$

The average error of the whole data was as follows.

$$f=\frac{1}{nm}\sum_{i=1}^{n}(\sum_{j=1}^{m}\left|e_i(t_j)\right|) \tag{13}$$

Generally, the quality of new information model MGM (1,1) is evaluated using the average relative error between simulation value and raw data sequence in final model. Therefore, it is feasible to establish an optimum new information model MGM (1, 1) of average relative error, which is a nonlinear programming model. (See Equation (12))

$$f=\frac{1}{nm}\sum_{i=1}^{n}(\sum_{j=1}^{m}\left|e_i(t_j)\right|) \tag{14}$$

When $t_k=k(k=1,2,\cdots,n)$, then the non-equidistant model can be transformed into equidistant model. The design variables were the elements of matrixes A and B. As A was the $n\times n$ matrix, and B the $n\times 1$ matrix, the design variable X should be a $n\times(n+1)$ column vector. For example, if $n=2$, then the design variable X was a six—dimesional column vector.

If $A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}$, $B=\begin{bmatrix}B_{11}\\ B_{21}\end{bmatrix}$, then $X=[A_{11}\quad A_{12}\quad A_{21}\quad A_{21}\quad B_{11}\quad B_{21}]$.

Taking the average error f as the objective function, and X as design variable, the solution was obtained using the optimization function fmincon in Matlab7.5 or other optimization methods [13]. However, special attention should be paid to Equation (4) for its singularity in the solution. Therefore, it is important to give the initial value while solving by function fmincon. Generally, the matrixes A and B of the solution of non-equidistant model MGM are taken as the initial value of the optimum model.

Therefore, if $n=1$, then the optimum non-equidistant new information model MGM(1,n) will degenerate as the optimum non-equidistant new information GM(1,1) model; if $B=0$, then the optimum non-equidistant new information model MGM(1,n) is the combination of n optimum non-equidistant new information GM(1,1) models. Besides its function of modelling, the optimum non-equidistant new information model MGM (1, n) can also be used for prediction or data fitting and processing. Based on the value of n under concrete conditions, different optimum non-equidistant new information models can be obtained, such as MGM (1, 2), MGM (1, 3) and MGM (1, 4).

3. 3. Application Examples

Example 1: Table 1 [14] showed the measured cutting force with the use of carbide cutter YT14 on common lathe CA6140 for the turning of outside round. The data was obtained with different cutting depths while constant geometric parameters and cutting speed of the tool.

Table 1 The data table of cutting experiments f=0.02mm/r

No.	1	2	3	4	5
ap / mm	1.00	1.25	1.50	1.75	2.00
F_{1z} / N	838.98	1060.45	1261.79	1483.25	1704.72
F_{1y} / N	255.10	290.16	355.22	420.28	469.08

Regarding cutting depth a_p as t_j, main cutting force F_{1z} as x_1 and F_{1y} as x_2, the optimum non-equidistant new information model MGM(1,2) was built using the method in the work. The parameters of this model were as follows.

$$\mathbf{A}=\begin{vmatrix}1.6966 & -3.7283\\ 1.3922 & -4.1810\end{vmatrix},\quad \mathbf{B}=\begin{vmatrix}697.8527\\ 121.3384\end{vmatrix},$$

The fitted value of F_{1y} was $\hat{F}_{1y}$ =[255.1, 289.1829, 353.8429, 420.2459, 493.5264].

The absolute error of F_{1y} was $q=$ [4.62307e-05, -0.977118, -1.37706, -0.034142, 24.4464].

The relative error of F_{1y} (%) was $e=$[1.8123e-05, -0.33675, -0.38766, -0.0081236, 5.2116].

The average relative error of F_{1y} was 1.1888%.

The average relative error of this model was 2.4334%, representing the high precision of this model.

Example 2: Reference [8] showed that water absorption had an influence on the mechanical properties of pure PA66. Based on the data in Reference [8], the mechanical properties of pure PA66 were measured under different water absorptions. The flexural strength, flexural modulus and tensile strength of PA66 under different water absorptions were obtained in the work. Table 2 showed the values of flexural strength $X_1^{(0)}$ (Mpa), flexural modulus $X_2^{(0)}$ (Gpa) and tensile strength $X_3^{(0)}$ (Mpa)

Table 2 The influence of water absorption on the mechanical properties of PA66

No.	1	2	3	4	5	6	7	8	9
Water absorption t_j/%	0	0.0607	0.1071	0.1662	0.2069	0.4344	0.5243	0.8524	0.9756
$X_1^{(0)}$	83.4	84.9	84.5	84.2	84.4	78.4	75.4	59.5	54.1
$X_2^{(0)}$	2.63	2.64	2.61	2.65	2.66	2.52	2.32	1.90	1.72
$X_3^{(0)}$	84.2	84.4	86.3	84.3	81.3	74.9	75.7	73.2	66.9

The optimum non-equidistant new information model MGM (1, 3) built using the method in the work was shown as follows.

$$\mathbf{A}=\begin{vmatrix}0.0669 & -0.1645 & -0.5427\\ 0.2062 & -3.2111 & -0.1064\\ 0.3995 & -0.1283 & -0.6252\end{vmatrix},\quad \mathbf{B}=\begin{vmatrix}85.0560\\ 3.1114\\ 86.0730\end{vmatrix},$$

The fitting value was as follows.

$\hat{X}_3^{(0)}$ =[84.94,84.3753,83.38,82.3871,81.4381,78.8377,75.7066,71.4942,66.9017]

The absolute error of $X_3^{(0)}$ was as follows.

q =[0.73998,-0.024693,-2.92,-1.9129,0.13808, 3.9377, 0.0065505, -1.7058, 0.0016596]

The relative error of $X_3^{(0)}$ (%) was as follows.

e = [0.87884,-0.029258,-3.3835,-2.2692, 0.16984, 5.2573, 0.0086532, -2.3303, 0.00248]

The average relative error of $X_3^{(0)}$ was 1.5922 %, representing the high precision of this model.

4. Conclusion

In this work, an optimum non-equidistant multi-variable new information model MGM (1, n) was built according to non-equidistance sequences with mutually influenced and restricted variables. Such model was built using the minimum average relative error between restored value of new information MGM (1, 1) model and real value as objective function. Besides, the m-th component of model MGM (1, n) was taken as the initial value of solution of grey differential equations, and matrixes A and B as design variables. With the help of mathematic software MATLAB, the optimum solution of this model could be directly obtained. This optimum new information model MGM (1, n) was suitable for both equidistance and non-equidistance model, with high accuracy and strong stability. Furthermore, for its strong practicality and reliability, this model is of great theoretical value and application value for practice.

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