Principle and Application of Triangular Arch Bridge
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Abstract
Aiming at the shortcoming of large deformation of arch ribs under live load, a new type of arch bridge is proposed to solve it. The principle is to use triangular structures to constrain the deformation of the arch bridge, thereby improving its overall stiffness significantly. Specifically, six rigid diagonals are added in the structure. Each diagonal connects the arch rib to the girder. The girder and diagonals form three triangles as the bases and the waists, respectively. The top corners of the triangles are clamped to the positions that the maximum displacement of the arch rib under the dead and live load, respectively. Since the diagonals are installed after major bridge structure is completed, the structural characteristics of the arch structure are maintained. The added three triangles have strong anti-deformation performance and constrain the deformation of the structure significantly. Meanwhile, the triangular structures provide extra elastic supports for the girder, which greatly improves the overall stiffness of the bridge. Finite element analysis proves that the stiffness, strength, dynamic performance and stability of the new arch bridge are improved significantly and only a small amount of material is added. Therefore, the new arch bridge is a competitive new type of bridge and has a certain aesthetic value.

Keywords
Arch Bridge, stiffness, rigid diagonal, triangular structure, dynamic characteristics.

1. Introduction
The arch bridge is a long-standing and competitive bridge type in bridge construction because of its strong spanning ability, high bearing capacity, wide application range and rich structural modeling [1-3]. The arch rib is its main bearing member with simple and reasonable force which is stress-based. Its stress distribution is uniform with high stiffness, so it is a superior structure [4-7]. Along with the increase of engineering practice and the deepening of scientific research, arch bridge has some problems to be solved because of its structural and loading features. First of all, different from that of the beam bridge, the maximum-deflection section of the arch bridge under live load is not in mid-span but in the vicinity of 1/4 and 3/4 span, that is, the vertical deformation of the arch bridge is the largest when the live load is arranged in a half span [8-10]. The 1/4 and 3/4 span of the cross section are the weak parts of the arch bridge. How to strengthen them is the key to improving the stiffness of the arch bridge. What’s more, the arch bridge is generally regarded as a bridge type with high bearing capacity, however, with the increase of span, the cross sectional area (CSA) of the arch ribs rises rapidly and the proportion of stress generated by the self-weight of the arch ribs to the total stress increases as well [11-14]. In recent years, the bridge has been multi-functionalized and heavy-loaded, and many multilane and multi-railway combined arch bridges have been built. Therefore, it has become the key point to improve the bearing capacity for the design of super long-span arch bridges [15-18].

In order to improve the stiffness and bearing capacity of the arch bridge, this paper proposes a new type of arch bridge—triangle arch bridge, that is, on the basis of the existing flexible boom arch bridge, 6 diagonal-connection between arch ribs and girders are added to each arch rib. Diagonals and girders form three triangles which are arranged in the position of the largest deflection of the arch bridge under dead load and live load. The triangular structure of good anti-deformation ability conducts operative constraint on the deformation of the arch rib, and strengthens the linear stiffness of
the girder so that the integrity of the new arch bridge is enhanced, the stiffness and bearing capacity is greatly improved. This paper mainly introduces the structural style and mechanics principle of triangular arch bridge, studies the strength, stiffness, stability and dynamic property by finite element analysis, and analyzes the influence of CSA of diagonal and the rise-to-span ratio of the arch ribs on the mechanical property of arch bridge. For convenience of description, the triangular arch bridge is hereinafter referred to as \( \Delta \) arch bridge.

2. Structural Style and Mechanics Principle of \( \Delta \) Arch Bridge

2.1 Structural Style

\( \Delta \) arch bridge, based on flexible boom arch bridge, is a new kind of arch bridge structure that makes up for the deficiency of large deformation of arch ribs and girders under half-span load. Its structural style is as shown in Fig. 1 and design sketch is as shown in Fig. 2. The main components of \( \Delta \) arch bridge include arch ribs, girders, diagonals, diagonal bracings, the arch rib bracings and suspenders. Diagonals and diagonal bracings are installed after completion of bridge (staggered with suspenders), both ends of which are respectively connected to the arch ribs and the girders. Three triangles are formed by diagonals and girders, so the new arch bridge is called \( \Delta \) arch bridge. In addition, the flexible boom is arranged in the same way as ordinary arch bridge and the arch rib is consolidated with the bridge abutment, and both ends of the girder support on the abutment by bridge bearings.

