A Create Design of Tollbooth

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Abstract

According to the shape, size and merging pattern of toll plaza, we established an improved Re-Vascular-Branche Model and BPR model based on the dynamics principles. After that, we presented a design of "curved" toll station and gave the comparative results of the performance of "curved" scheme and the traditional "linear" one by MATLAB and Cellular Automaton. Besides, we analyzed our design with the influence of different traffic flow and combinations by multi-purpose programming, and gave the proportions of three types of toll stations under different situations by Lingo. Finally, the model was properly evaluated and an improved scheme was given. Firstly, we use the dynamics formulas to analyze the running process of vehicles between the toll plaza and the carriageways in detail. After upgrading and improving the Vascular-Branche Model， we established the "Re-Vascular-Branche Model". Combing with the traffic dynamics model and the improved function of BPR, we optimized the merging pattern between the toll station and carriageways . On this basis, we put forward a design of "curved" toll station, and analyzed its performance and feasibility. Secondly, as for these two plans, "curved" toll station and traditional "linear" toll station, we use MATLAB and Cellular Automaton to simulate the operation process of these two different types of stations. With the collected data, we analyzed and compared these two plans in three different aspects. The reaults showed that "curved" toll station is better than traditional "linear" one, no matter in accident prevention, throughput or cost.(The results is shown in figure 8 and figure 12). Then, in view of the influence of the "curved" tollbooths which traffic volume and combination mode put on, we have set up MOP models for the analysis of various traffic conditions, such as the joining of the self-driving cars and the proportion of the three different types of tollbooths. Taking advantage of the Lingo, we finally made the conclusion that the proportion of the human-staffed tollbooths, exact-change tollbooths and ETC has changed from 2:1:1 to 1:1:2 after the joining of self-driving cars. Finally, we made the sensitivity analysis about the models and put forward the further improvement. After that, we writed a letter to New Jersey Turnpike Authority in which our design was included.

Keywords

Re-Vascular-Branche Model, BPR model, MOP, Cellular Automaton.

1. Introduction

1.1 Background

Currently, the increase of road traffic results in the loss of country and people for the highway construction lagging development, which also leads to the traffic accident happened frequently and pressure rise sharply. In some major cities around the word, congested traffic lead to speed reduction on the highway in the rush hour, which caused a long time delay. Traffic problems have been a prominent problem all over the word.

Barrier toll is the main reason for traffic jams in the tollbooths. A barrier toll is the most common way of charging in highway. It is a row of tollbooths placed across the highway, perpendicular to the direction of traffic flow. As there are usually more tollbooths than there are incoming lanes of traffic, vehicles must "fan in" from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier (Figure 1[1]).L

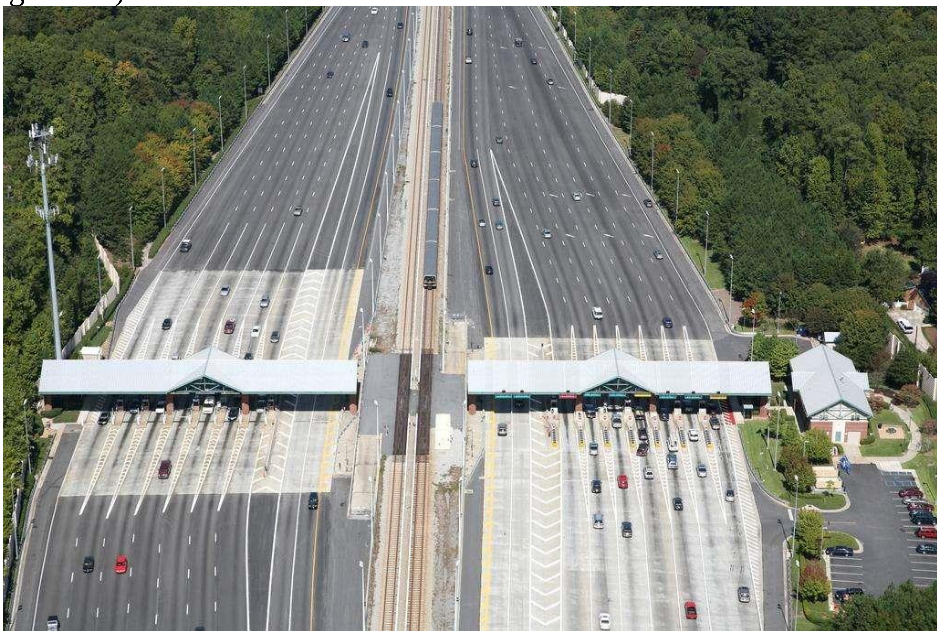

Fig.1 The General Shape of Tollbooths

1.2 Restatement of the Problem

The main point is to determine the best merging pattern when vehicles "fan in" from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. The problem aims at designing the shape, size, and merging pattern of the toll plaza based on the consideration of throughput, cost, and accident prevention. And then determine the proportions of human-staffed tollbooths, automated tollbooths, and electronic toll collection booths in different traffic flows and traffic compositions.

We are required to solve the following problems:

● Establish a model to analyze the shape, size, and merging pattern of the toll plaza following the toll barrier. There are three factors which should be taken into consideration:

1. The accident prevention;

2. Throughput;

3. Cost.

● Determine the performance of our solution in different cases:

1. In light and heavy traffic;

2. More autonomous (self-driving) vehicles added to the traffic mix;

3. The proportions of conventional tollbooths, exact-change tollbooths, and electronic toll collection booths changed.

1.3 Our work

The overview of our model is shown in figure 2.

