The force analysis of the crank and connecting rod mechanism

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Abstract

In this paper, the stress of a crank and connecting rod mechanism is studied. The key is to analyze the action of various forces in the crank and connecting rod mechanism. According to these forces, the strength, stiffness and wear are analyzed, calculated and designed for the main parts of crank and connecting rod mechanism, so as to achieve the requirements of engine output torque and speed.

Keywords

Crank connecting rod mechanism, Force analysis, Design.

1. Introduction

The crank and connecting rod mechanism is a mechanism for transmitting and moving the engine. It transforms the reciprocating straight motion of the piston into the rotary motion of the crankshaft and outputs power. Therefore, the crank connecting rod mechanism is the main force component in the engine, and its reliability determines the reliability of the engine work. With the continuous improvement of the engine strengthening index, the working conditions of the mechanism are more complex. Under the action of various cyclic loads, how to ensure adequate fatigue strength and stiffness and good dynamic and static mechanical properties in the design process becomes the key problem of crank link mechanism design.

2. Motion and force analysis of crank connecting rod mechanism

2.1 The kinematics of the crank linkage mechanism

The schematic diagram of the central crank and connecting rod mechanism is shown in Figure 1. In Figure .1, the center line of the cylinder passes through the crankshaft center O, OB for crank, AB for connecting rod, B for crank pin center, and A for connecting rod small head center or piston pin center. When the crank is rotated at the same angular speed, any point on the crank OB will rotate at the same speed as the center of the O point, and the A point of the piston will do reciprocating motion along the central line of the cylinder. The connecting rod AB will do compound planar motion. The B point of the big head is connected to the end of the crank, making the rotational motion of the constant speed, and the connecting rod small head is connected to the piston to do the reciprocating motion. In practical analysis, in order to simplify the problem, the general linkage is simplified to two lumped masses, which are concentrated on the large and small ends of the connecting rod respectively. They are considered to rotate and reciprocate separately, so that no needs to study the motion law of the connecting rod separately.

When the piston moves in and out, its speed and acceleration are changed. The speed and acceleration of the crank linkage and the whole work of the engine are greatly influenced by its speed and acceleration. Therefore, the main task of studying the movement rule of the crank and connecting rod mechanism is to study the motion rule of the piston.

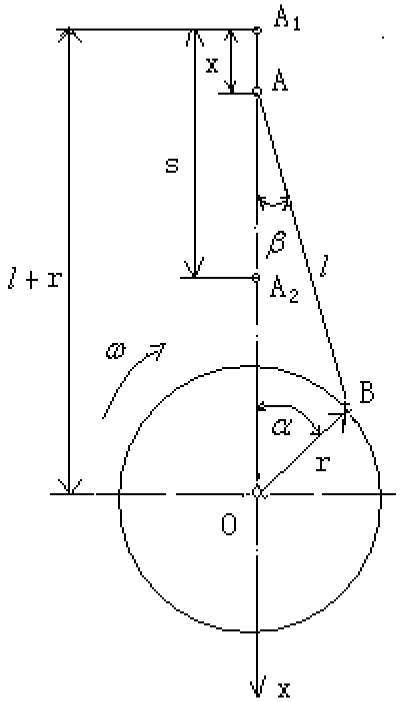

Fig. 1 motion diagrams of crank and connecting rod mechanism

2.1.1 Piston displacement

Suppose that at a certain time, the crank angle is α and rotated clockwise. The axis of the connecting rod deviates from the cylinder axis in its motion plane is β, as shown in Figure 1.

When $\alpha = 0^{\circ}$, the piston pin center A is at the top of the position A1, which is called the top stop. When $\alpha = 180^{\circ}$, the A point is at the bottom of the position A2, which is called the lower stop point. The displacement of the piston at this time x is:

$$x = \overline{A_1A} = \overline{A_1O} - \overline{AO} = (r + l) - (r\cos\alpha + l\cos\beta)$$

$$= r[(1-\cos\alpha) + \frac{1}{\lambda}(1-\cos\beta)] \quad (1)$$

Type: λ—connecting rod ratio. The formula (1) can be further simplified, as can be seen from figure 1 is:

$$r\sin\alpha = l\sin\beta$$

That is

$$\sin\beta = \frac{r}{l}\sin\alpha = \lambda\sin\alpha$$

Also because of

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\lambda^2\sin^2\alpha} \quad (2)$$

The formula (2) is brought into the form (1).:

$$x = r[1-\cos\alpha + \frac{1}{\lambda}(1-\sqrt{\lambda^2\sin^2\alpha})] \quad (3)$$

