Dynamical characters analysis of nonlinear ship rolling in rough sea based on Empirical Mode Decomposition and Hilbert-Huang Transform

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Abstract
This paper attempts to simulate the nonlinear rolling movements of ships under random ocean waves according to empirical mode decomposition and Hilbert-Huang Transform. Since the roll movement of ship is a very important factor to ship capsizing and it belongs to the strong nonlinear problem, the general linear methods are not suitable for the analysis. The Hilbert-Huang transform is a new two-step time-frequency analytic method to analyze the nonlinear and non-stationary signal. In this paper, the roll angles, which are simulated from the established model, are looked as the analytic signals, and then some of intrinsic mode functions are obtained by means of empirical mode decomposition on the simulated signals. According to the Hilbert time–frequency transform, the nonlinear characters of ship rolling are analyzed. Examples from the numerical results of signals are given to demonstrate the power of this new method and these results can clarify the advantage and efficiency of the method, which brings a new means to analysis the ship-rolling problem.

Keywords
Empirical Mode Decomposition (EMD); Hilbert-Huang transform (HHT); nonlinear rolling;

1. Introduction
Capsizing of ships constitutes a primary group of casualties that leads to loss of life and money. Unfortunately, its mechanism has yet to be fully resolved due to underlying complex dynamics and parameters. Upon studying the causes in more detail, designing safer ships against capsizing may become a reality. [1] The most dangerous motion to ship capsizing is severe rolling. In the paper the nonlinear damping moment, restoring moment, random wind load and random excitation are taken into accounted in the nonlinear rolling-motion differential equation. Ship rolling is simulated by a single degree-of-freedom system. The governing differential equation of motion is integrated by applying the Runge-Kutta method. Then the EMD method and is applied to the different sea states and the instantaneous states of ship motion. The calculation results shows that Hilbert-Huang transform is easy to distinguish the nonlinear phenomenon.

2. Hilbert-Huang Transform [2,3]
Recently, a number of new methods have been proposed to analyze vibration signals. One of the promising methods is the Hilbert-Huang Transform (HHT). The HHT is derived from the principals of empirical mode decomposition (EMD) and the Hilbert Transform. When applying the HHT, first, the EMD will decompose the acquired signal into a collection of intrinsic mode functions (IMF). The IMF is a kind of complete, adaptive and almost orthogonal representation for the analyzed signal. Since the IMF is almost monocentric, it can determine all the instantaneous frequencies from the nonlinear or non-stationary signal. Second, the local energy of each instantaneous frequency can be derived through the Hilbert Transform. Hence, the result is an energy-frequency-time distribution of the signal. Since applying the process of HHT is not computational intensive, the HHT becomes a promising method to extract the properties of nonlinear and non-stationary signal. However, after the completion of a thorough experiment, the result generated by the HHT has its deficiency. First, the
EMD will generate undesirable IMFs at the low-frequency region that may cause misinterpretation to the result. Second, depends on the analysed signal, the first obtained IMF may cover too wide a frequency range such that the property of monocomponent cannot be achieved. Third, the EMD operation cannot separate signals that contain low energy components. In this study, new techniques have been applied to improve the result of HHT. In the improved version of HHT, the wavelet packet transform (WPT) is used as preprocessing to decompose the signal into a set of narrow band signals prior to the application of EMD. With the help from WPT, each IMF derived from the EMD can truly become monocomponent. Then, a screening process is conducted to remove unrelated IMFs from the result. Both simulated and experimental vibration signals of having a rotary system with the fault of rubbing occurred have proven that the improved HHT does show the rubbing symptoms more clear and accurate than the original HHT. Hence, the improved HHT is a precise method for nonlinear and non-stationary signal analysis.

3. Nonlinear ship rolling

3.1 Nonlinear ship rolling model

In this paper ship rolling is simulated by a single degree-of freedom system. Non-linear viscous damping and the restoring moment, presented by a higher order polynomial, are taken into account. According to the theory of moment balance the governing differential equation of motion can be described as below: \[ M_I(t) + M_D(t) + M_S(t) = M(t) + M_p \] (1)

Where \( M_I(t) \) is the virtual moment of inertia. \( M_D(t) \) is the damping moment. \( M_S(t) \) is the restoring moment. \( M(t) \) is the wave excitation moment. \( M_p \) is the random wind load.

\[ M_I(t) = I_\phi = (I_S + I_W) \phi \] (2)

The virtual moment of inertia consists of the ship mass moment \( I_\phi \) and the added mass moment of the surrounding water \( I_W \). Where \( \phi \) is the roll angle and \( \dot{\phi} \) is the roll angle acceleration.

\[ M_D(t) = D_1 \dot{\phi} + D_3 \phi^3 \] (3)

The damping moment is viscous and may be approximated by a cubic polynomial as an analytical function. Where \( \phi \) is the roll angle and \( \dot{\phi} \) is the roll angle speed.

\[ M_S(t) = K_1 \phi + K_3 \phi^3 + K_5 \phi^5 \] (4)

The restoring moment is hydrostatic and given by a non-linear antisymmetric function with three roots for an intact ship. It may be represented by a fifth-order polynomial.

