New Teaching Equipment Based on Bernouli Large Numbers Law

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Abstract

Because of the lack of sensorial teaching equipment in daily teaching, we design a new sensor attached Galton nailing board. In the passage, firstly we give out the mathematical principles of Galton nailing system. Then we design a new type of Galton nailing system. After that, we do the experiment and analyze practical data with the help of software program. At last, we find our equipment has fairly good usability.

Keywords

Sensorial teaching equipment; Galton nailing board; Bernouli large numbers law.

1. Introduction

Human senses include listening, speaking, reading and writing. According to research data from Harvard Business School [1], the ratio of the human brain receiving external information through five senses per day is: taste 1%, touch sense 1.5%, sense of smell 3.5%, 11% hearing, and 83% vision. Using the traditional "chalk plus lecture" method, most of the information can only be obtained through listening . To obtain a comprehensive understanding of objective things, these five senses should be coordinated to complete the event. In daily teaching, intuitive statistical teaching aids are scarce. Galton nailing board is a relatively intuitive teaching apparatus. We combine Galton's nailing board with sensors to help us count and code a corresponding software to make the phenomenon more intuitive and facilitate the use of everyday teaching.

A domestic patent numbered CN205177231U [2] shows a new demonstration teaching aid on the normal distribution in the statistical series. The demonstration prop has a box and a hopper. Inside the box, there are multiple layers of crossbars, with observation windows on the opposite side of the nailing plate. This is a simple, cleverly designed teaching aid that enables students to more fully understand the actual concept of normal distribution. This is a interesting physical Galton nailing board aids.

A patent numbered CN205405940U [3] shows a new mathematics teaching aid in the probability of cognitive statistics. The teaching aids include a plate, a display slot, a telescopic slot, a slide bar corresponding to the telescopic slot. When the slide bar rises in the telescopic tank , the indication color mark in the telescopic tank is displayed through the display trough, which can directly demonstrate the probabilistic demonstration. The adjacent display through uses different colors of the indication color logo, the presentation is more intuitive and more eye-catching, which is conducive to students' understanding and learning statistical probability. This course is characterized by eye-catching and easy operation during presentation.

In foreign patent research, a patent numbered KR1020170049624A [4] ,used in the statistical golf teaching tool. A system and a method were described for applying to various golfer based on data generated during their respective golf swings. It can provide training and equipment specifications. Through the electronic interaction between golfers and electronic machines, it described a data center based on golfer training information and service, reflected the practicability of electronic interaction.

2. The theoretical basis of Galton nailing board

2.1 The distribution of ball positions in Galton nailing boards

We should first assume that the ball will fall to the left or right with equal probability each time it hits the Galton nail plate, that is, the probability of both left and right is 0.5. Then we look at the n-th row of the nail board. The nth row has n+2 impact bolts. As the first row shows three nails (as shown in Figure 1), there is an empty space between the two nails. One line has n+1 empty space. For the n+1 empty spaces number from left to right. Observe the probability that the i-th space will drop the ball.

Let the random variable X be the position of the space that falls in. The value of the random variable X is 0 1 2 ...n . The results are shown in Table I.

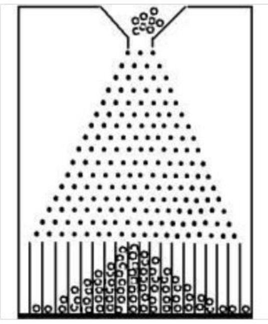


Figure 1 Traditional Galton nail board

Table I spaces number and random variable X correspondence table

Number of spaces	0	1	2	...	n
The value of the random variable X	0	1	2	...	n

When i=0, the ball falls into the space under the condition that the ball falls continuously to the left after the collision with the nail for the first time, that is, consecutively n times to fall to the left, then it falls into this space (i=0). The probability is:

$$P\{X=i\}=P\{X=0\}=C_n^0\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^0 \tag{1}$$

Further, we can know that when the ball falls into the i-th space, it means that during the n collisions after the first collision with the nail, there are i choices that fall to the right, and the rest of the collisions fall to the left. , so the probability of falling into the i-th space is:

$$P\{X=i\}=C_n^i(\tfrac{1}{2})^{n-i}(\tfrac{1}{2})^i=C_n^i(\tfrac{1}{2})^n \quad \mathrm{i}=0,1,2,3\ldots\mathrm{n} \tag{2}$$

By formula (2) we can see that the random variable X obeys the binomial distribution.