Type (3) is the exact formula for calculating the displacement of the piston of X, for the convenience of calculation, the formula (3) in the root according to Newtonian binomial theorem, too:

$$\sqrt{1-\lambda^2\sin^2\alpha} = 1-\lambda^2\sin^2\alpha - \frac{1}{8}\lambda^4\sin\alpha - \frac{1}{16}\lambda^6\sin^6\alpha - \ldots$$

Taking into account $\lambda \leq 1/3$, the numerical method above is very small, can be neglected. Only the first two items are retained.

$$\sqrt{1-\lambda^2 \sin^2 \alpha} \approx 1-\frac{1}{2}\lambda^2 \sin^2 \alpha \tag{4}$$

The formula (4) is brought into the form (3).

$$x = r(1-\cos\alpha + \frac{\lambda}{2}\sin^2\alpha) \tag{5}$$

2.1.2 The speed of the piston

The exact value of the piston velocity v can be obtained by the differential equation of the piston displacement (1) to the time t.

$$v = \frac{dx}{dt} = \frac{dx}{da} \times \frac{da}{dt} = r\omega(\sin\alpha + \frac{\lambda}{2}\frac{\sin 2\alpha}{\cos\beta}) \tag{6}$$

The approximate formula for the velocity of the piston can be obtained by means of the equation (5) to the time differential:

$$v \approx r\omega(\sin\alpha + \frac{\lambda}{2}\sin 2\alpha) = r\omega\sin\alpha + r\omega\frac{\lambda}{2}\sin 2\alpha = v_1 + v_2 \tag{7}$$

As you can see from the formula (7), the velocity of the plug is considered to be composed of $v_1 = r\omega\sin\alpha$ and $v_2 = (\lambda/2)r\omega\sin 2\alpha$ two parts of simple harmonic motion.

When $\alpha = 0^{\circ}$ or 180°, the piston speed is 0, and the piston changes the direction of motion at these two points. When $\alpha = 90^{\circ}$, $v = r\omega$ the speed of the piston is equal to the circumferential velocity of the crank pin center.

2.1.3 The acceleration of the piston

The exact value of the acceleration of the piston can be obtained by means of the equation (6) to the time differential.:

$$a = \frac{dv}{dt} = \frac{dv}{da} \times \frac{da}{dt} = r\omega^2[\cos\alpha + \lambda\frac{\cos 2\alpha}{\cos\beta} + \frac{\lambda^3}{4}\frac{\sin^2 2\alpha}{\cos^3\beta}] \tag{8}$$

The approximate value of the acceleration of the piston can be obtained by the equation (7) for the time differential.

$$a \approx r\omega^2(\cos\alpha + \lambda\cos 2\alpha) = r\omega^2\cos\alpha + r\omega^2\lambda\cos 2\alpha = a_1 + a_2 \tag{9}$$

Therefore, the acceleration of the piston can also be considered as the sum of two harmonic acceleration, which is composed of $a_1 = r\omega^2\cos\alpha$ $a_2 = r\omega^2\lambda\cos 2\alpha$.

2.2 The force in the crank connecting rod mechanism

The forces acting on the crank rod mechanism are divided into the pressure in the cylinder, the inertia force of the motion quality, the friction resistance and the load resistance on the engine crankshaft. It is difficult to master the friction force and it is difficult to master the change law. The friction resistance is ignored when the force is analyzed. The load resistance and the active force are in equilibrium state, so there is no need for additional calculation. Therefore, the effect of the change rule of the gas pressure and the motion mass inertia force on the mechanism components is mainly studied. The relevant data required for the calculation refer to the EA1113 gasoline engine, as shown in the attached table 1.

2.2.1 The force of the working fluid in the cylinder

The gas force acting on the piston is equal to the product of the gas pressure difference between the upper and the lower sides of the piston and the area of the piston top., That is:

$$P_g = \frac{\pi D^2}{4}(p - p') \tag{10}$$

P_g —The gas force on the piston, N ;

p —Absolute pressure in cylinder, MPa ;

p' —atmospheric pressure, MPa ;

D —Piston diameter, mm .

Because the diameter of the piston is certain, the gas force depends on the piston on the piston and the lower sides of the space pressure difference for the four stroke engine, general $p' = 0.1 MPa$, $D = 80.985 mm$, the absolute pressure of 1 cylinder, four stroke engine, the calculation results are shown in table 1:

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The pressure force P_g is calculated by the formula (10), as shown in table 1.