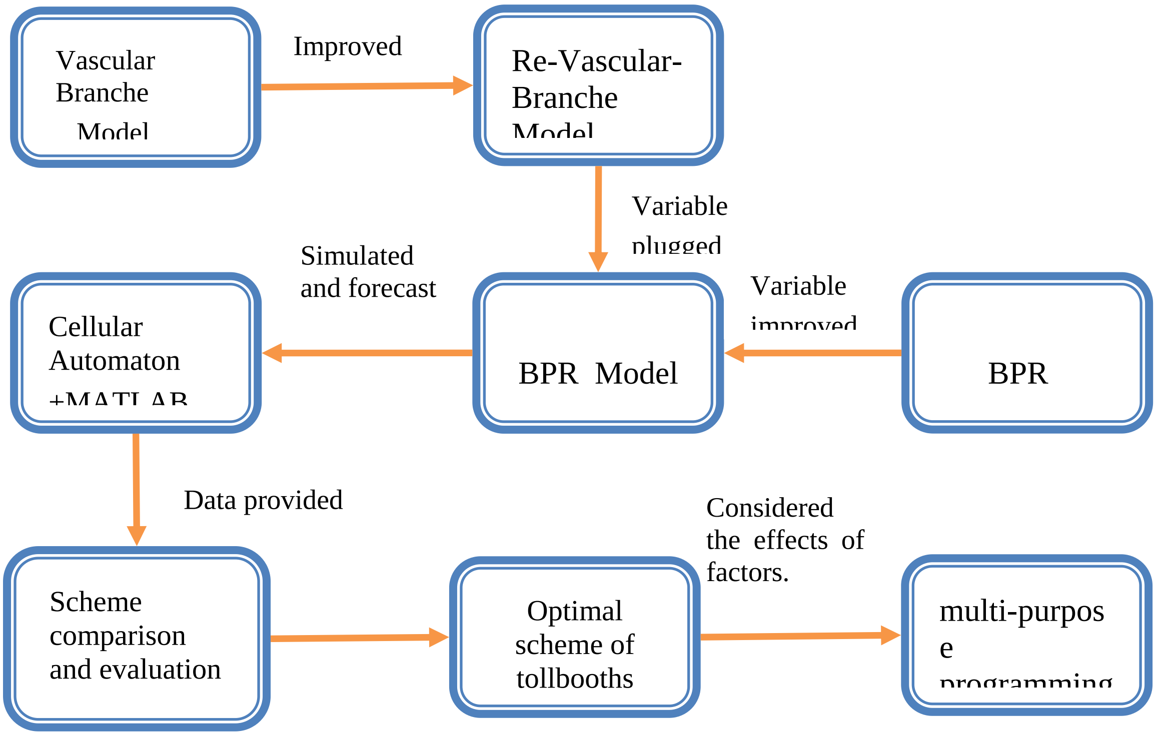

Fig.2 The General Shape of Tollbooths

In order to solve the problems above, we will proceed as follows:

- Stating assumptions. We will make some reasonable assumptions through analyzing the problems and consulting a lot of literature. This step aims at narrowing the focus of our approach towards the problems.
- Explaining terminologies. We will give a detailed explaination about the terms and give some notations which are important for us to clarify our models.
- Presenting our models. In order to solve the problems above, we will establish improved Re-Vascular-Branche Model and BPR model, present a design scheme of "curved" toll station, gave the comparative results of the performance in accident prevention, throughput, and cost of "curved" scheme and traditional "linear" scheme by using MATLAB and Cellular Automaton to model and simulate.
- Adding variables to our models. In this part, we will consider the performance of our design under the influence of accident prevention, throughput, and cost.
- Model testing and sensitivity analysis. Sensitivity analysis is a effective method that can determine which uncertainties to a value model will produce the greatest effects. With some criteria defined, we will evaluate the reliability of our models and do the sensitivity analysis.
- Evaluating the model. In this part, we will provide an honest, objective evaluation about the strengths and weaknesses of our models.
- Further discussion. Based on the weaknesses, we will discuss about different ways to strengthen our models. What's more, we will put forward the future direction of improvement and the ways to apply them in reality.

2. Assumptions and Justification

In order to simplify the problem and make it convenient for us to establish models, we make the following basic assumptions, each of which is properly justified.

- The nature of all the vehicles are same and they receive the same services from the toll stations.
- The number of tollbooths which provide services to vehicles are same everyday.

- The vehicles and roads are ideal. That is to say, all the vehicles have the same length and the road is flat. Besides, in the driving of vehicles, there does not exist any rolls, yaws and other sports.
- Ignore the different types of vehicles. That is to say, all cars obey the same standard..
- There is no queue at the start of the analysis period.
- Ignoring the acceleration and deceleration behavior of the vehicles in the change process of lane. If the vehicles meet the change conditions, they will give a direct conversion to the adjacent lane.

3. Terminology

3.1 Terms

- Throughput: Different fields have different definition on throughput. In this paper, throughput means the number of vehicles per hour passing the point where the end of plaza joins the outgoing traffic lanes.
- ETC: The full name of ETC is Electronic Toll Collection, also named by automated tollbooths. It is the most advanced collection system in the world. This charging system allows the car to go through the toll station without parking to pay. ETC of highway is being internationally popular.
- Traffic capacity[5]: It is the maximum sustainable flow rate at which vehicles or persons reasonably can be expected to traverse a point or uniform segment of a lane or roadway during aspecified time period under given roadway, geometric, traffic, control conditions and environmental; usually expressed as vehicles per hour, passenger cars per hour, or persons per hour[2].

3.2 Notations

Table 1 Notation

Symbols	Definition	Unit
B	the total number of tollbooths	pc
L	the total number of carriageways	pc
K	the number of ETC	pc
D	the distance from the tollbooths to merge area	m
C	the vertical length of merge area	m
θ	the angle between the side lines of merge area and the driveway	degree

Note: Other symbols will be given detailed explainations in the following text.

4. Basic Model

The construction of tollbooths is linked closely to the factors such as accident prevention, throughput, and cost. In order to improve the design of the tollbooth's shape, size, and merging pattern, we need to establish a model to analyze and evaluate the tollbooths with a consideration of the factors above.

4.1 Improved Re-Vascular-Branche Model

Vascular branche model[3] is about the minimal energy of blood flowing from a single vessel to multiple vessels. The merging pattern of toll plaza can be regard as the reverse process of Vascular branche model, and we name it a *Re-Vascular-Branche Model*.

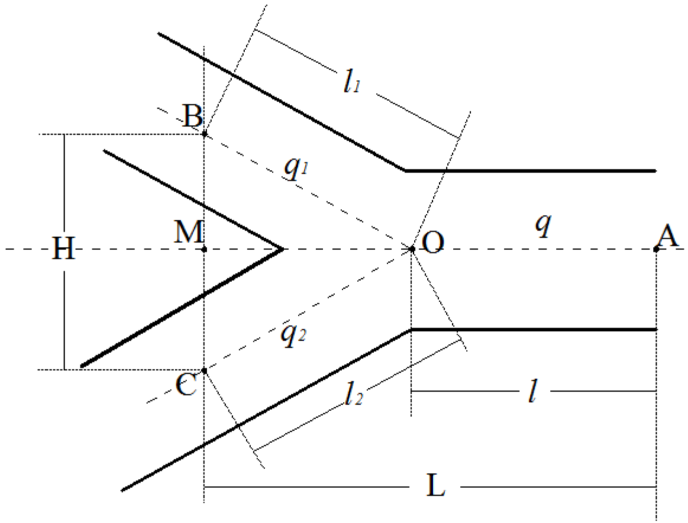

Fig.3 The sketch Diagram of Re-Vascular-Branche Model

The sketch map of Re-Vascular-Branche Model is shown in figure 2. We set time as an index, evaluating the impact of merge mode on traffic flow by calculating the time spent per unit of traffic. The process is as follows:

$$\frac{H}{tan\theta_1+tan\theta_2}+l=v_m t+\frac{1}{2}at^2 \quad (4\text{-}1)$$

$$\overset{solve}{\Longrightarrow} t=\frac{-v_m+\sqrt{v_m^2+2a(\frac{H}{tan\theta_1+tan\theta_2}+l)}}{a} \quad (4\text{-}2)$$

Therefore , the average time per unit distance is :

$$t_0=\frac{t}{l+\frac{H}{tan\theta_1+tan\theta_2}} \quad (4\text{-}3)$$

Where :

O is the point of intersection of the two roads;

H is the width of the toll station;

v_m is the speed at point M;

L is the length from point A to point M;

q is the traffic volume.

