Kinematic Analysis of Human Lower Limbs Based on Homogeneous Coordinate Transformation and D-H Method

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Abstract

In the structure based on the analysis of the lower limb of the human body, we establish a 7 rod model of human body, and the human lower extremity kinematics equation is built by using the homogeneous coordinate transformation and the D-H method. Then the equations of motion are tested by using MATLAB. It provides a reference for studying the movement of human lower limbs and designing the exoskeleton robot of the lower limb.

Keywords

7rod model, homogeneous transformation, D-H method, exoskeleton.

1. Introduction

The lower extremity exoskeleton robot, as a human-machine integrated mechanical device, has tremendous application value in its theory of military individual combat and aids in walking and medical walking rehabilitation, and thus becomes a country in various countries of the world. One of the hot topics in the research robot. In addition, foreign studies on the exoskeleton of the lower extremities have made great progress. The most representative ones are the American BLEEX and HULC lower limb exoskeletons [1-2], the Japanese HAL exoskeleton [2], and Israel. ReWalk exoskeleton exoskeletons [3] et al. Domestic research on the exoskeleton of the lower extremities is relatively late. Although some research results have been obtained, most of them are still in the stage of theoretical analysis and laboratory testing [4]. The purpose of this paper is to analyze the structure characteristics of human lower limbs, and use homogeneity transformation method to establish a mathematical model of the lower limbs of the human body. Finally, it is proposed to use MATLAB to solve it, which can provide the necessary theoretical basis for the further design of lower extremity exoskeleton robots.

2. Homogeneous coordinate method - establishment of human lower limb model

Fig.1 The lower limb model of human body
According to the human body structure of the lower limb structure of the human body to establish a human body model, as shown in Fig.1 lower limb model[5].

To define the pose of each link, first set the local coordinate system of each rod. Considering that each connecting rod moves around the corresponding joint, the origin of the established local coordinate system is established on each joint of the lower limb, and its direction is consistent with the division of the human body in the three-dimensional plane direction. In addition, in the established 7-link human lower limb model, the parameters of each connecting rod are also important parts of the model description, and are the basis for kinematics and dynamic analysis[6]. These parameters include the length, mass, center of mass position, and joint of the connecting rod angle.

3. Homogeneous Coordinate Method - Analysis of Right Kinematics of Human Lower Limb

The kinematics of the lower limbs of the human body studies the displacement relationship, velocity relationship and acceleration relationship between the various links of the human lower limbs. Based on the symmetry of the left and right legs of the human body, in addition, the normal left and right legs are often alternated periodically during the walking process. Therefore, the study of the lower limb movement of the human body can be performed from only one leg[7]. This article takes the right leg of the human lower extremity as the research object, and makes the following assumptions: 1. The upper limb of the human body (the seventh bar in Fig.1) always maintains the state of the established model in the walking gait; 2. The movement of the human lower limb can be seen as relative The movement of the center point O of the upper extremity segment; 3. In the analysis of exercise, it is considered that the point O is the fixed point, which is equivalent to the person being dangled, and the upper body is fixed while the lower extremities are relative to the treadmill. Thus, for the right leg lower extremity model in Fig.1, its motion equation is obtained using homogeneous coordinate transformation:

\[
\begin{align*}
\alpha T &= \begin{bmatrix} \alpha R & \alpha P_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\beta T &= \begin{bmatrix} \beta R & \beta P_C \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\gamma T &= \begin{bmatrix} \gamma R & \gamma P_E \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{align*}
\]

among them:

\[
\begin{align*}
\alpha P_A &= [0 \ -l_{OA} \ 0]^T, & \beta P_C &= [0 \ 0 \ -l_1]^T, & \gamma P_E &= [0 \ 0 \ -l_3]^T, \\
\alpha R &= \text{Rot}(x,y,z;\theta_1,\theta_2,\theta_3), & \beta R &= \text{Rot}(y,\alpha), & \gamma R &= \text{Rot}(x,y,z;\beta_1,\beta_2,\beta_3)
\end{align*}
\]

From (1), (2) and (3)Right knee position and posture :

\[
\begin{align*}
\alpha P_c &= \alpha P_A + \alpha R \beta P_C \\
\beta R &= \alpha R \beta R
\end{align*}
\]

Right ankle position and posture:

\[
\begin{align*}
\alpha P_e &= \alpha P_A + \alpha R (\beta P_c + \beta R \gamma P_E) \\
\beta R &= \alpha R \beta R \gamma R
\end{align*}
\]

Among them: \(\alpha P_A\) is the right hip position, \(\beta R\) is the right hip position.

The obtained equations of motion (1)~(7) can be seen that when the length of each rod is known, there are 7 control variables for the right leg, namely 3 rotation variables around the hip at the hip joint, and
1 at the knee joint. There are three rotation variables around the axis and three rotation variables at the ankle joint[8]. Therefore, when these 7 variables are given a specific value, the corresponding position of the entire right leg of the lower limb is determined.

### 4. D-H method - establishment of the right side member model for lower limb

According to the characteristics of the movements of the various lower limbs of the human body[9], the D-H method is used to establish a coordinate system model, as shown in Fig.2.

![Fig.2 Right D-H model of human lower limb](image)

From the coordinate system established in Fig.2, the corresponding link parameters in Table 1 can be given.