2.2.2 The inertia force of the mechanism

The inertia force is caused by the inhomogeneous motion. In order to determine the inertia force of the mechanism, it is necessary to know the distribution of the acceleration and the mass first. As the acceleration is known in kinematics, it is now necessary to know the mass distribution. The quality distribution of the actual mechanism is very complex and must be simplified. The quality conversion is carried out for this purpose.

1, Quality conversion of mechanism motion parts

The principle of mass conversion is to maintain the dynamic equivalence of the system. The purpose of mass conversion is to calculate the motion quality of the parts so as to further calculate the inertia force produced by them in motion.

Table 1. p calculation results of absolute pressure in cylinder

Four stroke end point pressure	Calculation formula	Calculation results / MPa
Inlet endpoint pressure p_{de}	$p_{de} = (0.75 \sim 0.90) p'$	0.08
Compression end pressure p_{co}	$p_{co} = p_{de} \varepsilon_e^{n_1}$	1.46
Expansion end pressure p_{ex}	$p_{ex} = \frac{p_{\max}}{\delta^{n_2}}$	0.45
Exhaust end pressure p_r	$p_r = 1.15 p'$	0.115

n_1 —Average compression index, n_1 =1.32 ~ 1.38; ε —Compression ratio, ε =9.3; n_2 —Average expansion index, n_2 =1.2 ~ 1.30; $\delta = \frac{\varepsilon}{\rho}$; $p_{\max}$ —Maximum burst pressure; $p_{\max}$ =3 ~ 5 MPa ;order $p_{\max}$ =4.5 MPa ;Pressure angle at this time $\alpha = 10^\circ \sim 15^\circ$,order $\alpha = 13^\circ$.

Table 2. Calculation results of pressure force P_g

Four strokes	P_g / N
Intake end point	77.23
Compression end point	-102.97
Expansion end point	7001.933
Exhaust end point	1801.968

(1) Conversion of connecting rod quality

A connecting rod is a part of a complex plane motion. In order to facilitate the calculation, the whole connecting rod (including related accessory) the quality of m_L by two and m_1 to m_2 mass conversion and substitution, m_1 assumptions are concentrated in the center of the top end of the connecting rod, and only the quality of the reciprocating motion of the m_1; is concentrated in the center of the connecting rod, and only do quality rotation along the circumference the movement, as shown in figure 2.2:

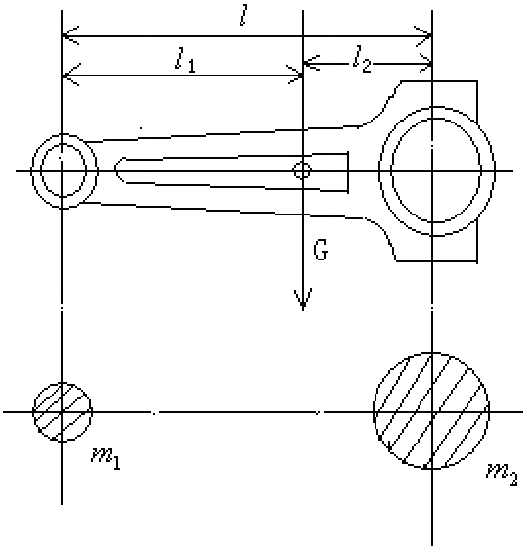

Figure 2. connecting rod quality conversion diagrams

In order to ensure that the replacement quality system is mechanically equivalent to the original quality system, the following three conditions must be met:

① the total mass of the connecting rod is constant that $m_L = m_1 + m_2$.

② the position of the center of gravity of the connecting rod is constant, that $m_1 l_1 = m_2 (l - l_1)$.

③ the moment of inertia of the connecting rod relative to the center of gravity G is I_G, that $m_1 l_1^2 + m_2 (l - l_2)^2 = I_G$.