Substituting from equations (2)-(4) into equation (1) and dividing by the moment of inertia. The final form of differential equation of motion is obtained:

\[ \ddot{\phi} + d_1 \dot{\phi} + d_3 \phi^3 + k_1 \phi + k_3 \phi^3 + k_5 \phi^5 = m(t) + m_p(t) \] (5)

where \( d_i = D_i / I, i=1,3,k_i = K_i / I, i=1,3,5 \); \( m(t) = M(t) / I \); \( m_p(t) = M_p(t) / I \)

According to the theory of harmonic waves and random waves, the relative excitation moment equation can be described as below:

\[ m(t) = a_0^2 a_0^2 \sin^{n} x \sum_{n=1}^{\infty} \frac{h_n}{\lambda_n} \cos(\omega_n t + \xi_n) \] (6)

Where \( a_0 \) is the effective wave slope coefficient. \( \omega_0 \) is the initial roll natural frequency. \( x \) is the heading angle. \( n \) is the index of wave component. \( h_n \) is the wave height. \( \lambda_n \) is the wavelength. \( \omega_n \) is the encounter frequency for a wave component. \( \xi_n \) is the random phase angle in the range 0 to \( 2\pi \).
whose probability of occurrence $1/2\pi$ is uniformly distributed. Based on the wave energy spectrum the wave height can be described as

$$h_n = 2\sqrt{2\bar{\omega}S(n\bar{\omega})}$$  \hspace{1cm} (7)

where $\bar{\omega}$ is the step of wave frequency. $S(\omega)$ is the energy spectrum, which is expressed as:

$$S(\omega) = 0.862 \frac{0.0135}{\omega^5} \exp[-\frac{5.186}{\omega^{1.63}h_{1/3}^3}] \quad \text{(8)}$$

where $h_{1/3}$ is the significant wave height, the remaining quantities are defined as follows:

$$p = \exp[-\frac{(\omega-\omega_m)^2}{2\delta^2\omega_m^2}]$$  \hspace{1cm} (9)

$$\omega_m = 0.32 + \frac{1.80}{h_{1/3}^3} + 0.6$$  \hspace{1cm} (10)

$$\delta = \begin{cases} 0.08, \omega < \omega_m \\ 0.10, \omega > \omega_m \end{cases}$$  \hspace{1cm} (11)

The wavelength is given by

$$\lambda_n = \frac{2\pi g}{(n\bar{\omega})^2}$$  \hspace{1cm} (12)

Furthermore, the encounter frequency for a wave component takes the form as:

$$\omega_{en} = n\bar{\omega} - \frac{(n\bar{\omega})^2U}{g} \cos \chi$$  \hspace{1cm} (13)

Where $U$ is the ship’s speed.

The wind load moment is defined in the appendix [6].

### 3.2 Roll parameters

Rolling motion is illustrated in the case of a fishing vessel with the parameters shown in reference [5].

### 3.3 Roll analysis

Using MATLAB to establish the simulation model, which is shown below in figure 1. The figure 1 model is built in MATLAB SIMULINK BLOCK. The other functions are all written in m-function of MATLAB. The results of different methods are shown below in figure 2 and 3.

![Figure 1: the simulation](image1.png)

![Figure 2: the EMD results](image2.png)
4. Conclusion

The EMD method is used to process the simulated roll signals, Hilbert transform is made for each of the IMFs to obtain Hilbert spectrum of the signals. From the figure 2, we can’t observe the characters of the time series. Then the HHT results show us the periodic response, quasi-periodic, chaotic motions respectively. Only by observing the time series information, we can’t observe the different motion characters, but when the signals are transformed by EMD and HHT, the characters are clearly showed.

EMD and HHT methods are introduced to the analysis of the dynamical behaviors of the ship rolling in rough sea. Examples from the numerical results of signals are given to demonstrate the power of this new method and these results can clarify the advantage and efficiency of this method.

References