4.2 Improved BPR Model

The BPR function[19] is the key to master the change law of road impedance in urban traffic assignment process. At present, there are many research models on road impedance, among which, the road function model of Federal Highway Bureau (BPR) is of great representative. The mathematical formula of BPR is:

$$T_{(q)}=T_0[1+\alpha(\frac{q}{c})^{\beta}] \quad (4\text{-}4)$$

Where :

$T_{(q)}$ is the time travel time per unit road when the traffic volume is q;

T_0 is the time travel time per unit road when the traffic volume is 0;

c is the capacity of the road;

α and β are the parameter of the model.(Usually, α is valued by 0.15 and β by 4)

q is the traffic volume.

Taken the formula (4-3) into (4-4), then we get the formula (4-5)

$$T(\theta_1,\theta_2,q) = \frac{t}{l + \frac{H}{tan\theta_1 + tan\theta_2}}\left[1 + \alpha\left(\frac{q}{c}\right)^{\beta}\right] \qquad (4\text{-}5)$$

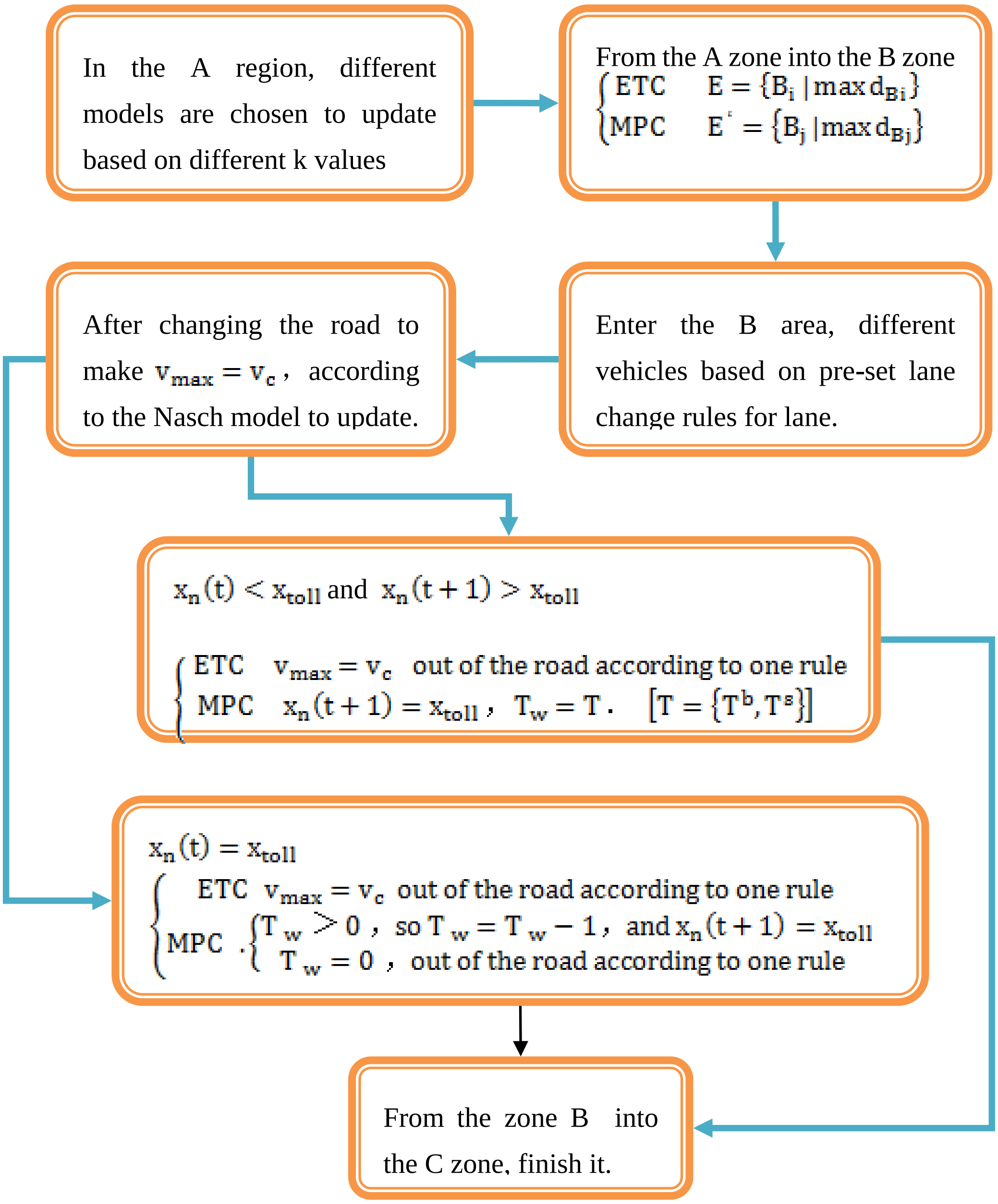

Fig.4 The Flowchart of Cellular Automation

4.3 The Simulation of Cellular Automation.

Cellular Automaton is a time and space are discrete power system. Each cell distributed in the lattic e grid takes a finite discrete state, following the same rules of action, and constitutes the evolution o f the system through a simple interaction. We use the cellular automata to simulate the shape, size a nd merge mode of "straight line" and "arc" toll station, and evaluate the accident prevention, throug hput, and cost according to the simulation results.

4.3.1 local assumption

●There are three kinds of cells in the toll plaza: the vehicle cell, the control cell, the forbidden cell.

●A cell represents a physical space that can hold a standard vehicle and a buffer area

the vehicle.

●All cars have the same size.

4.3.2 The Rules of Cellular Automation

According to the process of vehicle flow, cellular automata simulation run as follows:

4.3.3 Simulation Diagram

The simulation diagrams of the two types is shown in figure 4 and figure 5:

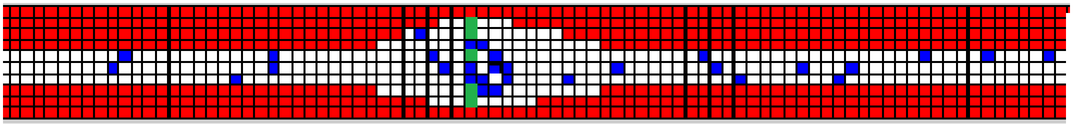

Fig.5 The Simulation Diagram of “linear” Tollbooths

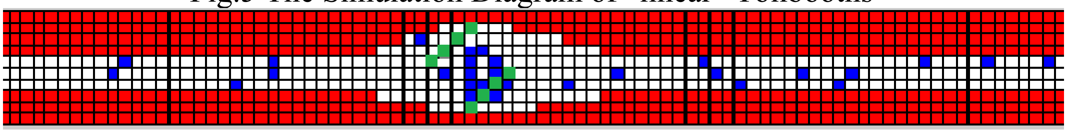

Fig.6 The Simulation Diagram of “Curved” Tollbooths

In the pictures above, the green cells represent the tollbooths, the white cells represent the roadways, and the blue ones represent the vehicles.