2.2 Bernouli Large Numbers Law and De Moivre-Laplace Central Limit Theorem

Let the random variable X be subject to the binomial distribution with the parameter n, p $(0<p<1)$

$$\lim_{n\to\infty}P\left\{\frac{X-np}{\sqrt{np(1-p)}}\le x\right\}=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{t^2}{2}}\mathrm{d}t=\Phi(x) \tag{3}$$

It shows that when n is sufficiently large, the $\frac{X-np}{\sqrt{np(1-p)}}$ distribution approximately obeys the standard normal distribution, that is, the binomial distribution converges to the normal distribution, and the normal distribution is the limit distribution of the binomial distribution.

According to Bernouli Large Numbers Law and De Moivre-Laplace's central limit theorem, the distribution of the balls that falls into the position X is approximately obeys normal distribution, and its density curve is similar to the peak curve.

3. Traditional nailing board improvements

3.1 MCU counting system

Our wooden Galton nailing board is shown in Figure 2.In order to expand the function of the nail plate, we add a counting system to the nail plate. This system is shown in Figure 3. We combine microprocessors, LCD screens and photoelectric door with counting systems. We use photoelectric door to count the number of small balls then we use LCD screen to display it. After that we attach sensor with PC and build a communication line between PC and sensors. PC could get the data in time and draw a curve to fit the data. Our data collecting system can get the data and analyze it in time.

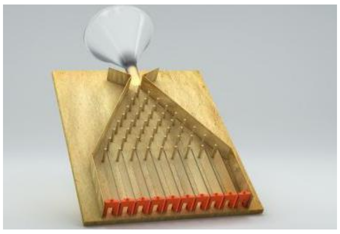


Figure 2 Wooden Galton nail board

Figure 3 Electronic Counting System

The actual system measurement steps are as follows:

1. Release the ball;

2. The ball passes through the Galton plate and falls to the sampling area, achieving normal distribution during the process;

3. ball presses sensor at sampling area, the total data is transmitted to the PC;

4. LCD shows the number of small balls at the same time PC draw the fitting curve;

In the actual measurement, we measured 10 sets of data. We will conduct statistics and analysis of the data in the next section.

3.2 Counting Programs and Drawing

In order to more intuitively present the statistical results of the current Galtonian nail board, we designed a PC-based QT (software) graphical human-computer interaction program.This program interface is shown in Figure 4.The table on the left of this program is the input for the statistics of the number of balls obtained from each slot of Galton's nailing board, and the corresponding bar chart is drawn on the right according to the data on the left. In order to realize the function of changing the image in real time, we have adopted the signal and slot mechanism in QT. This mechanism can modify one or several objects (right bar graph chart object) when a visible object (statistical data on the number of balls) changes in some way. Through this mechanism, as the number of balls continues to increase, the right bar graph can be dynamically changed. And it is more convenient and intuitive to display real-time statistics in the Galton nail plate.

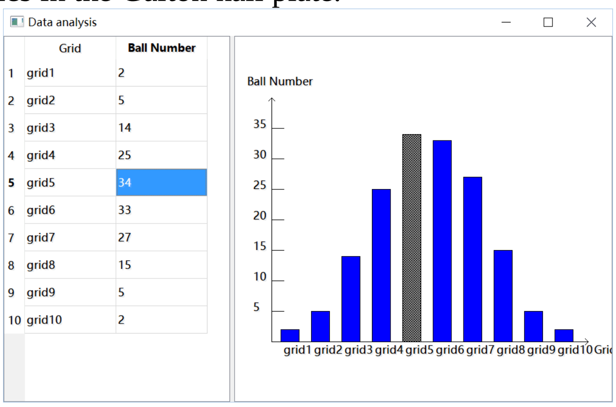


Figure 4 Statistics program interface

4. Comparison between measured data and theory

We counted the number of actually measured balls, and counted them in the following Table II data table.

Table II Number of balls in sampling point

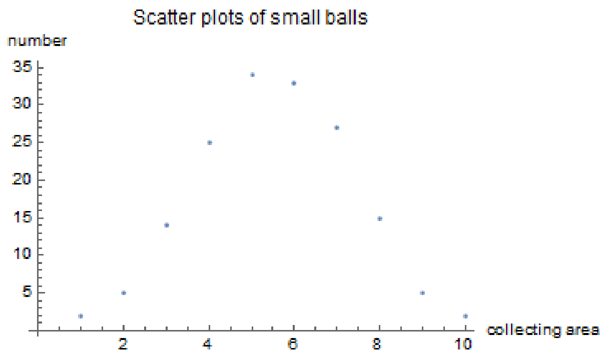
Sampling point\times	1	2	3	4	5	6	7	8	9	10
1	3	3	2	2	2	2	2	3	2	2
2	6	9	6	5	4	3	6	8	3	2
3	15	17	14	14	12	13	11	10	16	17
4	24	17	20	25	25	27	29	19	23	26
5	28	32	27	34	36	34	38	32	33	32
6	31	27	30	33	37	36	30	31	33	33
7	25	21	30	27	26	25	28	24	24	25
8	13	13	16	15	15	16	17	13	14	11
9	7	6	4	5	3	4	3	6	5	4
10	2	1	3	2	1	1	2	2	1	2