2.2 Structure Principle

Based on the stability principle of triangular structure, \( \Delta \) arch bridge takes the girder as the “base” of the triangle and adds the diagonal as the “waist” to form three triangles that restrain the deformation of the arch ribs and enhance the linear stiffness of the girder. As the span increases, the stiffness of the diagonals decrease more rapidly than that of the girders, which affects the cooperative work of the triangle, so it is necessary to add a bracing between the two diagonals. Characterized by good anti-deformation, the triangle can restrain the deformation of the arch rib and the girder, greatly improving the overall stiffness.
3. Design Consideration of Position of Diagonal

3.1 Deformation Analysis of Ordinary Arch Bridge

In order to study the deformation characteristics of the arch bridge, a flexible boom through steel box arch bridge model with a span of 420 m has been built with MIDAS/CIVIL software, as shown in Fig. 3, where the rise-to-span ratio is 1:5 and arch-axis coefficient is 1.3; the arch rib is 6 m high and 4 m wide, and multiple longitudinal stiffeners are arranged in the steel box to increase the stiffness of the arch ribs; the girder is 24 m wide and is arranged with two-way 6-lane.

The results show that the maximum vertical displacement of the arch rib occurs in mid-span under dead load; the displacement envelope diagram is a "W" curve under live load and the maximum vertical displacement appears near 1/4 and 3/4 span of sections. Vertical displacement of ordinary arch bridge under dead load is shown in Fig. 4, the maximum vertical displacement of the arch rib and the girder is 281.7 mm and 641.2 mm, respectively. Vertical displacement of ordinary arch bridge under live load is shown in Fig. 5, the maximum vertical displacement of the arch rib and the girder is 140.5 mm and 185.2 mm, respectively. The results show that the mid-span of sections are the structural weakness of the ordinary flexible boom arch bridge under dead load and near the 1/4 and 3/4 span of sections under the live load.

3.2 Deformation Analysis of $\Delta$ Arch Bridge

For the structural weakness position of the ordinary arch bridge above, 6 diagonal-connections between arch rib and girder are added to each arch rib, the girder is taken as the “base” of the triangle and the added diagonal as the “waist” to form three triangles to strengthen structural weakness position and improve the integrity of the structure. The calculation model of $\Delta$ arch bridge is as shown in Fig. 6. The amount of steel used for diagonals and diagonal bracings is 471.1t, only 3.73% of the ordinary arch bridge.

The results show that the vertical displacement of $\Delta$ arch bridge is much smaller than that of ordinary arch bridge under live load, that is to say, the stiffness of $\Delta$ arch bridge has greatly improved. Under dead load, the deformation diagram of the arch rib of $\Delta$ arch bridge is the same as that of ordinary
arch bridge, and the maximum vertical displacement appears in the mid-span. However, under live load, the deformation diagram is a curve with 9 displacement peaks, and the maximum vertical displacement appear in the vicinity of 1/4 and 3/4 span of sections. The vertical displacement of the \( \triangle \) arch bridge under dead load is shown in Fig. 7, the maximum vertical displacement of the arch rib is 493.0 mm, and the maximum vertical displacement of the girder is 830.3 mm. Fig. 8-9 show the displacement envelope diagrams of the \( \triangle \) arch bridge and the ordinary arch bridge, respectively. The results show that the maximum vertical displacement of the arch rib and girder are 38.8 mm and 54.0 mm, which is 27.6% and 29.2% of that of ordinary arch bridge, respectively.

