Modeling and Analysis of Mecanum Wheel and Its Four-Wheel System

Dongfang Xu [a], Yang Gao [b]

School of Shandong University of Science and Technology, Shandong 266590, China

[a]dongfangxu1995@163.com, [b]1582629943@qq.com

Abstract

Mecanum wheel is compact and flexible in movement. It is a successful Omnidirectional wheel. In this paper, an elliptic arc approximation method is proposed to establish the roller curve equation, and the roller curve is fitted by using the method of interval picking, so that the Mecanum wheel can be accurately modeled. The kinematics analysis of the four-wheel system based on Mecanum wheel was performed to study the relationship between the speed of the mobile platform and each Mecanum wheel, the effect of each parameter change on the Mecanum wheel speed was also analyzed. These provide a basis for further research on the kinematics of omnidirectional mobile platforms.

Keywords

Mecanum Wheel,Roller Curve, Four Wheel System,Kinematic Analysis.

1. Introduction of Mecanum Wheel

The Mecanum wheel was invented by Swedish engineer Bengt Ilon in 1973[1]. It has a compact structure, flexible movement, and a very successful Omnidirectional wheel. Its structure is shown in Fig.1. It is composed of a hub and a group of turning rollers uniformly arranged around the hub. The roller axis N is at an angle (usually 45°) with the hub axis M. During the movement[2], these angled rollers passively rotate under the friction of the ground, which will generate an axial force component. By adjusting the angular velocity and steering of each Mecanum wheel, the resultant force and slewing moment in any direction in the plane can be synthesized. So the mobile device can be rotated under the conditions of zero radiusand without changing the attitude of the vehicle body.

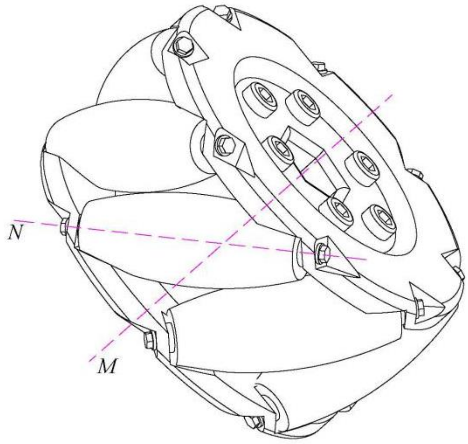


Fig.1 The structure of Mecanum wheel

2. The Design and Modeling of Mecanum Wheel

The rollers of the Mecanum wheel are special. When the wheel rotates around the axle, the envelope formed by the outer contour of all the rollers is a cylindrical surface, so the wheel can continuouslyand stably roll forward. Combining with the research results of scholars at home and abroad, this paper adopts the elliptic arc approximation method to establish the roller curve equation, and uses the method of interval picking to draw the roller generatrix. Taking into account the deformation of the roller and giving corresponding compensation, a three-dimensional model of the

Mecanum wheel was established, which improved the running continuity and movement accuracy of the Mecanum wheel.

2.1 The Establishment of Roller Curve

As shown in Fig.2, the width of Mecanum wheel hub is b, the angle between roller axis and wheel axis is α , and both are known parameters. The wheel radius is R, and the roller curve (curve MN) is projected on the XOY plane. The central angle corresponding to the arc M'N' is γ_0, the effective length of the roller is a,the vertical distance between the Mecanum axis A'B' and the line AB is s, and s' is the vertical distance between the axis of the roller and the axis of the hub.

