Feedback-Aided PD-Type Iterative Learning Control

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Abstract

This paper presents a feedback-aided PD-type(FAPD) iterative learning control(ILC) algorithm for linear time-invariant systems. The convergence analysis of the proposed learning algorithm is conducted, and sufficient convergence condition are derived. Furthermore, the initial rectifying strategy is applied to eliminate the effect of fixed initial errors. This result is generalized to higher order systems. It is shown that the system output converges to the desired one in finite time, whatever value the fixed initial shift takes. Numerical simulations are given to demonstrate effectiveness of the proposed learning algorithms.

Keywords

Iterative learning control, feedback-aided, initial rectifying, finite time.

1. Introduction

Iterative learning control (ILC) strategy applied to the systems which undertaken perform tasks repetitively over a pre-specified finite time interval [1-2]. This method via learning repetitively, to realize the tracking error converges to zero, that is, achieve completely tracking effect, and this strategy is widely employed in assembly lines, automatic welding machines, laser cutting machines, linear motors, injection moulding machines, rapid thermal processing systems, batch chemical processes, etc, owe to which design simply, high control precision, need less pre-knowledge and smaller amount of on-line calculation.

A reset is required at the beginning of each iteration. In the early stage, ILC require strict initial position condition, that is, initial state $x_k(0)$ coincide with desired state $x_d(0)$. But as the system suffer disturb and restricted by repeated positioning accuracy, initial error may be generated, i.e., $x_k(0) \neq x_d(0)$, and which accumulates as iterations times goes to infinity, then the system may be divergent. [3] design learning law to track the desired trajectory when there exists initial error, and the limit trajectory is proposed. [4] introduce the pulse function to eliminate the effects generated by fixed initial error. However, this method is unattainable. [5] discuss the limit trajectory under the PD-type learning law, when there exists initial error, then completely tracking result can not be obtained in finite time. [6] introduce initial rectifying item to the PD-type learning algorithm in dealing with uncertain time-delay system. [7] design high order PD and PID type learning law to handle fixed initial error, and limit trajectories are presented, in order to eliminate the effect by initial error, add initial rectifying item in D-type learning law to realize the completely tracking in the given interval. [8] based on the learning law, introduce initial state learning law to handle with initial state error to obtain the complete tracking effect.

As we know, PD-type learning algorithm overcome the effect produced by initial error, assure the output trajectory converges to the desired one asymptotically. Most published results show open- or closed-loop learning law. In the open-loop learning law, utilize the output error which generated in the last iteration and all-order derivative of which to construct the rectifying item to update the controller. But the gain is difficult to choose to improve tracking precision. Otherwise, in closed-loop learning law, the differential output error signal may not be obtained accurately, which has to be replaced by estimated value. [9] design P-type learning law, introduce error signal in the present iteration to compose closed-loop to improve the tracking performance. In this paper, we propose feedback-aided PD-type (FAPD) learning law, which utilize the derivation of the tracking error of the last iteration, and the tracking error of the current iteration to construct the rectifying item, inspired by [10], we propose a kind of initial rectifying strategies, and the concept of finite time attractor is introduced, the output error converges to zero in the prescribed interval is realized. In order to verify the effectiveness of the proposed algorithms, theoretical analysis and the numerical simulations are stated in this paper.

In this paper, FAPD learning law is proposed first, and the control process as Fig.1, then a kind of initial rectifying strategy is stated to eliminate the tracking error invoked by initial position error. Furthermore, the result is generalized to higher-order system.

At last, numerical results are given to demonstrate effectiveness of the proposed learning algorithms.

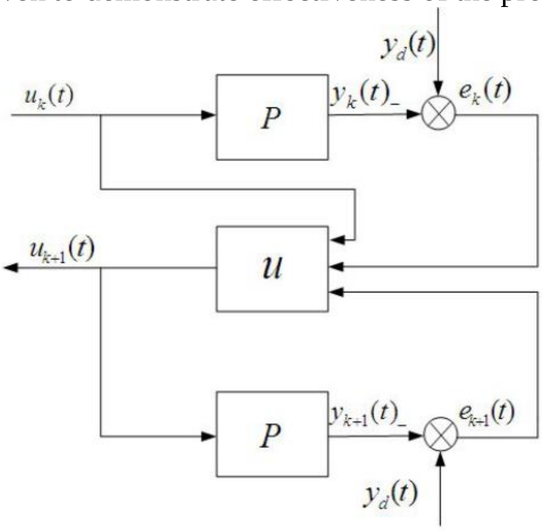

Fig. 1 the algorithm of FAPD

2. FAPD Learning Algorithm

2.1 Basic FAPD Learning Algorithm

Consider the system described as below

$$\begin{cases} \dot{x}_k(t) = Ax_k(t) + Bu_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{1}$$