Fig. 6 Model of \( \triangle \) Arch Bridge

Fig. 7 Vertical displacement of \( \triangle \) arch bridge under dead load (unit: mm)

Fig. 8 The displacement envelope diagram of arch rib of \( \triangle \) arch bridge and ordinary bridge under live load

Fig. 9 The displacement envelope diagram of the girder of \( \triangle \) arch bridge and ordinary bridge under live load
4. Mechanical properties analysis of Δ arch bridge

In order to study the mechanical properties of Δ arch bridge, force analysis of the Δ arch bridge model in Part 2 is further conducted and compared with the ordinary flexible boom arch bridge under the same calculation condition. The main loads taken into consideration include dead load, live load, temperature load and differential settlement of bridge foundation. The temperature range is ±25 °C; the live load is arranged according to two-way 6 lanes of Grade I highway; differential settlement of bridge foundation is set as 2 cm height change. When the load combination is calculated, the factor for importance of structure is 1.1, the factor for dead load is 1.2, the factor for live load is 1.4, and the factor for live load combination is 0.7.

4.1 Internal force analysis of Δ arch bridge

The results of internal force analysis show that the diagonal and the girder have good cooperation ability, and the triangle composed of the two has strong anti-deformation ability, which effectively restrains the deformation of the arch rib and the girder. Fig. 10 and 11 are the axial force diagram of diagonals under dead and live load, respectively and the axial force of diagonals are as shown in Tab. 1. Because the diagonals are installed after the completion of the major bridge, the upper part of the diagonal is subjected to the axial tension while the lower part is subjected to the axial pressure under dead load; under live load arrangement when extracting the maximum vertical displacement of the girder, the 1st, 3rd and 5th diagonals are all subjected to the axial tension while the 2nd, 4th and 6th diagonals are all subjected to the axial pressure, which has proved it mechanics characteristic of the triangular structure.

<table>
<thead>
<tr>
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<th>Dead load</th>
<th>Live load</th>
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<tr>
<td>Maximum axial tension of diagonal (kN)</td>
<td>1071.5</td>
<td>2140.8</td>
</tr>
<tr>
<td>Maximum axial pressure of diagonal (kN)</td>
<td>-110.8</td>
<td>-1067.5</td>
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</tbody>
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![Fig. 10 Axial force diagram of diagonal under dead load](image)

![Fig. 11 Axial force diagram of diagonal under live load](image)

4.2 Stress analysis of Δ arch bridge

To study the load response and structural strength of Δ arch bridge, calculation is carried out under single load and combined load respectively. The results of calculation are as shown in Tab. 2, the results show that the strength bearing capacity of Δ arch bridge is much higher than that of ordinary flexible boom arch bridge, under combined load and live load, the maximum stress of Δ arch bridge decreases by 17.5% and 62.0% compared with that of contrast arch bridge, respectively. Although the added diagonals improve the bearing capacity and stiffness of arch bridge, they also increase the
degree of statical indeterminancy of the structure, which makes the influence of temperature load and differential settlement of the foundation of \( \Delta \) arch bridge slightly bigger than that of ordinary arch bridge.

<table>
<thead>
<tr>
<th>Tab. 2 Summary Sheet of Stress (unit: MPa)</th>
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| Linear elastic buckling analysis is carried out on the structure and the calculation results are shown in Table 3, the results reveal that the setting of the diagonal enhances the stability of the arch bridge, the in-plane stability and out-of-plane stability of \( \Delta \) arch bridge are higher than that of contrast arch bridge. The in-plane stability coefficient of the \( \Delta \) arch bridge is 23.71, which is 196.6% higher than that of contrast arch bridge; the out-of-plane stability coefficient of \( \Delta \) arch bridge is 6.66, which is 6.9% higher than that of contrast arch bridge. When the geometric nonlinear influence is considered, the stability analysis results of the structure are as shown in Fig. 12 and 13, where the stability coefficient of the \( \Delta \) arch bridge is 6.40, which is 4.2% higher than that of contrast arch bridge. Although the setting of diagonal increases the self-weight of the structure, the triangular structure composed of diagonals and girder not only enhances the in-plane anti-deformation ability, but also brings the positive effect of non-orientedly conservative loadings to the arch ribs, which makes both in-plane and out-of-plane stability of the arch bridge higher than that contrast arch bridge.