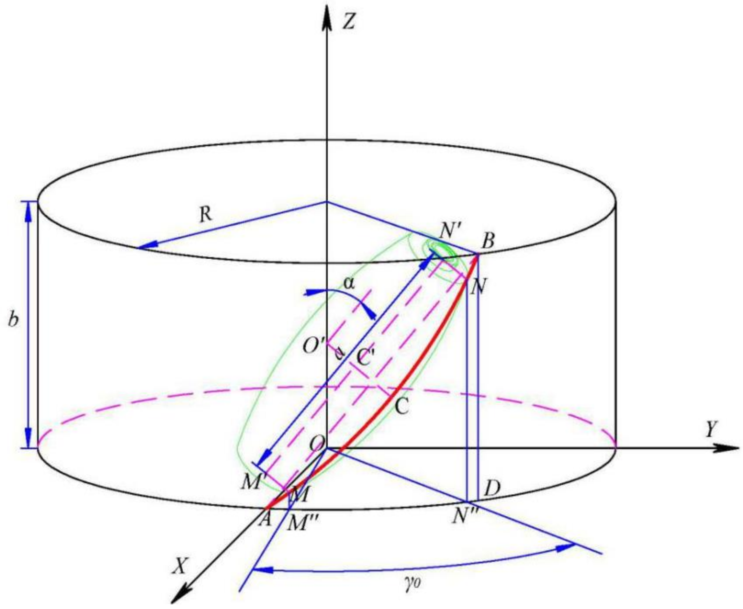


Fig.2 Elliptic arc approximation

Taking the a=80mm as an example to design the Mecanum wheel roller. Because the end point of the roller axis can not be in point contact with the roller edge, the line segment MN needs to be offset by 8mm toward the center of the Mecanum wheel. The resulting line segment M'N' is the roller axis, and there is $s-s'=8\text{mm}$. The specific relationship between the parameters can be expressed as:

$$
\begin{aligned}
&\alpha = \arctan\frac{2R\sin(\gamma_0/2)}{b}\\
&s' = R\cos(\gamma_0/2)-8\\
&\gamma_0 = \arcsin\frac{\sqrt{2}a}{2R}
\end{aligned}
\tag{1}
$$

From the analysis of Fig.2, set $\gamma_0 = 52°$, and substitute the values of known parameters a and α into equation (1), calculate and round, and take R=71mm, s=55mm, b=70mm (consider the thickness of the fixed roller portion) .

As shown in Fig.2, in the coordinate system O-XYZ, the curve MN is the roller curve, and its extension line crosses the end face of the Mecanum wheel at two points A and B. The plane ACB is at an angle of 45° to the plane XOY, and the projection of the curve ACB on the plane XOY is the arc AD. In the two-dimensional coordinate system O-XY, the curve equation of the arc AD can be expressed as:

$$x^2 + y^2 = R^2 \tag{2}$$

To find the equation of the bus MN, we need to establish a two-dimensional coordinate system C'-XY, as shown in Fig. 3, and the coordinate origin is selected at C'. Because the angle between the coordinate system C'-XY and the plane of the coordinate system O-XY is 45°, when all the points on the coordinate system O-XY are projected to the coordinate system C'-XY, its abscissa is unchanged, and Coordinates are enlarged $\sqrt{2}$ times.

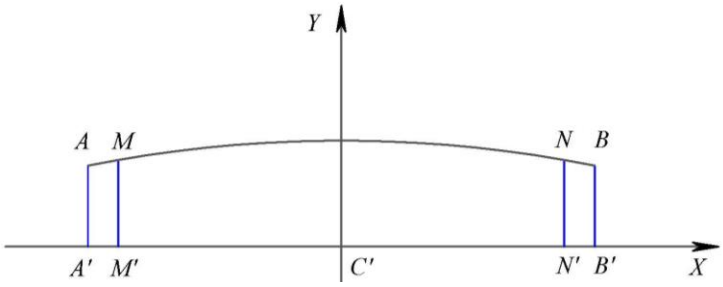


Fig.3 Anatomy of the roller curve

Thus the curve equation of the MN is:

$$\frac{X^2}{\left(R\sqrt{2}\right)^2}+\frac{(Y+O'C')^2}{R^2}=1 \tag{3}$$

By substituting the data, you can find the roller curve MN, using Matlab to paint the roller curve diagram, cause the roller is symmetrical structure, here only depicts the half of entire curve, the roller curve shown in Fig 4.