It can be seen from the two figures above, the improved toll station was significantly better than the toll station before the improvement.

4.4 The Design of “Curved” Tollbooths and Its Evaluation

4.4.1 The Design of “Curved” Tollbooths

The sketch map of tollbooths currently is shown in figure 2 (The number of the lanes and tollbooths in figure 3 have no actual meaning.) Highway congestion mainly occurs in the toll station, and the main influencing factor is the merging pattern that passengers access to less merger roadway from multi-channel tollbooths. According to our analysis, vehicles depart from every Toll Gate at almost every moment when traffic flow is big. Due to the short time lag of the vehicles depart from different tollbooths, the probability of traffic jams occur at the entrances of the roadway increases.Based on the analysis above, we consider the scheme of adding time lag of the vehicles from different charging channels to realize the purpose of easing traffic pressure.

In order to increase the time lag between vehicles, we change the traditional pattern of "linear" tollbooths design and changed it into innovative "arc" mode,which has certain vertical distance between each Tollbooths, creating time lag of the vehicles reaches the tollbooths through this way , and the time lag of vehicles access to the entrance of roadway after leaving the tollbooths increases accordingly.

The shape of “curved” tollbooth we designed is shown in figure 4.

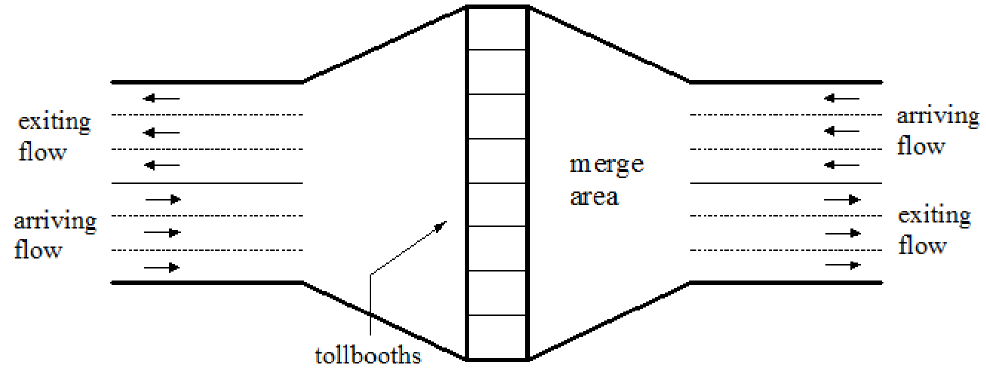

Fig.7 The shape of tollbooths currently

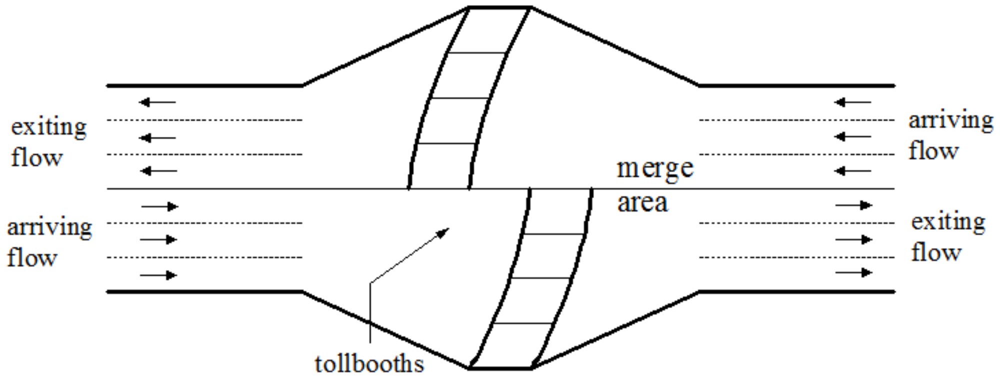

Fig.8 The shape of “curved” tollbooth

4.4.2 The Analysis and Comparison of the Two Schemes

According to the formulas in 4.2， we simulated the two designs with MATLAB, and obtained the average travel time of the two schemes under a series of different traffic volumes, as is shown in figure 9:

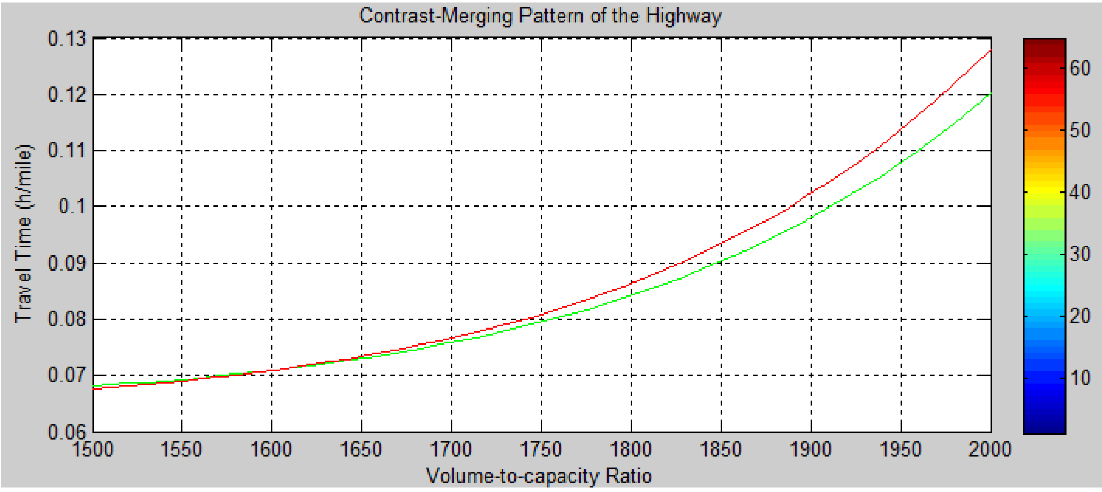

Fig.9 Contrast-Merging Pattern of the Highway

The picture above shows: with the increase in traffic flow, the average travel time of the "curved" toll station scheme was significantly lower than the average travel time of "straight-line" toll stations under the same conditions. This shows that the "curved" toll booths has a significant effect on reducing traffic congestion and easing the expressway traffic pressure at the entrance .

5. The Evaluations of the Designs Based on Simulated Data

5.1 The Evaluations of Accident Prevention

5.1.1 Local Assumption

- We assump that all the vehicles go straight the way if there is a carriageway in front.
- There are vehicles driving into the merge area from every tollbooths at every time.
- There only exist ETCs and human-staffed tollbooths.