where $x_k(t)$, $u_k(t)$ and $y_k(t)$ denote the state, the control input and the output of the system, respectively, and \$A, B, C\$ stand for the matrices with appreciate dimensions.

Design FAPD learning law

$$u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + L_1 e_{k+1}(t)) \tag{2}$$

where $e_k(t) = y_d(t) - y_k(t)$ is the tracking error.

PD-type(PID) learning law was stated in [2], where the tracking error and differential signal of which is adopted, that is open-loop learning pattern. Under the fixed initial state error, [3] proposed the limit trajectory under PD-type learning law. [8] discuss the PD-type learning law with the tracking error and the differential of which in current iteration to construct closed-loop learning law. In the formula (2), the derivative of tracking error is same as which in open-loop law, while tracking error itself is same as which in closed-loop law.

Theorem 1. Suppose the system described by (1), under the learning law formula (2), and hypothesis there exists a gain matrix s.t.

$$\frac{\|I - CB\Gamma\|_\infty + \|Ce^{A(\cdot)}AB\Gamma\|_1}{1 - \|Ce^{A(\cdot)}B\Gamma L\|_1} \le \rho < 1$$

and the initial state satisfies that

$$x_k(0) = x_0$$

then we obtain that

$$\lim_{k\to\infty} y_k(t) = y_d^*(t)$$

where $y_d^*(t) = y_d(t) - e^{-Lt}(y_d(0) - Cx_0)$.

Given the open-loop PD learning algorithm in the following

$$u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + L_1 e_k(t))$$

then conference the processing proposed above, we obtain that

$$\left\|e_{k+1}^*(t)\right\| \le \left\|I - CB\Gamma\right\|_\infty + \left\|Ce^{A(\cdot)}AB\Gamma\right\|_1 + 1 - \left\|Ce^{A(\cdot)}B\Gamma L\right\|_1 \left\|e_k^*(t)\right\|$$

It can be proved that

$$\frac{\left\|I - CB\Gamma\right\|_\infty + \left\|Ce^{A(\cdot)}AB\Gamma\right\|_1}{1 - \left\|Ce^{A(\cdot)}B\Gamma L\right\|_1} \le \left\|I - CB\Gamma\right\|_\infty + \left\|Ce^{A(\cdot)}AB\Gamma\right\|_1 + 1 - \left\|Ce^{A(\cdot)}B\Gamma L\right\|_1$$

that is, the convergent rate by FAPD maybe faster than which by open-loop PD learning algorithm.

From the formula (2), we see that the output trajectory converges to the desired one asymptotically via choosing decent matrix L. Through adjusting matrix L, the eigenvalue has smaller negative real part, then the tracking performance may be improved.

2.2 Initial Rectifying Strategy

In order to realize complete tracking, that is, the tracking error converges to zero in prescribed time, this paper introduce initial rectifying strategy in the learning law

$$u_{k+1}(t) = u_k(t) + \Gamma(\dot{e}_k(t) + L_1 e_{k+1}(t) + r(t)) \tag{3}$$

where $r(t) = e^{-L^T(h-t)} W_c^{-1}(0,h) e^{-Lh}(y_d(0) - Cx_0), W_c(0,t) = \int_0^t e^{-L(t-\tau)} e^{-L^T(t-\tau)} d\tau$.

Theorem 2. Under the algorithm (3), it is derived that the limit trajectory

$$y_d^*(t) = \begin{cases} y_d(t) - e^{-Lt}(y_d(0) - Cx_0) + \int_0^h e^{-L(t-\tau)} e^{-L^T(t-\tau)} d\tau W_c^{-1}(0,h) e^{-Lh}(y_d(0) - Cx_0), 0 \le t \le h \\ y_d(t), h < t \le T \end{cases} \tag{4}$$

From this theorem, we obtain that under the learning law (3), after the prescribed time h, the tracking error converges to zero.