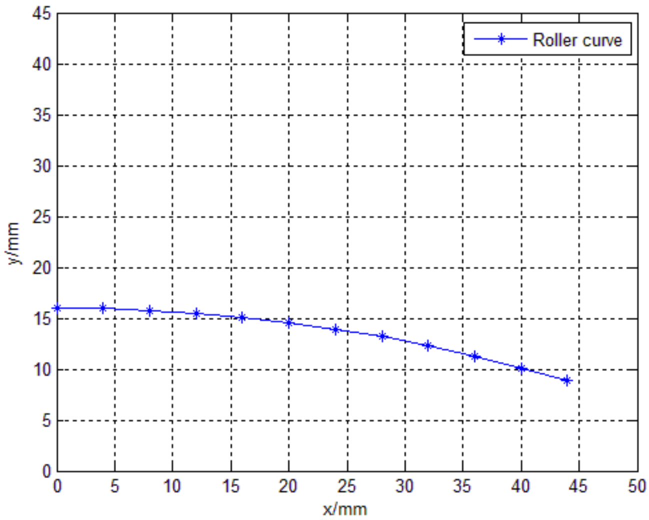


Fig.4 Roller curve

2.2 Calculation of Roller Coincidence

Fig.5 is an Anatomy of the Mecanum wheel roller.The projection of the curve AB on the XOY plane is AD, so AD=BD=70mm and AB=99mm. Set the arc angle corresponding to the arc AD to γ. In the ΔAOD, AO=OD=R=71mm, AD=70mm, according to the cosine theorem, $\gamma=59.07°$. Because MM', NN' are the boundary lines of both ends of the roller, and are all perpendicular to the roller axis M'N', known M'N'=80mm, so AM=9.5mm, then AM_1=38.5mm. Draw a straight line M1M2, make it perpendicular to the AD, MM" vertical to the plane XOY at the point M", so M_2M'' must be perpendicular to the line segment AD; In the $Rt\Delta AM_1M_2$, $AM_1=9.5mm$, so $AM_2=6.7mm$, you can find $M_2M''=9.2mm$; In the $Rt\Delta AM_1M_2$: $AM_2=6.7mm$, $M_2M''=9.2mm$, then $AM''=11.4mm$, connecting OM", in the $\Delta AOM''$: $AO=OM''=R=71mm$, $AM''=11.4mm$, then by the cosine theorem can obtain the arc center angle $\beta=9.21°$.

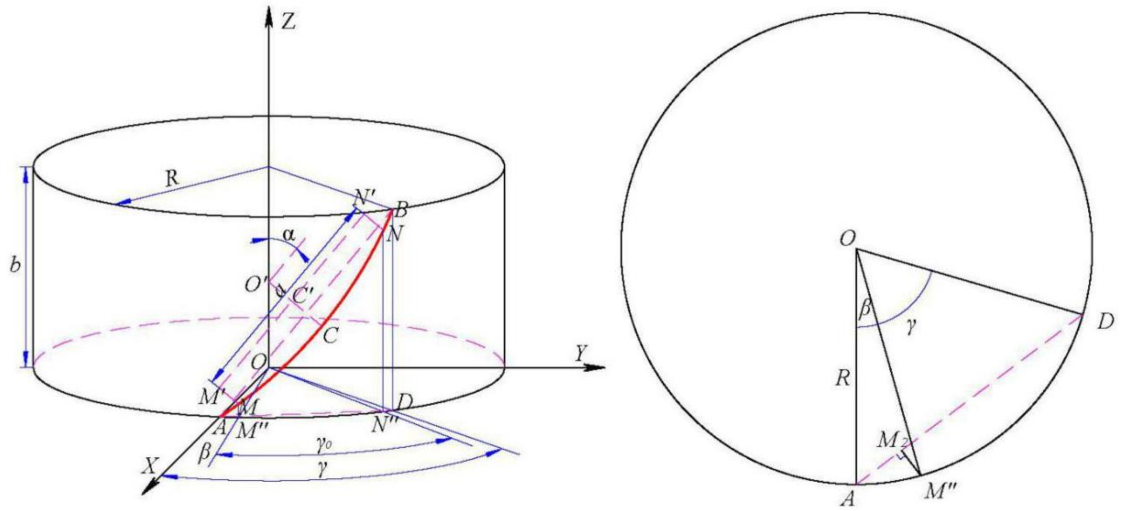


Fig.5 Anatomy of the Mecanum wheel roller

Calculation of Roller Coincidence:

$$\varepsilon = \frac{N(\gamma - 2\beta)R}{2\pi R} = \frac{N}{2\pi}(\gamma - 2\beta) = 1.014 \quad (4)$$

In the formula: N ——the number of rollers;

ε ——the degree of coincidence of the Mecanum wheel.

If ε <1, the continuity between the rollers is not good, there will be greater vibration during the wheel movement; if ε =1, only one roller is in contact with the ground; when ε >1, there will be at least two rollers contact with the ground at the same time, in the rolling process. When the rollers are in contact with the work surface at the same time,and the continuity is ideal, but it will interference occurs between the rollers when the coincidence is too large, and affecting the normal rotation of the entire wheel. The most reasonable degree of coincidence is in the range of 1.01 to 1.03[3]. The coincidence degree of the Mecanum wheel in this paper is 1.014, which is within the range of the value, so it meets the requirements.