5.1.2 "Linear" Tollbooths

The emphasis of this project is to determine the shape, size, and merging pattern of the toll plaza in which vehicles must fan in from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. For that reason, we simplify the process and make mathematical definitions of relevant data.(As shown in figure 3)

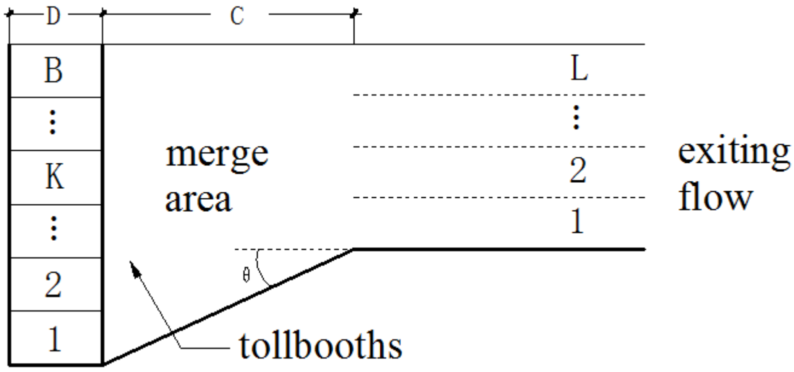

Fig.10 The Sketch Diagram of "Linear" Tollbooths

In the picture above:

B is the total number of tollbooths;

L is the total number of carriageways;

K is the number of ETC;

D is the distance from the vehicles left the tollbooths to enter the merge area.

C is the vertical length of merge area;

θ is the angle between the side lines of merge area and the carriageway.

A lot of data have shown that ETC usually sets in the middle of the toll station within the current design models, and the conventional tollbooths sets on both sides of the toll station. There is a short period between the vehicles leaving the tollbooths and arriving at the entrance of carriageways from different tollbooths. If the period is too short, it will cause congestion at peak times. Based on the analysis above, we set the time between the vehicles leaving the tollbooths and arriving at the entrance of carriageways as a index for evaluation. For different K and L, there are three different cases. We will make a detailed analysis in the following sections.

(1) K=L

K=L, that is to say, the number of ETC is equal to roadways, and the number of human-staffed tollbooths is (B-K). According to the first assumption above, the vehicles which leaving from the ETC going straight to the roadways, while vehicles which leaving from the (B-K) human-staffed tollbooths will first pass through a curve before entering the carriageway. We set the initial speed of the vehicles which leaving from the ETC is v_0; the initial speed of the vehicles which leaving from the human-staffed tollbooths is 0; the acceleration is a. According to the dynamic formulas[12]:

$$D + C = v_0 t_1 + \frac{1}{2} a t_1^2 \overset{\text{solve}}{\Longrightarrow} t_1 = -2v_0 + 2\sqrt{v_0^2 + 2a(D+C)} \qquad (5\text{-}1)$$

$$D + \frac{C}{cos\theta} = \frac{1}{2} a t_2^2 \overset{\text{solve}}{\Longrightarrow} t_2 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} \qquad (5\text{-}2)$$

Then :

$$\Delta t_1 = t_2 - t_1 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} + 2v_0 - 2\sqrt{v_0^2 + 2a(D+C)} \qquad (5\text{-}3)$$

Where：

v_0 is the initial speed of the vehicles which leaving from the ETC;

a is the acceleration of all vehicles;

t_1 is the time between the vehicles leaving from the ETC and going straight to the entrence of driveways;

t_2 is the time between a vehicle exiting from the human-staffed tollbooths and traveling through the curve to the entrance of roadway.

Δt_1 is the difference between t_1 and t_2

(2) K<L

When K<L, all vehicles leaving from the ETC going straight into the driveway, and the vehicles leaving from the (L-K) human-staffed tollbooths going straight into the driveway as well. The remaining (B-L) vehicles entering the roadway from the curve. According to the dynamic formulas:

$$D + C = v_0 t_3 + \frac{1}{2} a t_3^2 \overset{\text{solve}}{\Longrightarrow} t_3 = -2v_0 + 2\sqrt{v_0^2 + 2a(D+C)} \qquad (5\text{-}4)$$

$$D + C = \frac{1}{2} a t_4^2 \overset{solve}{\Longrightarrow} t_4 = \sqrt{\frac{2(D+C)}{a}} \qquad (5\text{-}5)$$

$$D + \frac{C}{cos\theta} = \frac{1}{2} a t_5^2 \overset{\text{solve}}{\Longrightarrow} t_5 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} \qquad (5\text{-}6)$$

Then :

$$\Delta t_2 = t_4 - t_3 = \sqrt{\frac{2(D+C)}{a}} + 2v_0 - 2\sqrt{v_0^2 + 2a(D+C)} \qquad (5\text{-}7)$$

$$\Delta t_3 = t_5 - t_4 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} - \sqrt{\frac{2(D+C)}{a}} \quad (5\text{-}8)$$

Where：

t_3 is the time between the vehicles leaving from the ETC and going straight to the entrence of driveways;

t_4 is the time between the vehicles leaving from human-staffed tollbooths and going straight to the entrence of driveways;

t_5 is the time between a vehicle exiting from the human-staffed tollbooths and traveling through the curve to the entrance of roadway;

Δt_2 is the difference between t_3 and t_4;

Δt_3 is the difference between t_4 and t_5.

(3) K>L

When K>L, the vehicles leaving from the *L* ETCs going straight into the driveway; the vehicles leaving from the (K-L) ETCs entering the roadway from the curve and the vehicles leaving from the (B-K) human-staffed tollbooths entering the roadway from the curve too. According to the dynamic formulas:

$$D + C = v_0 t_6 + \frac{1}{2} a t_6^2 \overset{solve}{\Longrightarrow} t_6 = -2v_0 + 2\sqrt{v_0^2 + 2a(D+C)} \quad (5\text{-}9)$$

$$D + \frac{C}{cos\theta} = v_0 t_7 + \frac{1}{2} a t_7^2 \overset{solve}{\Longrightarrow} t_7 = -2v_0 + 2\sqrt{v_0^2 + 2a\left(D + \frac{C}{\cos\theta}\right)} \quad (5\text{-}10)$$

$$D + \frac{C}{cos\theta} = \frac{1}{2} a t_8^2 \overset{solve}{\Longrightarrow} t_8 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} \quad (5\text{-}11)$$

Then :

$$\Delta t_4 = t_7 - t_6 = 2\sqrt{v_0^2 + 2a\left(D + \frac{C}{\cos\theta}\right)} - 2\sqrt{v_0^2 + 2a(D+C)} \quad (5\text{-}12)$$

$$\Delta t_5 = t_8 - t_7 = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} + 2v_0 - 2\sqrt{v_0^2 + 2a\left(D + \frac{C}{\cos\theta}\right)} \quad (5\text{-}13)$$

Where：

t_6 is the is the time between the vehicles leaving from the ETCs and going straight to the entrence of driveways;

t_7 is the time between the vehicles leaving from ETCs and going straight to the entrence of driveways;

t_8 is the time between a vehicle exiting from the human-staffed tollbooths and traveling through the curve to the entrance of roadway;

Δt_4 is the difference between t_6 and t_7;

Δt_5 is the difference between t_7 and t_8.