3. Numerical Results

In this section, we prove the effectiveness of FAPD learning law and initial rectifying strategy via numerical simulations.

Consider the system (1), where

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 \end{pmatrix},$$

$$y_d(t) = 0.25t^2(5 - t)$$

under the control law (2) and (3), where $\Gamma = 1, L = 6, x_0 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, h = 0.2$

Define the index function $J_k = \max_{t\in[0.4,5]} \left|e_k(\mathrm{t})\right|$, Comparing the results as shown in figure 2-5.

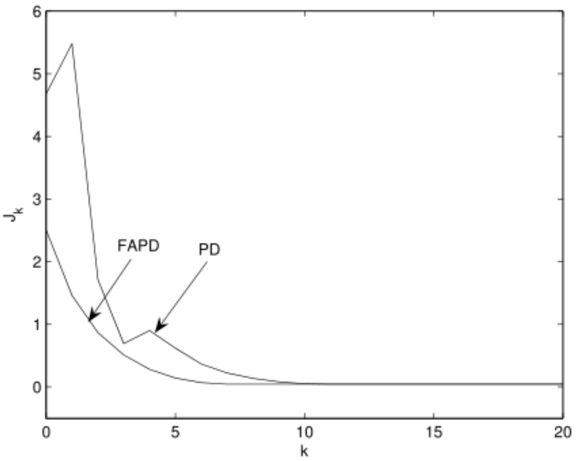

Fig. 2 the comparison of J_k

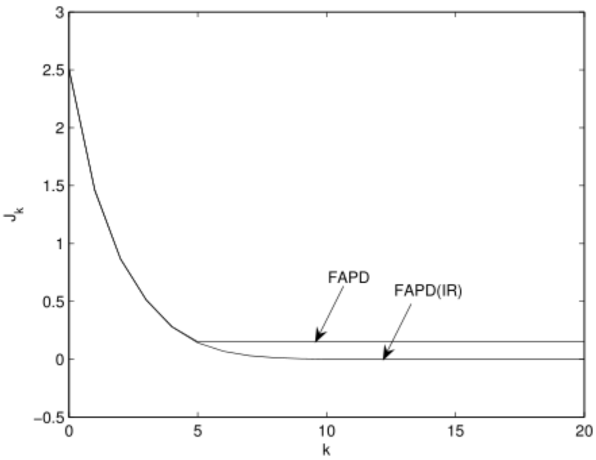

Fig. 3 the comparison of J_k

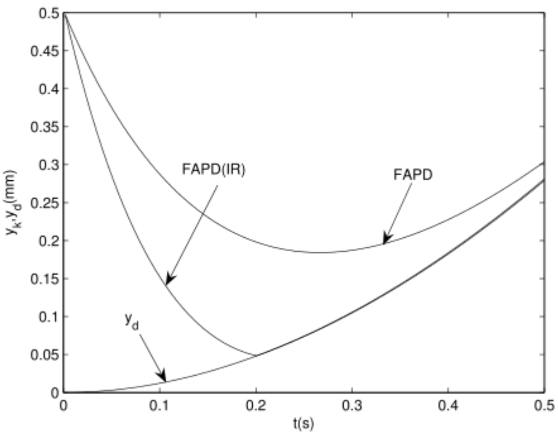

Fig. 4 the comparison of output at $k = 20$

From fig.2, tracking error rapidly converges to the stable value via FAPD learning law rather than PD learning law, but which can not converges to zero in finite time. From the fig.3, tracking error rapidly convergent via FAPD learning law and two kinds initial rectifying ones, while tracking error converges to a smaller boundary under initial rectifying algorithms. From Fig.4, when iteration number k=20, the output trajectory converges to the desired one through algorithm (3), and the tracking error converges to a smaller boundary, but output trajectory can not completely tracking the desired one under the learning law (2).

4. Conclusion

In this paper, FAPD learning law is proposed to track the desired trajectory, and initial rectifying strategy is introduced to obtain the completely tracking results in the prescribed finite time. The higher order system is analyzed, and the performance of the proposed algorithms are stated in this paper, the effectiveness of the proposed algorithms are demonstrated via numerical simulations.

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