2.3 3D Modeling

Roller solid modeling was established based on the curve of the roller, and the three-dimensional solid model of the Mecanum wheel was obtained by evenly arranging eight rollers along the hub with an angle of 45°.As shown in Fig.6 , it can be seen that the envelope formed by the eight rollers is a cylindrical surface. The Mecanum wheel can continuously scroll forward.

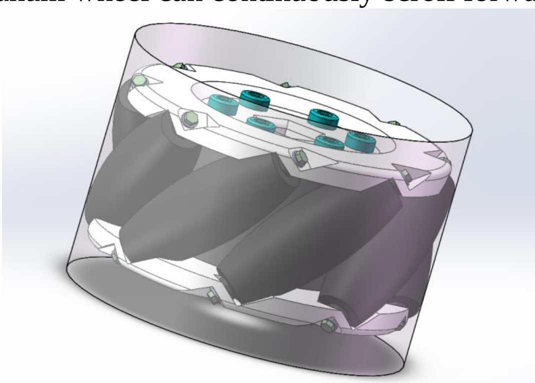


Fig.6 Three-dimensional model of Mecanum wheel

3. Force Analysis and Typical Movement of Mecanum Wheel

The most typical omnidirectional mobile platform is a four-wheel system based on the Mecanum wheel. To achieve omnidirectional motion, there are also requirements for the assembly of four Mecanum wheels. The wheel assembly structure is shown in Fig.7[4].

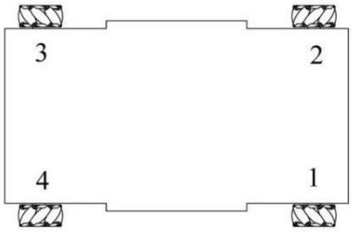


Fig.7 The assembly structure of Mecanum wheel

Each Mecanum wheel has 3 degrees of freedom: rotation about the axle, rotation about the roller axis, and rotation about the contact point between the wheel and the ground. Each Mecanum wheel is independently driven by a motor to rotate around the axle, and the remaining 2 degrees of freedom follow[5]. The wheel is forced to rotate, and the roller in contact with the ground can decompose the force into two directions along the axis of the wheel and the radial direction of the wheel due to the angle between the roller and the axle.Through the vector sum of the two directional components, the omnidirectional motion of the platform is finally achieved[6]. The omnidirectional movement principle of the four-wheel system based on the Mecanum wheel is shown in Fig. 8.