Among them, l connecting rod length, l_1 for connecting rod center of G. The following conversion formula can be obtained by the conditions:

$$m_1 = m_L \times \frac{l - l_1}{l} \quad m_2 = m_L \times \frac{l_1}{l}$$

The cable polygon method of balancing force is used to find the center of gravity G . The connecting rod is divided into several simple geometric figures. The weight of each link and its center of gravity are calculated respectively. According to the polygon drawing method, the center of gravity of the connecting rod is calculated, and the weight of the center of the connecting rod is calculated to be G_1 and G_2, as shown in figure 3:

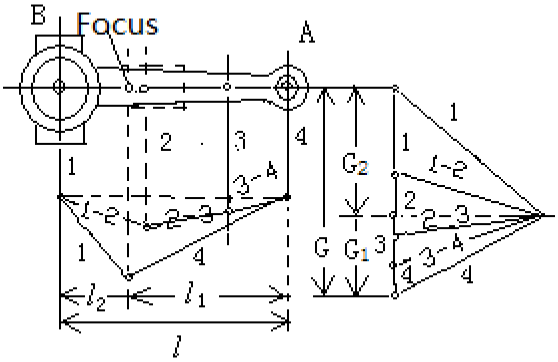

Figure 3. Cable polygon method

(2) The mass of the reciprocating linear motion part m_j

The piston (including the parts on the piston) is moving in a straight line along the center of the cylinder. Their quality can be considered to be concentrated on the center of the piston pin and expressed in m_h . The quality m_h and the sum of m_1 the mass conversion to the center of the connecting rod is called the reciprocating motion quality m_j , that $m_j = m_h + m_1$.

(3) Mass of unbalanced rotation m_r

The unbalance quality and replacement quality of the crutches are shown in Figure 4.

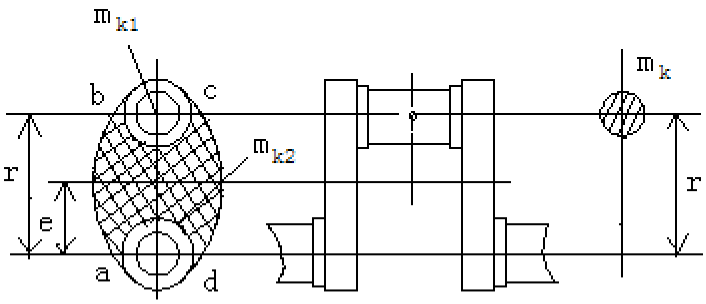

Figure 4. The unbalance mass and its substitution quality

The mass of a crank pin and a part of the crank arm will produce an unbalanced centrifugal force, known as the unbalance mass of the curved crank when the crank is rotated around the axis. For convenience of calculation, all these qualities are converted to the center of the connecting rod journal with a turning radius of r according to the condition of equal centrifugal force, and the conversion mass m_k is m_k :

$$m_k = m_g + 2m_b \frac{e}{r}$$

Type: m_k —Conversion quality of flexure, kg ;

m_g —The quality of the connecting rod journal, kg ;

m_b —The quality of a crank arm, kg ;

e —The distance between the centroid position of the crank arm and the center of the crank, m .

The mass of m_2 and the connecting rod quality m_k to the center of the large head is called the mass of unbalanced rotation, that $m_r = m_k + m_2$.

Calculated by the above conversion method:

The mass of the reciprocating linear motion part is m_j =0.583 kg ,Mass of unbalanced rotations m_r =0.467 kg .

2, the inertia force of the crank connecting rod mechanism

After the mass of the crank rod mechanism is simplified to two mass m_j and m_r , the mass inertia force can be calculated from the motion condition and come down to two forces. Reciprocating inertia force m_j of reciprocating mass of P_j and rotational inertia force of rotating mass of m_r :

(1) inertia force

$$P_j = -m_j a = -m(r\omega^2 \cos\alpha + r\omega^2 \cos 2\alpha) = -m_j r\omega^2 \cos\alpha - \lambda m_j r\omega^2 \cos 2\alpha \quad (11)$$

Type: m_j —Quality of reciprocating movement, kg ;

λ —Connecting rod ratio;

r —Crank radius, m ;

ω —Crank rotation angular velocity, rad/s ;

α —Crankshaft angle.

P_j the central line of the cylinder direction, formula (11) before the negative sign direction P_j and acceleration of the piston a on the contrary.

The angular velocity of the crank is ω :

$$\omega = \frac{2\pi n}{60} = \frac{\pi n}{30} \quad (12)$$

Type: n —Crankshaft number, $r/\min$;

Known rated number n =5800 $r/\min$, so $\omega = \dfrac{\pi \times 5800}{30} = 607.07$ rad/s ;

Referring to appendix Table 2: four cylinder machine working cycle tables, the crankshaft angle α of each working condition is substituted for (11). The reciprocating force of P_j is calculated, and the result is shown as shown in Table 3:

Table 3 calculation results of reciprocating inertia force

Four strokes	P_j / N
Intake end point	-10519.68
Compression end point	6324.5
Expansion end point	-10519.68
Exhaust end point	6324.51

(2) Rotating inertia force

$$P_r = -m_r r\omega^2 = -0.467 \times 0.04023 \times 607.07^2 = -6923.799\ N \quad (13)$$

3. The total force acting on the piston

As mentioned above, at the center of piston pin, gas interaction force P_g and reciprocating inertia force P_j at the same time, because the direction of force is all along the center line, so only algebraic addition is required, resultant force can be obtained

$$P_\Sigma = P_g + P_j \tag{14}$$

The results of the calculation are shown in Table 4.