The high stability of \( \Delta \) arch bridge is propitious to the construction of super long-span arch bridge. As the span of the arch bridge increases, the stability decreases rapidly and the in-plane stability decreases faster than out-of-plane stability. Thus, the problem of insufficient in-plane stability is more serious. However, the proposed \( \Delta \) arch bridge provides a new effective way to improve the stability of super long-span arch bridges.

**4.3 Stability analysis of \( \Delta \) arch bridge**

Tab. 3 Summary Sheet of Stability Coefficient

<table>
<thead>
<tr>
<th>Tab. 3 Summary Sheet of Stability Coefficient</th>
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<tr>
<td>Stability coefficient</td>
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<td>1 6.23</td>
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<tr>
<td>2 7.33</td>
</tr>
<tr>
<td>3 7.993</td>
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<tr>
<td>4 11.38</td>
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<tr>
<td>5 12.22</td>
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<tr>
<td>6 12.45</td>
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<tr>
<td>7 18.04</td>
</tr>
<tr>
<td>8 19.22</td>
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</tbody>
</table>
4.4 Dynamic characteristics analysis of Δ arch bridge

The in-plane dynamic performance of Δ arch bridge is much better than that of contrast arch bridge while the out-of-plane dynamic performance is almost equal to that of contrast arch bridge. The structural dynamic response analysis results are as shown in Tab. 4, where the in-plane natural frequency of Δ arch bridge is 0.72366 while contrast arch bridge is only 0.39195. The results reveal that Δ arch bridge has better in-plane dynamic performance, which ensures driving comfortableness.

Tab. 4 Summary Sheet of Natural Frequency

|pairs | 
|--- | --- | --- | --- |
|Contrast arch bridge | Δ arch bridge | | |
|In-plane | Natural frequency | Mode of vibration | Natural frequency | Mode of vibration |
| | 0.39195 | Dissymmetry of first order | 0.72366 | Dissymmetry of first order |
|Out-of-plane | 0.29473 | Symmetry of first order | 0.29532 | Symmetry of first order |
5. Influence of CSA of diagonal on mechanical property of Δ arch bridge

5.1 Calculation parameter

CSA of diagonal is an important parameter in the design of Δ arch bridge, which affects the strength, stiffness, stability and dynamic characteristics of the structure. By studying the influence of CSA of diagonal on the structural performance, a good balance between amount of steel used and structural performance can be achieved so that the structural performance can be greatly improved by using small amount of steel. Therefore, on the basis of model of Δ arch bridge in section 2.2, the finite element model of Δ arch bridge with variable CSA of 0.02-0.22 m² is established to study the deformation, internal force and stress of arch ribs. The calculation parameters are as follows: the span is 420 m, the width of bridge is 24 m and arch axis coefficient is 1.3; the arch rib is steel box-section structure with height of 5-7 m, width of 4 m and wall thickness of 28 mm. The loading condition is the same as section 3.

5.2 Calculation result

The calculation results show that diagonals with appropriate CSA should be selected to maximize the comprehensive performance of the structure in the design of Δ arch bridge. Too small CSA not only effectively improves the strength, bearing capacity and stiffness of the structure but also causes local buckling and vibration. Too large CSA not only increases burden for the arch rib but also causes material waste.