<table>
<thead>
<tr>
<th>#</th>
<th>i</th>
<th>(a_{i-1}(mm))</th>
<th>(a_i(\degree))</th>
<th>(d_i(mm))</th>
<th>(\theta_i(\degree))</th>
<th>(q)</th>
</tr>
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<td>L3</td>
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<td>0</td>
<td>01 (0)</td>
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<tr>
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<td>0</td>
<td>-90</td>
<td>0</td>
<td>02 (-90)</td>
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</tr>
<tr>
<td>2-3</td>
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<td>0</td>
<td>-90</td>
<td>0</td>
<td>03 (-180)</td>
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<td>0</td>
<td>04 (0)</td>
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<td>0</td>
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<td>06 (-90)</td>
<td>06</td>
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<tr>
<td>6-7</td>
<td>7</td>
<td>0</td>
<td>-90</td>
<td>0</td>
<td>07 (0)</td>
<td>07</td>
</tr>
</tbody>
</table>

### 5. Analysis of Right Kinematics of Human Lower Limb by D-H Method

In the above, the parameter list of the right link of the lower limbs of the human body was established. According to the principle of "from left to right", the link transformation between adjacent coordinate systems is denoted as \(i_{i}\), and the operation formula is:

\[
^i_{i-1}T = \text{Trans}(x, a_{i-1})\text{Rot}(x, \alpha_{i-1})\text{Trans}(z, d_i)\text{Rot}(z, \theta_i)
\]

(8)

Expand formula by:
\[ (c \theta_i \cos \theta_i, \quad s \theta_i \sin \theta_i) \]

According to the link parameters listed in Table 1, it is brought into equation (9) to obtain each link transformation matrix:

\[
0_7^7T = \begin{bmatrix}
    c \theta_1 & -s \theta_1 & 0 & L_1 \\
    s \theta_1 & c \theta_1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
0_4^2T = \begin{bmatrix}
    c \theta_2 & -s \theta_2 & 0 & 0 \\
    s \theta_2 & c \theta_2 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
0_3^3T = \begin{bmatrix}
    c \theta_3 & -s \theta_3 & 0 & 0 \\
    s \theta_3 & c \theta_3 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
0_6^6T = \begin{bmatrix}
    c \theta_6 & -s \theta_6 & 0 & 0 \\
    s \theta_6 & c \theta_6 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

According to the relative coordinate system transformation chain multiplication rule, we can get the transformation formula of the coordinate system relative to the \(0\) system at any joint:

\[
0_7^T = 0_7^1T \cdot 0_4^2T \cdot 0_3^3T \cdot \ldots \cdot 0_{13}^{12}T \cdot 0_{13}^{14}T \cdot 0_{14}^{15}T \cdot 0_{15}^{16}T \cdot \ldots \cdot 0_n^T \quad (10)
\]

Similarly, a transformation formula between arbitrary joint coordinate systems can be obtained.

\[
\begin{align*}
0_j^T &= 0_j^iT \cdot 0_{i+1}^jT \cdots k_i^T, \quad k - j \geq 2 \\
0_j^w &= 0_j^T \cdot 0_{k+1}^T \cdots 0_{w-1}^T, \quad v < w
\end{align*}
\]

Each of the obtained link transformation matrices is brought into equation (10) to obtain the right kinematics equation of the human lower limbs, i.e., the transformation matrix \(0_7^T\)

\[
0_7^T = 0_7^1T(\theta_1)0_7^2T(\theta_2)0_7^3T(\theta_3)0_7^4T(\theta_4)0_7^5T(\theta_5)0_7^6T(\theta_6)0_7^7T(\theta_7)
\]

In order to verify the correctness of the established kinematics equations, the initial values of the various joint variables in Table 1 are brought into equation (12).

Calculate the value of \(\theta_1 = 0^\circ, \quad \theta_2 = 0^\circ, \quad \theta_3 = 90^\circ, \quad \theta_4 = -90^\circ, \quad \theta_5 = 0^\circ, \quad \theta_6 = 0^\circ, \quad \theta_7 = 0^\circ\). The result is

\[
0_7^T = \begin{bmatrix}
    1 & 0 & 0 & L_3 \\
    0 & -1 & 1 & 0 \\
    0 & 0 & -1 & -L_4 - L_5 \\
    0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
Contrary to the position and orientation of the waist center coordinate system relative to the coordinate system {7} shown in Fig. 2, the model is established correctly.

6. MATLAB kinematics simulation

According to the D-H parameters established in Table 1, the robot linkage parameters were programmed in the MATLAB programming environment, and the human body’s lower limb right simulation model was obtained, as shown in Fig. 3. Comparing Fig. 2 with Fig. 3, it can be seen that the established model is correct.

Fig.3  human body’s lower limb right simulation model

7. Summary

This paper analyzes the structural characteristics of human lower limbs, establishes a 7 rod model of human lower limbs, establishes its kinematics equations on the right side of human lower limbs using homogeneous coordinate transformation and DH method, and validates the establishment of kinematics using MATLAB. Therefore, it can provide basis for further study of human lower limb movement and design of human lower extremity exoskeleton robot.

References


Shanghai Science and Technology Press, 2008.