4. Decomposition and transfer of the total force on the piston

As shown in Figure 5, first, P_Σ decomposed into two forces: the force acting along the connecting rod axis K and the lateral force of the piston pressure to the cylinder wall N .

The force along the connecting rod is K :

$$K = P_\Sigma \frac{1}{\cos\beta} \tag{15}$$

And the lateral force is N :

$$N = P_\Sigma \tan\beta \tag{16}$$

Table 4. on the total force on the piston P_Σ

Four strokes	Pressure force P_g / N	Inertia force P_j / N	Total force P_Σ / N
Intake end point	77.23	−10519.681	−10442.45
Compression end point	-102.97	6324.5	6221.54
Expansion end point	7001.933	−10519.681	−3517.747
Exhaust end point	1801.968	6324.5	8126.478

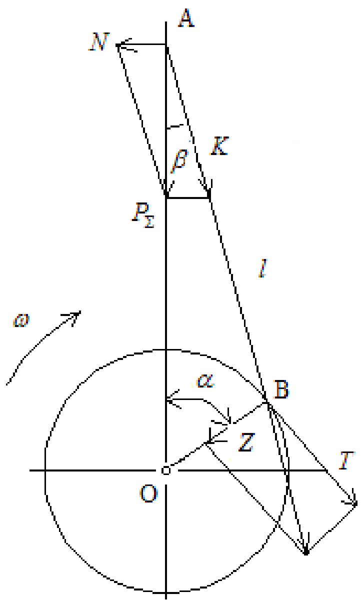

Figure 5. forces and moments on an mechanism

The provisions are as follows: K the force direction connecting rod of the connecting rod of the connecting rod is a compression, tension is negative, the lateral force wall N of the symbol is defined as: when the lateral force of anti-torque with the crankshaft rotating in the opposite direction, the lateral force is positive and negative.

When $\alpha = 13^{\circ}$, according to the sine theorem, it can be obtained:

$$\frac{l}{\sin\alpha} = \frac{r}{\sin\beta}$$

Obtained $\beta = \arcsin\frac{r\sin\alpha}{l} = \arcsin\frac{40.23\times\sin 13^{\circ}}{149} = 3.48^{\circ}$

β Substituting (15) and formula (16) respectively, the results are shown as shown in table 2.5:

Table 5. connecting rod force K and lateral force N

Four strokes	Connecting rod force K/N	lateral force N/N
Intake end point	-10717.128	-2410.83
Compression end point	6385.19	1436.356
Expansion end point	-3610.278	-812.136
Exhaust end point	8340.237	1896.923

Force K acts on the crank arm of the crankshaft through the connecting rod, and the force is also decomposed into two forces, that is, the tangential force that drives the crankshaft to rotate T,

$$T = K\sin(\alpha+\beta) = P_{\Sigma}\frac{\sin(\alpha+\beta)}{\cos\beta} \tag{17}$$

The radial force of the crank arm and the compression of the crank arm Z, that:

$$Z = K\cos(\alpha+\beta) = P_{\Sigma}\frac{\cos(\alpha+\beta)}{\cos\beta} \tag{18}$$

The force T is consistent with the rotation direction of the crankshaft, and the force Z points to the crankshaft as positive. The tangent force T and radial force Z are shown as shown in table 2.6:

Table 6. calculation results of tangent force T and radial force Z

Four strokes	Tangential force T/N	Radial force Z/N
Intake end point	-3040.242	-10276.856
Compression end point	1811.355	6122.8789
Expansion end point	-1024.17	-346.964
Exhaust end point	2365.96	7997.61

3. Conclusion

This chapter first analyzes the movement of the crank and connecting rod mechanism, analyzed the movement of the piston, based on the analysis of the change of gas pressure in each working process, calculation formula deduced the process gas force theory, the conversion mechanism in motion quality, and according to the specific structural parameters of EA113 type gasoline engine to calculate the gas force of each process, and provides a theoretical basis for the dynamic simulation data of the following chapters.

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