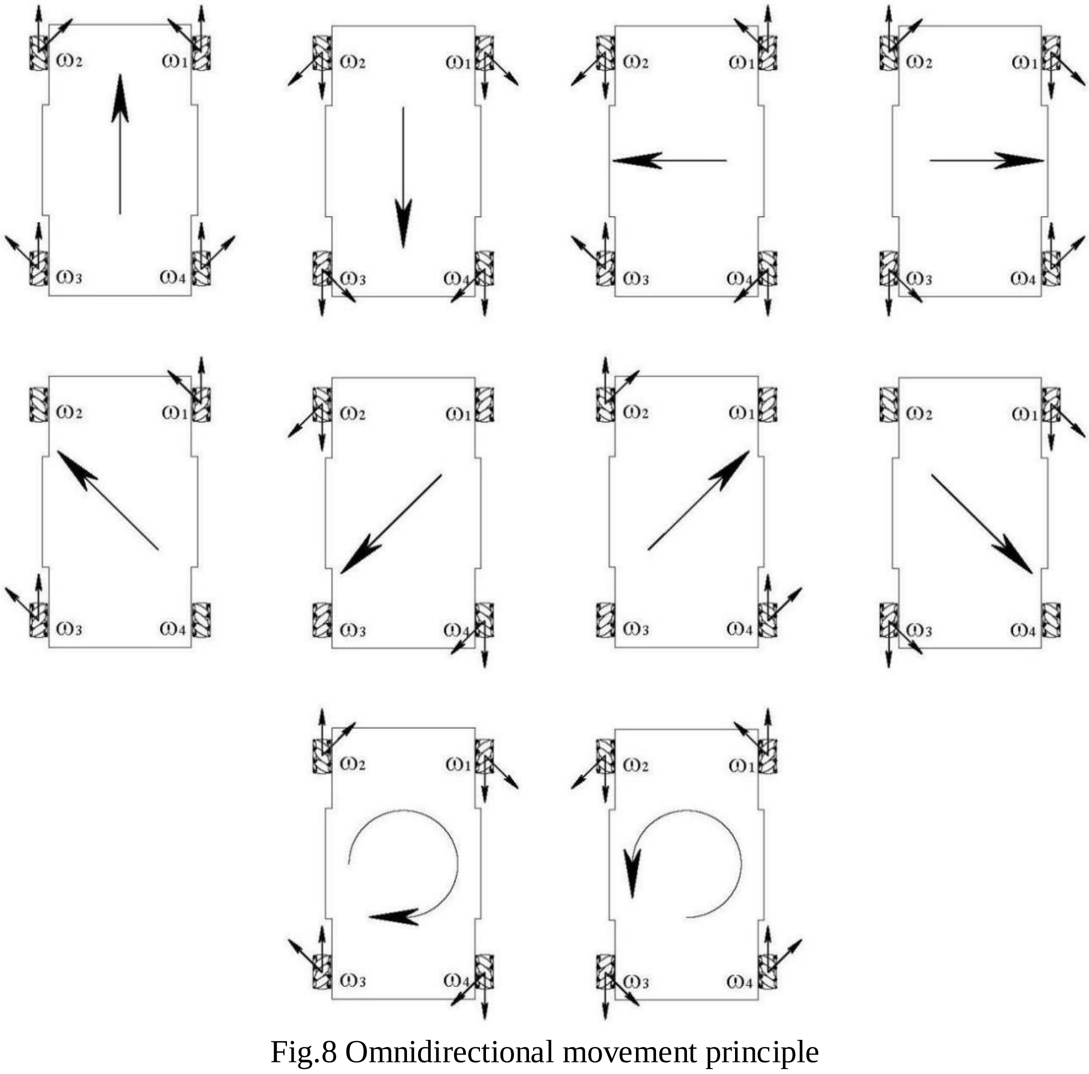


Fig.8 Omnidirectional movement principle

4. Kinematic Analysis of Mecanum Wheel Four-Wheel System

Fig.9 is a schematic diagram of a coordinate relationship between the center of the mobile platform and the center of the i-th Mecanum wheel. O-YX is a coordinate system fixed on the mobile platform, and $o_i - xy$ is a coordinate system fixed at the center of the i-th Mecanum wheel. V_x, V_y and ω are the generalized velocities of the center of the mobile platform in the O-YX coordinate system, V_{xi}', V_{yi}' and ω_i' represent the generalized velocities of the center of the wheel in the coordinate system $o_i - xy$, and V_{xi}, V_{yi} and ω_i indicate the generalized velocity of the wheel center in the O-YX coordinate system. V is the offset angle of the i-th Mecanum wheel relative to the wheel; N is the angle between the direction of the i-th Mecanum wheel and the direction of the moving platform; M is the angle between the line connecting the center of the Mecanum wheel with the center of the mobile platform and the movement direction of mobile platform.

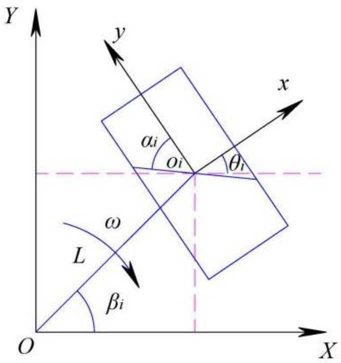


Fig.9 Schematic diagram of a coordinate relationship

According to the established coordinate system, the relational expression between each quantity can be derived:

$$\begin{aligned}
&V_{xi}' = V_i \cdot \sin\alpha_i \\
&V_{xi}' = V_i \cdot \cos\omega_i + \omega_i \cdot r \\
&V_{xi} = V_{xi}' \cdot \cos\theta_i - V_{yi}' \cdot \sin\theta_i \\
&V_{yi} = V_{xi}' \cdot \sin\theta_i - V_{yi}' \cdot \cos\theta_i \\
&V_{xi} = V_x - L_{yi} \cdot \omega \\
&V_{yi} = V_y - L_{yi} \cdot \omega
\end{aligned} \tag{5}$$