5.1.3 "Curved" Tollbooths

As the situation of the "Curved" tollbooths is more complex, we only discuss about the situation when K<L, and the other cases are similar to this.

We set B=4, K=2, and L=3, and the tollbooths set into a distribution of ladder. The angle is equal to the angle between the side of toll plaza and the driveway. (As is shown in figure)

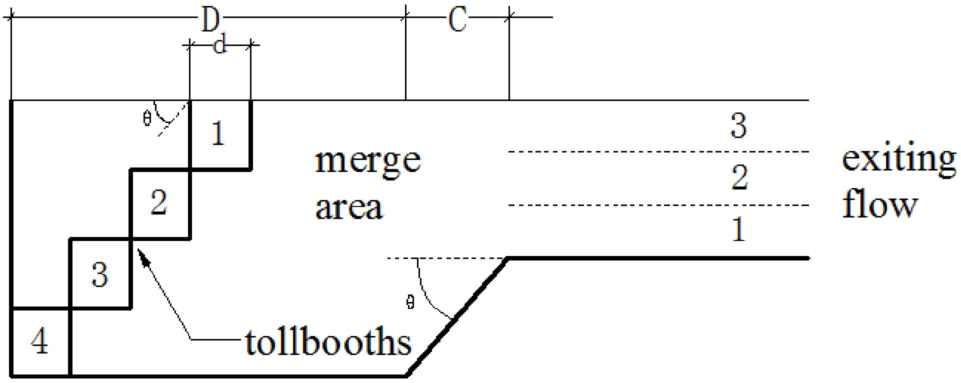

Fig.11 The Sketch Diagram of "Curved" Tollbooths

The derivation process is similar to the one in 6.1.2. According to the dynamic formulas, we get the following formulas:

$$D + C - 3b = v_0 t_9 + \frac{1}{2} a t_9{}^2 \quad (5\text{-}14)$$

$$D + C - 2b = v_0 t_{10} + \frac{1}{2} a t_{10}^2 \quad (5\text{-}15)$$

$$D + C - b = \frac{1}{2} a t_{11}^2 \quad (5\text{-}16)$$

$$D + C - \frac{H cos\theta}{2sin\theta} + \frac{H}{2sin\theta} = \frac{1}{2} a t_{12}^2 \quad (5\text{-}17)$$

Then :

$$\Delta t_6 = t_{10} - t_9 = \frac{\sqrt{v_0^2 - 2a(2b - D - C)} - \sqrt{v_0^2 - 2a(3b - D - C)}}{a} \quad (5\text{-}18)$$

$$\Delta t_7 = t_{11} - t_{10} = \sqrt{\frac{2(D + C - b)}{a}} + \frac{v_0 - \sqrt{v_0^2 - 2a(2b - D - C)}}{a} \quad (5\text{-}19)$$

$$\Delta t_8 = t_{12} - t_{11} = \sqrt{\frac{2(D + C - \frac{H cos\theta}{2sin\theta} + \frac{H}{2sin\theta})}{a}} - \sqrt{\frac{2(D + C - b)}{a}} \quad (5\text{-}20)$$

Where :

t_9 and t_{10} are the times between the vehicles leaving from the ETC and going straight to the entrence of driveways;

t_{11} is the time between the vehicles leaving from human-staffed tollbooths and going straight to the entrence of driveways;

t_{12} is the time between a vehicle exiting from the human-staffed tollbooths and traveling through the curve to the entrance of roadway.

Δt_6 is the difference between t_9 and t_{10}

Δt_7 is the difference between t_{10} and t_{11}

Δt_8 is the difference between t_{11} and t_{12}

When the toll station is “Liner”, we put these values (B=4，K=2， L=3) into the equations and get the following results:

$$\Delta t_1' = 0 \tag{5-21}$$

$$\Delta t_2' = \sqrt{\frac{2}{a}(b + \frac{C}{cos\theta})} + 2v_0 - 2\sqrt{v_0^2 + 2a(D + C)} \tag{5-22}$$

$$\Delta t_3' = \sqrt{\frac{2}{a}(D + \frac{C}{cos\theta})} - \sqrt{\frac{2(D + C)}{a}} \tag{5-23}$$

5.2 The Evaluations of Throughput

5.2.1 The Rules of Evaluating Throughput

Throughput is the number of vehicles per hour passing the point where the end of the plaza joins the outgoing traffic lanes. For the convenient of evaluating, We assume that the vehicle is stationary, and the cross section of the entrance of driveway moving in the direction of the vehicle. We assume that the average length of all the vehicles is *l* and the average space between the vehicles is *hd* (As is shown in figure 8)

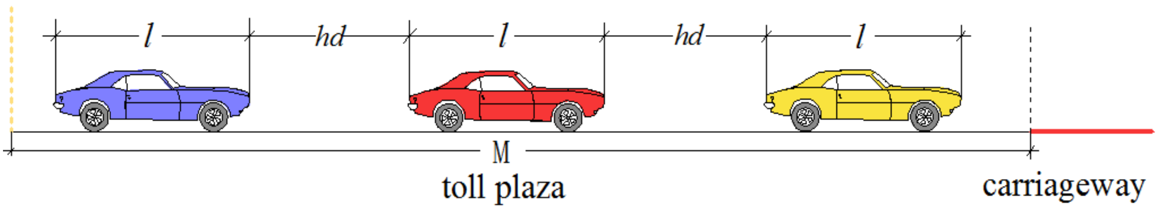

Fig.12 The Sketch Diagram of Evaluating Throughput

According to the dynamic formulas, we get the following formulas:

$$n(hd + l) + M = v_i t + \frac{1}{2}at^2 \tag{5-24}$$

$$\xrightarrow{solve} n = \frac{v_i t + \frac{1}{2}at^2 - M}{hd + l} \qquad (5\text{-}25)$$

Where :

l is the average length of all the vehicles;

hd is the average space between the vehicles;

n is the number of vehicles passing the section per hour.

M is the distance between the tollbooths and the entrance of driveways.

a is the acceleration of vehicles;

v_i is the speed at which the *i*-th car passes the cross section .

n is the number of vehicles passing the cross section per hour;

T is equal to 1 hour.