Figures 14-19 shows that the change of the main mechanical indexes of Δ arch bridge with the variable CSA of diagonal. It can be seen from Fig. 14-16 that the stress peak of the arch rib increases linearly with the increase of CSA of diagonal under dead load; under live load, the maximum stress appears minimal value when CSA is 0.10 m²; under combined load, the maximum stress also has minimal value when CSA is 0.14 m². Fig. 17 shows that the stability of Δ arch bridge increases gradually with the increase of CSA of diagonal and out-of-plane buckling appears except for the model whose CSA of diagonal is 0.02 m². When CSA is 0.02 m², the stability is lowest due to the local buckling of diagonal. As shown in Fig. 18, the fundamental frequency of Δ arch bridge gradually increases with the increase of CSA of diagonal and out-of-plane vibration appears except for the model whose CSA of diagonal is 0.02 m². When CSA is 0.02 m², the fundamental frequency is lowest because the diagonal vibrates prior to the arch rib. As can be seen from Fig. 19, with the increase of CSA of diagonal, the deflection of girder of Δ arch bridge decreases gradually and the decreasing tendency of stiffness gradually slows down under live load. When CSA of diagonal exceeds 0.10 m², the influence of increasing CSA on deflection is small. The above analysis indicates that the comprehensive performance of Δ arch bridge is the best when CSA of diagonal is 0.10 m².

![Fig.14 Stress change curve of arch rib with variable CSA of diagonal under dead load](image-url)
Fig. 15 Stress change curve of arch rib with variable CSA of diagonal under live load

Fig. 16 Combined stress change curve of arch rib with variable CSA of diagonal under combined load

Fig. 17 Stability change curve with variable CSA of diagonal
6. Influence of rise-to-span ratio on stiffness of ∆ arch bridge

6.1 Calculation parameter

The rise-to-span ratio of arch rib is an important parameter in the design of ∆ arch bridge, which affects the strength, stiffness, stability and dynamic performance of the arch structure. In order to study the influence of the change of rise-span ratio on the mechanical property of the arch bridge, a finite element model with variable rise-to-span ratio of 1/4.0-1/6.5 is established to calculate deformation, internal force and stress of arch rib. Then the calculation result is compared with that of the ordinary flexible boom arch bridge under the same conditions. The calculation results are as follows: the span of model is 350 m, bridge roadway width is 24 m, and arch axis coefficient is 1.3; the arch rib is steel box-section structure with height of 5-7 m, width of 4 m and wall thickness of 28 mm; diagonal is I-shaped section with CSA of 219.5 cm². The loading condition is the same as in section 3, and the model is shown in Fig. 20.
6.2 Calculation result

The calculation results show that the strength, bearing capacity and stiffness of Δ arch bridge are higher than those of ordinary flexible boom arch bridge when the rise-to-span ratio is 1/4-1/6.5, which is featured in stable performance and strong applicability.

The curves of the main mechanical indexes of Δ arch bridge with the variable rise-to-span ratio of arch rib are shown in Fig. 21-26. As we can see from Fig. 21-23, under live load and combined load, the maximum stress of Δ arch bridge is much smaller than that of contrast arch bridge under any rise-span ratio. Under all loading condition, the maximum stress of arch ribs of them increases gradually with the decrease of rise-span ratio. As can be seen from Fig. 24, the stability of both Δ arch bridge and contrast arch bridge decrease gradually with the decrease of the rise-to-span ratio and both appear out-of-plane buckling. When the rise-span ratio is large, the stability of Δ arch bridge is larger than that of contrast arch bridge; when the rise-to-span ratio is smaller than 1/5.5, the stability of both is roughly similar. As shown in Fig. 25, the fundamental frequency of both Δ arch bridge and contrast arch bridge gradually increases as the rise-to-span ratio decreases. When the rise-to-span ratio is 1/4.5-1/6.5, the fundamental frequency of Δ arch bridge is slightly smaller than that of ordinary arch bridge; when the rise-span ratio is 1/4, diagonal vibrates prior to arch rib because of long and thin diagonal and the stiffness of diagonal should be strengthened. It can be seen from Fig. 26 that the stiffness of the Δ arch bridge is much higher than that of contrast arch bridge at any rise-span ratio, and the promotion effect of diagonal on the stiffness is less affected by the variable rise-to-span ratio, which prove it strong applicability.