Thus:

$$\begin{bmatrix} V_{xi} \\ V_{yi} \end{bmatrix} = R_{i2} \times R_{i1} \times \begin{bmatrix} \omega_i \\ V_i \end{bmatrix} = R_{i3} \times \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} \tag{6}$$

In the formula: $R_{i1} = \begin{bmatrix} 0 & \sin\alpha_i \\ r_i & \cos\alpha_i \end{bmatrix}$; $R_{i2} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$; $R_{i3} = \begin{bmatrix} 1 & 0 & -L_{yi} \\ 0 & 1 & L_{xi} \end{bmatrix}$

Get:

$$R_{i3} \times R_{i1}^{-1} \times R_{i2}^{-1} = \frac{1}{-r_i \cdot \sin\alpha_i} \times \begin{bmatrix} \cos(\alpha_i - \theta_i) & \sin(\theta_i - \alpha_i) & -L_{yi} \cdot \cos(\alpha_i - \theta_i) + L_{xi} \cdot \cos(\theta_i - \alpha_i) \\ -r \cdot \cos\theta_i & -r \cdot \sin\theta_i & r_i \cdot L_{yi} \cdot \cos\theta_i - r_i \cdot L_{xi} \cdot \sin\theta_i \end{bmatrix} \tag{7}$$

Among them, L_{xi} , L_{yi} represents the position and posture of the wheel, according to the formula: $L_{xi} = L_i \times \cos\beta_i; L_{yi} = L_i \times \sin\beta_i$

The speed expression for any wheel can be:

$$\begin{bmatrix} \omega_i \\ V_i \end{bmatrix} = \frac{1}{-r_i \cdot \sin\alpha_i} \times \begin{bmatrix} \cos(\alpha_i - \theta_i) & \sin(\theta_i - \alpha_i) & -L_{yi} \cdot \cos(\alpha_i - \theta_i) + L_{xi} \cdot \cos(\theta_i - \alpha_i) \\ -r \cdot \cos\theta_i & -r \cdot \sin\theta_i & r_i \cdot L_{yi} \cdot \cos\theta_i - r_i \cdot L_{xi} \cdot \sin\theta_i \end{bmatrix} \times \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} \quad (8)$$

It can be inferred that the overall movement formula of the four-wheeled mobile platform is:

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = J \times \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} \quad (9)$$

In equation (9), J is a four-wheel system kinematics Jacobian matrix:

$$J = -\frac{1}{r} \begin{bmatrix} \frac{\cos(\alpha_1 - \theta_1)}{\sin\alpha_1} & \frac{\sin(\theta_1 - \alpha_1)}{\sin\alpha_1} & \frac{L_1 \cdot \sin(\theta_1 - \alpha_1 - \beta_1)}{\sin\alpha_1} \\ \frac{\cos(\alpha_2 - \theta_2)}{\sin\alpha_2} & \frac{\sin(\theta_2 - \alpha_2)}{\sin\alpha_2} & \frac{L_2 \cdot \sin(\theta_2 - \alpha_2 - \beta_2)}{\sin\alpha_2} \\ \frac{\cos(\alpha_3 - \theta_3)}{\sin\alpha_3} & \frac{\sin(\theta_3 - \alpha_3)}{\sin\alpha_3} & \frac{L_3 \cdot \sin(\theta_3 - \alpha_3 - \beta_3)}{\sin\alpha_3} \\ \frac{\cos(\alpha_4 - \theta_4)}{\sin\alpha_4} & \frac{\sin(\theta_4 - \alpha_4)}{\sin\alpha_4} & \frac{L_4 \cdot \sin(\theta_4 - \alpha_4 - \beta_4)}{\sin\alpha_4} \end{bmatrix} \quad (10)$$

From equation (9), we can see that when given the moving speed of the mobile platform, we can get the speed and direction of the four wheels. When a = 0, let b and c be the lateral distance and longitudinal distance from the center of the vehicle body to the center of the wheel respectively. Then the overall kinematics matrix of the four-wheel system is:

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -\frac{1}{\tan\alpha} & 1 & \frac{-(l_1 \cdot \tan\alpha + l_2)}{\tan\alpha} \\ \frac{1}{\tan\alpha} & 1 & \frac{l_1 \cdot \tan\alpha + l_2}{\tan\alpha} \\ -\frac{1}{\tan\alpha} & 1 & \frac{l_1 \cdot \tan\alpha + l_2}{\tan\alpha} \\ \frac{1}{\tan\alpha} & 1 & \frac{-(l_1 \cdot \tan\alpha + l_2)}{\tan\alpha} \end{bmatrix} \times \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} \quad (11)$$