5.2.1 Comparison of the Two Schemes

According to data from the cellular automaton simulation, We calculated the "linear" and "curved " tolls throughput and made a line chart for comparison (As is shown in figure 9)

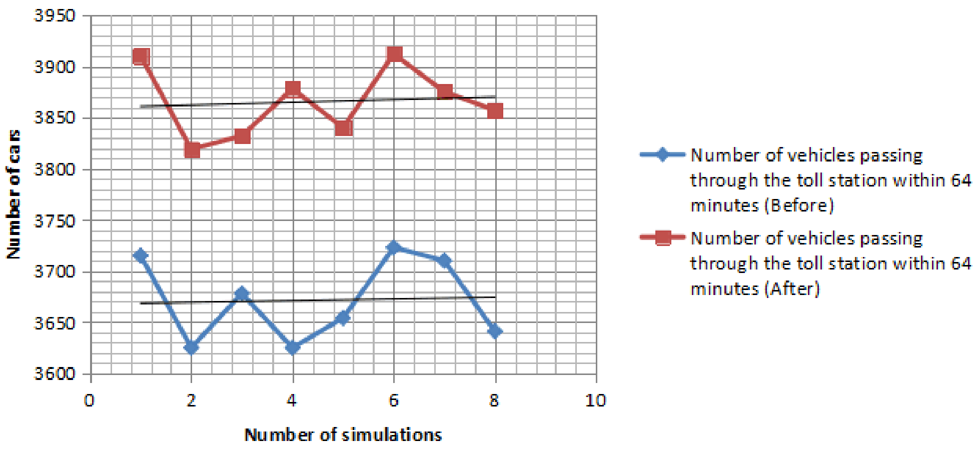

Fig.13 The Comparison of Throughput

As is shown in figure 9, the red line shows the number of vehicles passing through the “curved” toll station within an hour; and the green line shows the “linear”one. It can be seen clearly and intuitively from the figure: when the shape of the toll booth changed from the “linear” to the “curved” one, the throughput has increased significantly. So it can be concluded that the “curved” toll station is practical and feasible.

5.3 5.3 The Evaluations of Cost

The cost of building a toll station is directly proportional to the area of the toll booth.

We assume that the average cost per unit area is P_0 ,and the total cost is P.The “curved” toll station changes to an angle of “γ” compared to a “linear” toll station.

Then :

$$P = P_0[K \times (D + C) - \frac{1}{2}C^2 sin\,(\theta + \gamma)] \qquad (5\text{-}26)$$

Where :

P is the total cost of building a toll station;

P_0 is the average cost per unit area;

γ is the difference of angle between the “curved” toll station and the “linear”one.

$\theta \in [0,90]$

It is shown in the equation above: with the growth of $(\theta+\gamma)$, $sin\ (\theta+\gamma)$ followed by growth,while P followed by decline. The BPR model in 5.2 shows that $\gamma > 0$. So in terms of cost, the design of “curved” toll station is optimal.

6. MOP Model

6.1 The Influence of Different Traffic Combinations

Because the problem 2 required ua to analyze the performance of our solution under the influence of traffic volume, the combination of vehicles and tollbooths, we consider using the multi-objective programming (MOP) to evaluate the impact of these factors on our design. We set the number of human-staffed tollbooths, exact-change tollbooths, and electronic toll collection booths are r_1, r_2, and r_3 respectively and the traffic volume of human-staffed tollbooths, exact-change tollbooths, and electronic toll collection booths are q_1, q_2, and q_3. Combining the formulas in 5.1 and 5.2, we obtain the multiobjective programming functions as follows:

$$maxQ(r_1,r_2,r_3)=r_1q_1+r_2q_2+r_3q_3 \quad (6\text{-}1)$$

$$s.t.\begin{cases} r_1+r_2+r_3=\dfrac{B}{2} \\ q_1=q_2=c\times\sqrt[\beta]{\dfrac{T-\dfrac{\sqrt{2a\delta}}{a\delta}}{\alpha}} \\ q_3=c\times\sqrt[\beta]{\dfrac{T-\dfrac{-v_0+\sqrt{v_0^2+2a\delta}}{a\delta}}{\alpha}} \end{cases} \quad (6\text{-}2)$$

Where :

$\delta=\frac{H}{tan\theta_1+tan\theta_2}+l$;

r_1, r_2, and r_3 is the number of human-staffed tollbooths, exact-change tollbooths, and electronic toll collection booths respectively.

q_1, q_2, and q_3 is the traffic volume of human-staffed tollbooths, exact-change tollbooths, and electronic toll collection booths respectively.

B is the number of lanes.

T is the time travel time per unit road when the traffic volume is q;

c is the capacity of the road;

α and β are the parameter of the BPR model.

Taking advantage of Lingo,we finally made the conclusion that the proportion of the human-staffed tollbooths,exact-change tollbooths and ETC is 2:1:1 in the optimal solution.

The equation (6-1) represents the ratio of the three toll booths without considering the combination self-driving car. A lot of data shows that self-driving cars go through ETC when passing the toll staion. Therefore, As more and more self-driving cars are added to the flow, the proporion of r_3 will increase. By changing the constraints, we can obtain the ratio of the three toll booths in the case of the different traffic combinatios. The solving process is same as before, and the result is: $r_1':r_2':r_3' = 1:1:2$.

6.2 The Influence of Different Traffic Volume

In order to evaluate the effect of different traffic volume on the two design schemes, we use cellular automata to simulate and to get a large amount of data to analyze the service level of expressway in two cases. The result is shown in table 2.

Table 2 Evaluation of Traffic Volume in Different Periods

Type	Off-Peak Period	V/C	Peak Period	V/C	Maximum Service traffic pcu/h (Standard)
Linear	2786	0.70	3645	0.91	2000
Curved	3044	0.44	3862	0.56	2300

According to the highway service level standard from United States in 2000 road capacity manual[4] , we can see: in the off-peak period, The service level of the straight-line toll station is class C and the curve is class B.So whether the traffic in liht or heavy, the performance of curved toll stations are better than the straight-line toll stations, which indicating that our program is feasible.

7. Model Testing and Sensitivity Analysis

7.1 Different Types of Vehicles

According to the highway engineering technical standard[6], we can know that in the highway highway generally take the automobile as the standard. Other types of vehicles can be converted to cars by multiplying the corresponding conversion factors. Therefore, the toll station throughput will change, and because of the nature of different types of vehicles there are differences, resulting in vehicles through the fan-shaped area of time has changed.