Calculate the mean value and variance of the 10 sampling points. We got the mean value and variance of the 10 sampling points:

Sample point mean values: 2,5,14,25,34,33,27,15,5,2

Sample point variance: 0.3, 4.6, 5.3, 15.1, 13, 8.7, 5.7, 3.5, 1.7, 0.5

Using Mathematica math software to draw and fit, the following graph in Figure 5,6 is obtained:



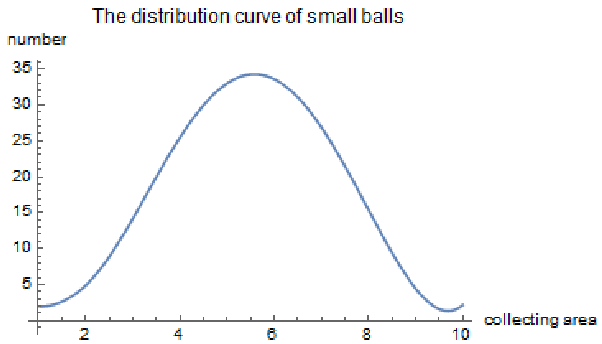
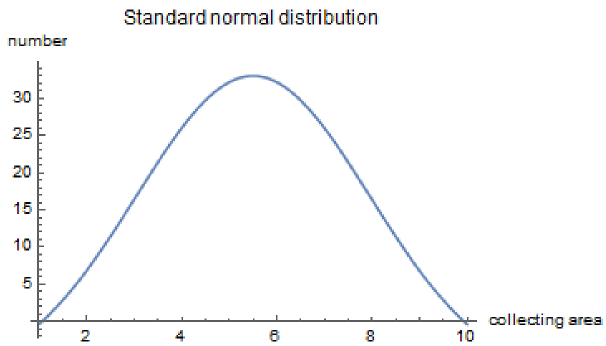


Figure5 Scatter plots of small balls Figure6 The distribution curve of small balls

We compare the obtained graph with the standard binomial distribution and its curves, and draw the image in Figure 7,8:



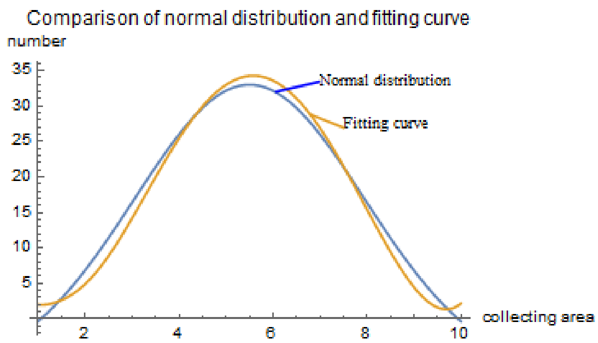


Figure7 Standard normal distribution Figure 8 Comparison of two curves

By comparison, we found that the shape and the curve of the two are basically the same as R^2= 0.82. The nail plate system we designed has good performance.

Further we analyze 10 variances, 0.3, 4.6, 5.3, 15.1, 13, 8.7, 5.7, 3.5, 1.7, 0.5. We observed that the variance of the observed values is not very large, indicating that our data is still relatively stable in the vicinity of the mean values. Our nail plate has good reliability.

5. Conclusion

Despite some errors, our nail board still meets the design requirements and has obtained good experimental results. We connect the traditional Galton nail board through a photoelectric door and a computer. The sensors collect the data and transmit it to the PC. The PC displays dynamic changes of the distribution through the QT graphical interface technology based on the known statistical data. And it intuitively describes the real-time situation of normal distribution. Based on mathematical reasoning, the nail plate can expand to display more types of distribution functions, expand the teaching content of teaching aids, and make traditional teaching aids digitized and intelligent. The visual information channel has the highest efficiency in all senses. Changes in the visual media can arouse students' freshness and interest in classroom teaching activities.

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