To achieve omnidirectional motion, the kinematics Jacobi matrix is required to rank full, thus:

$$\frac{l_1 \cdot \tan\alpha + l_2}{r \cdot \tan\alpha} \neq 0 \Rightarrow \tan\alpha \neq -\frac{l_1}{l_2} \quad (12)$$

Take $\alpha = \frac{\pi}{4}$, when $V_x \neq 0, V_y = 0$ and $\omega = 0$, the mobile platform will move straight along the X-axis direction, the speed of the four Mecanum wheels are:

$$W_1 = W_3 = -\frac{1}{r} V_x \qquad W_2 = W_4 = \frac{1}{r} V_x \quad (13)$$

When $V_x = 0, V_y \neq 0$ and $\omega = 0$, the mobile platform will move straight along the Y-axis direction, the speed of the four Mecanum wheels are:

$$W_1 = W_2 = W_3 = W_4 = \frac{1}{r} V_y \tag{14}$$

When $V_x = 0, V_y = 0$ and $\omega \neq 0$, the mobile platform will rotate around its center point O. The speeds of the four Mecanum wheels are:

$$\begin{aligned} W_1 = W_4 &= -\frac{l_1 \cdot \tan\alpha + l_2}{r \tan\alpha}\omega = -\frac{l_1 + l_2}{r}\omega \\ W_2 = W_3 &= \frac{l_1 \cdot \tan\alpha + l_2}{r \tan\alpha}\omega = \frac{l_1 + l_2}{r}\omega \end{aligned} \tag{15}$$

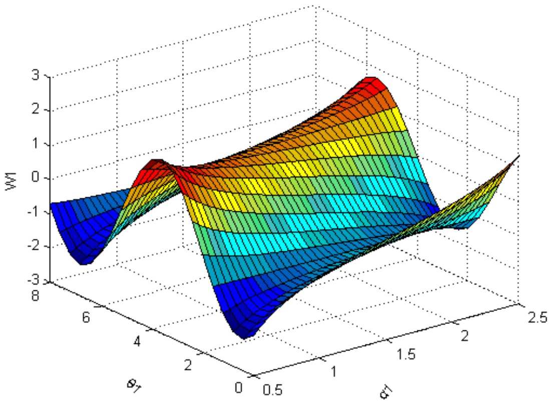
When $V_x = V\cos\theta, V_y = V\sin\theta$ and $\omega = 0$, the linear motion of the moving platform at a velocity *V*, and any angle θ will be achieved. At this time, the speeds of the four Mecanum wheels are:

$$\begin{aligned} W_1 = W_3 &= \frac{-V\cos\theta + V\sin\theta}{r} \\ W_2 = W_4 &= \frac{V\cos\theta + V\sin\theta}{r} \end{aligned} \tag{16}$$

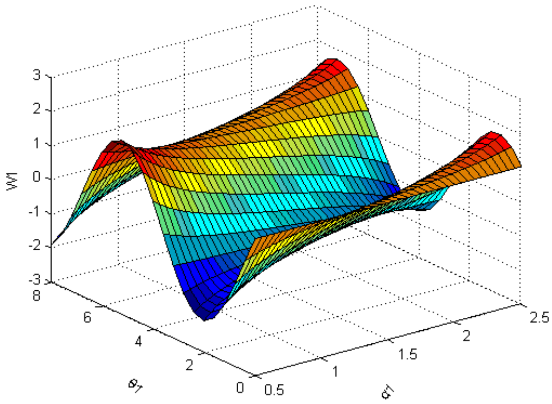
Therefore, when the motion of the four-wheel system is known, the speeds of the four Mecanum wheels required will be correspondingly obtained. From the kinematics matrix of the four-wheel system, it can be seen that the speed expressions of the four Mecanum wheels are the same, so the analysis method for the speed of each wheel is the same. Here we take the first wheel as an example to analyze α, β, θ, r Impact on mobile platform speed. From equation (9), the speed expression of wheel 1 is:

$$W_1 = \frac{\cos(\alpha_1 - \theta_1)}{-r_1 \cdot \sin\alpha_1} \cdot V_x + \frac{\sin(\theta_1 - \alpha_1)}{-r_1 \cdot \sin\alpha_1} \cdot V_y + \frac{L_1 \cdot \sin(\theta_1 - \alpha_1 - \beta_1)}{-r_1 \cdot \sin\alpha_1} \cdot \omega \tag{17}$$