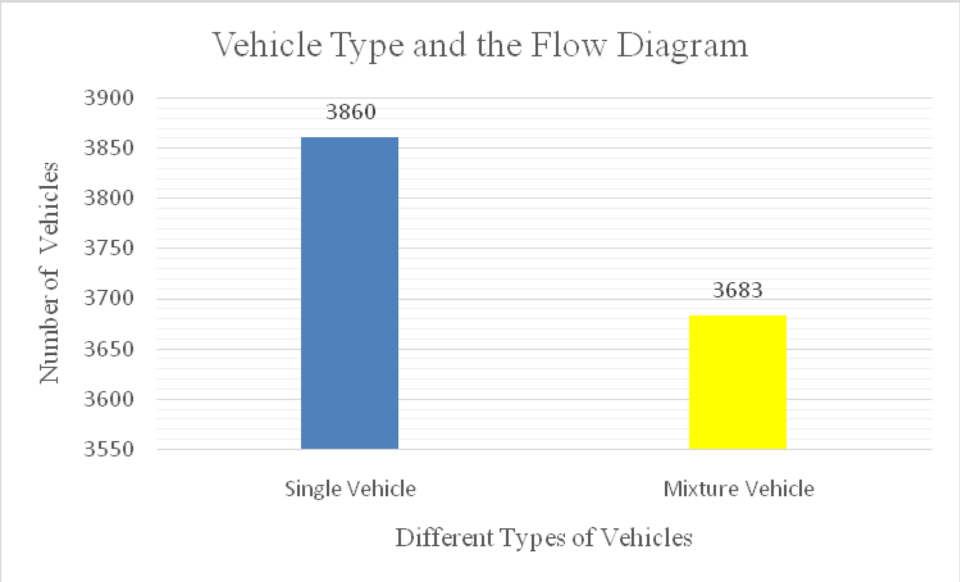

Fig.14 Vehicle Type and the Flow Diagram

7.2 Change the Road Horizontal Longitudinal Slope

Refer to the traffic industry standard highway toll stations and toll plaza design specifications we ca n see that the toll plaza area of the horizontal slope can not be greater than 0.5%. But in some areas due to terrain effects, can not meet the design specifications, vehicles in the course of the process wi ll be greatly affected. As a result, the throughput of the toll station is reduced and the time it takes fo r the vehicle to pass through the sector is prolonged.

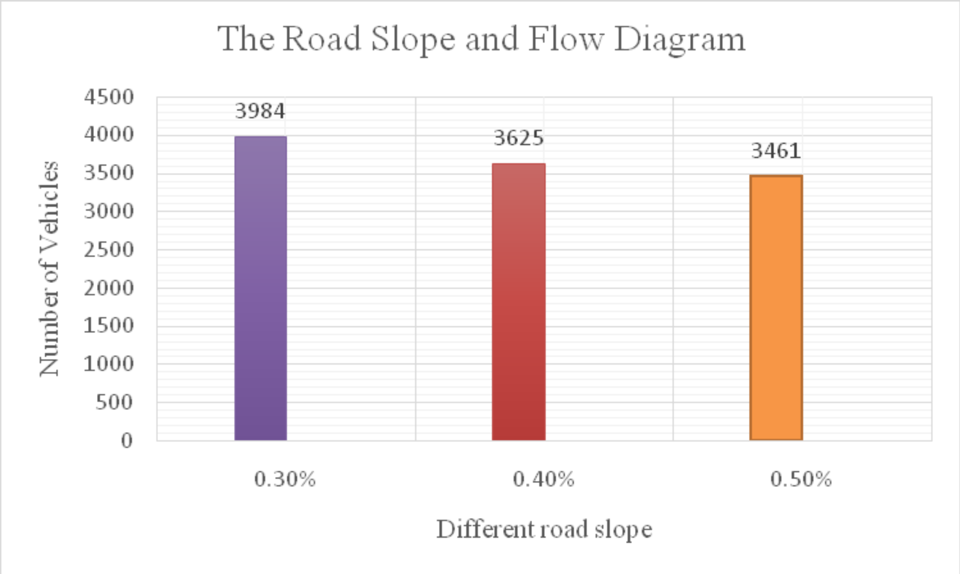

Fig.15 The Road Slope and Flow Diagram

7.3 Sensitivity Analysis of Lingo Multi-Objective Programming

Lingo for sensitivity analysis：

Global optimal solution found.

Objective value: *2500.0000*

Total solver iterations: *1*

Variable	Value	Reduced Cost
X1	1.00000	0.00000
X2	1.00000	0.00000
X3	2.00000	0.00000
Row	Slack or Surplus	Dual Price
1	215.00000	1.00000
2	0.00000	0.00000

Obviously, the objective function has a better stability, the multi-objective programming has a better advancement.

8. Strength and Weakness

8.1 Strength

- The model is clear and easy to understand, implement, and promote;
- Cellular automata simulation capability, to the greatest extent possible to simulate the reality of traffic flow changes, the actual number of vehicles on the road and time and other large amounts of data, to a certain extent, to ensure the quantity and quality of data.
- The reverse application of the vascular branch model, the more vivid description of the intersection mode, combined with the dynamic model, to improve and optimize the reality of traffic merging.
- The application of optimization model, combined with the size of the impact factor θ, traffic flow Q, etc., to meet the future development trend, based on obtained with the increase in self-driving, the proportion of toll booths. Is a toll booth that serves more vehicles at the same time.

8.2 Weakness

- Cellular automata in the programming has some difficulty in the formulation and implementation of the rules, because the actual or there are too many human disturbance factors, so there will be small data in the deviation.
- The output data only in the specified number of toll ports and the number of lanes to simulate the actual need to be based on the reality of the proportion of different vehicles to determine the results to adjust, so some parameters expressed with the function relationship.

9. Further Discussion

Through a deeply analysis on the weaknesses and assumptions which we ignored, we put forward the improvement scheme as follows:

●For the reverse branching model, we merge the traffic pattern with the reverse flow pattern. The fl ow is analogous to the fluid. However, in practice, the intertwined flow is more complex due to too much factors, so in order to further close and propose new merging patterns, the reverse merging pa ttern of branch bifurcation satisfies to a greater extent Situation, to achieve a more optimized merge r model, easy to promote.

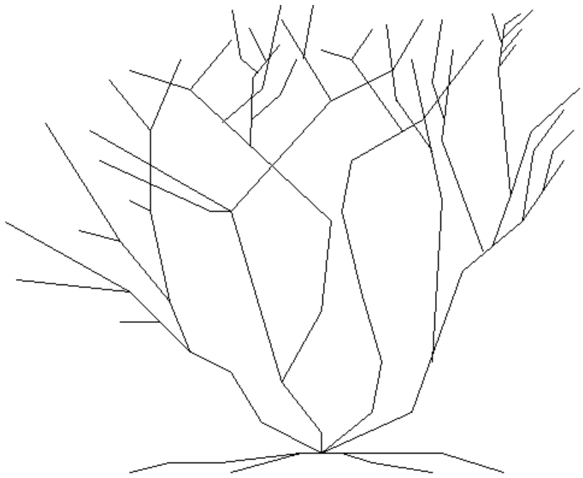

Fig.16 The sketch Diagram of Branches

●We have improved the shape of the toll barrier because it takes into account that there is no lane o ccupancy when the vehicle arrives at the cross-section in order to reduce the delay and thus reduce t he probability of simultaneous vehicle presence in the area most prone to traffic accidents.

●For the final optimization model, we only limit it to the proportion of various toll stations, which obviously limits the widespread use of the model, so the scope of the model by some way to expand to adapt to the overall application, to improve it.

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