Fig. 21 Stress change curve of arch rib with variable rise-to-span ratio under dead load
Fig. 22 Stress change curve of arch rib with variable rise-to-span ratio under live load

Fig. 23 Stress change curve of arch rib with variable rise-to-span ratio under combined load

Fig. 24 Stability change curve with variable rise-to-span ratio
Fig. 25 Fundamental frequency change curve with variable rise-span ratio

Fig. 26 Deflection change curve with variable rise-to-span ratio

7. Conclusion

This paper proposes a new type of arch bridge—△ arch bridge with high bearing capacity, high stiffness, high stability and good dynamic performance, expounds its mechanics principle and provides specific application examples and parametric analysis. Conclusions have drawn as follows:

(1) △ arch bridge with the characteristic of reasonable stress. △ arch bridge takes advantage of the strong anti-deformation ability of triangular structure and adds 6 diagonals to form three triangles together with girders, this restricts the maximum displacement section under dead and live load respectively and also enhances the linear stiffness of the girder, thus greatly increasing the overall stiffness of the structure. In addition, since the diagonal is installed after the major bridge is completed, good mechanical characteristic of the arch structure is maintained.

(2) △ arch bridge with the characteristic of high stiffness. The diagonal has strengthened the structural weakness of the arch rib to a great extent. Under live load, the maximum deflection of the girder of ordinary arch bridge with span of 420 m is 185.2 mm. On this basis, it can be reduced to 54.0 mm by adding only 3.73% of amount of steel used, which is only 29.2% of that of ordinary arch bridge, thus the stiffness has greatly improved.

(3) △ arch bridge with the characteristic of high carrying capacity. The strength and bearing capacity of △ arch bridge is much higher than that of contrast arch bridge. Under live load, the stress of arch rib of △ arch bridge with span of 420 m is 62.0% lower than that of contrast arch bridge. Under the
combined load, the maximum stress of arch rib of $\Delta$ arch bridge decreases by 17.5% compared with that of contrast arch bridge.

(4) $\Delta$ arch bridge with the characteristic of high stability. The stability of $\Delta$ arch bridge is higher than that of contrast arch bridge. The out-of-plane stability coefficient of $\Delta$ arch bridge with span of 420 m is 6.660, which is 6.9% higher than that of contrast arch bridge. The in-plane stability coefficient is 23.71, which is 196.6% higher than that of contrast arch bridge. If the geometric nonlinear influence is considered, the stability coefficient of $\Delta$ arch bridge is 6.40, which is 4.2% higher than that of contrast arch bridge.

(5) $\Delta$ arch bridge with the characteristic of good dynamic performance. The inner dynamic performance of $\Delta$ arch bridge is better than that of contrast arch bridge. The inner natural frequency of $\Delta$ arch bridge with span of 420 m is 0.72366 while contrast bridge is only 0.39195. The higher inner natural frequency ensures driving comfortableness.

(6) $\Delta$ arch bridge with the characteristic of stable structural performance and strong applicability. When the rise-to-span ratio is in the range of 1/4-1/6.5, the stiffness and strength bearing capacity of $\Delta$ arch bridge are higher than that of contrast arch bridge. The stiffness of $\Delta$ arch bridge is less sensitive to the variable rise-to-span ratio. The stiffness only changes 8.3% when the rise-to-span ratio decrease from 1/4.0 to 1/6.5.

(7) The diagonal with proper CSA can improve the comprehensive performance of $\Delta$ arch bridge to the greatest extent. Too small CSA causes local buckling and vibration, which will adversely affect the stability and dynamic performance of the structure. However, too large CSA will increase burden for the rib, resulting in material waste. Therefore, in the design of $\Delta$ arch bridge, it is necessary to select the appropriate CSA of diagonal so that fewer materials can achieve the better structural strength, bearing capacity, stiffness, stability, and dynamic performance.

To sum up, compared with ordinary flexible boom arch bridge, $\Delta$ arch bridge can greatly improve the strength, bearing capacity, stiffness, stability and dynamic performance of the structure by adding only a small amount of material, which is a new competitive bridge type.

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References