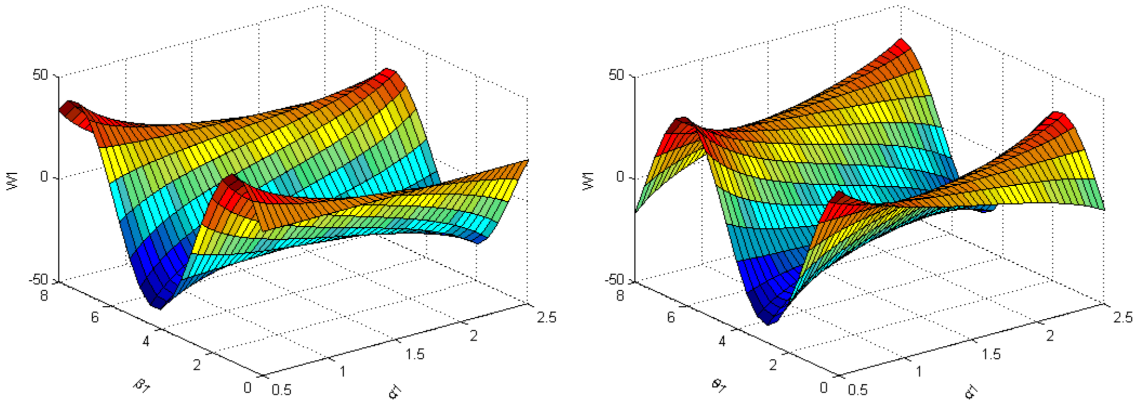
Fig. 10(a) shows the influence of α_1 and θ_1 on W_1 when the moving platform moves in the X-axis direction. Fig. 10(b) shows the effect of α_1 and θ_1 on W_1 when the moving platform moves in the Y-axis direction. Fig. 10(c)) shows the effect of α_1 and β_1 on W_1 when the mobile platform rotates around its center O, Fig. 10(d) is the effect of α_1 and θ_1 on W_1 when the mobile platform rotates around its center point O.



(a)



(b)



(c) (d)

Fig.10 The effect of parameters

5. Summary

In this paper, an elliptic arc approximation method is proposed to establish the roller curve equation, and the roller curve is fitted by using the method of interval picking. Taking the roller length a=80mm as an example, the Mecanum wheel is accurately modeled. Then the kinematics analysis of the four-wheel system based on Mecanum wheel was performed to study the relationship between the speed of the mobile platform and each Mecanum wheel, the effect of each parameter change on the Mecanum wheel speed was also analyzed. These provide a basis for further research on the kinematics of omnidirectional mobile platforms.

References

[1] Yanjie Zhu: *Research on Design and Performance of Mecanum Wheel Omnidirectional Vehicle*(MS. ,Yanshan University,China 2006),p22.

[2] Ming Yang,Yongjian Lv， Xinyong Cong: Mecanum Wheel Geometry Structure and Roller Generating Method[J],Mechanical Design and Research,Vol. 30 (2014),No.5, p.76-80.

[3] Xueling Zhang,Caihong Shi and Shaohua Kang: Mecanum Wheel Roller Outer Contour Surface Deformation Compensation Design[J], Chinese Journal of Construction Machinery, Vol.12 (2014) No.3, p.213-217.

[4] Diegel O, Badve A and Bright G: Improved Mecanum Wheel Design for Omni-directional Robots[C],Australasian Conferenceon on Robotiesand and Automation, Vol. 27 (2002) No.2, p.117-119.

[5] Muir P E and Neuman C P; Kinematic modeling for feedback control of an omnidirectional wheeled mobile robot [C], Proceedings of the 1987 IEEE International Conference On Robotics and Automation,Vol. 4 (1987) No.4, p.25-31.

[6] Zhongxin Liu,Chunliang Zhang and Zhanfu Wang:All-around Mobile Platform Motion Analysis and Simulation,Design and Research,Vol. 24 (2013) No.8, p.16-19.