Multiple Attribute Group Decision Making Method Based on type-2 Intuitionistic Fuzzy Sets

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Abstract

In this paper, we propose a new distance measure among type-2 intuitionistic fuzzy sets (T2IFSs), which satisfies classical properties for distance measure such as non-negativity, symmetry and triangular inequality. Then a method for multiple attribute group decision making with T2IFSs is presented. Finally, a numerical example is given to show the feasibility and efficiency of the proposed method.

Keywords

Type-2 intuitionistic fuzzy sets; distance measure; multiple attribute group decision making; intuitionistic fuzzy sets; type-2 fuzzy sets.

1. Introduction

The theory of fuzzy sets (FSs) proposed by Zadeh [1], has achieved great achievements in multiple attribute decision making (MADM). Atanassov [2] has introduced intuitionistic fuzzy sets (IFSs), which is a generalization of FSs. Atanassov and Gargov [3] presented the theory of interval-valued intuitionistic fuzzy sets (IVIFSs), which is an extension of the theory of intuitionistic fuzzy sets (IFSs), where the membership degree and the non-membership degree of each element belonging to an IVIFS are represented by interval-valued intuitionistic fuzzy values (IVIFVs), respectively. In order to enhance the system's ability to deal with uncertainty, more and more attention is paid to the study of multiple uncertainty. In 1975, Zadeh proposed type-2 fuzzy sets (T2FSs) [4], which is characterized by a fuzzy membership function. The membership function of (T2FSs) provide an additional degree of freedom to express uncertain and fuzzy information in real life. Based on it, Singh proposed some distance measures among (T2FSs) [5]. Due to the operation and information are more abstract and complex. Many researchers focus on some special fuzzy sets, such as interval type-2 fuzzy sets [6]. Guo and Yin put forward the concept of T2IFSs [7] and applied in multiple attribute decision making (MADM) problems. They defined the concepts of type-2 intuitionistic fuzzy positive ideal point and negative ideal point [7]. Singh redefined the concept of T2IFSs and presented the distance measure based on Hamming, Euclidean and Hausdorff metrics [8]. Considering the difference of the membership degree and non-membership degree. A new distance measure is proposed, and a ranking method for multiple attribute group decision making with T2IFSs is presented in this paper.

2. Basic concepts

Definition 2.1 [4, 9] Let X be a finite universe. A type-2 fuzzy set $A \subseteq X$, is characterized by the membership function

$$A = \{((x,u_A), \mu_A(x,u_A)) \mid x \in X, u_A \in j_x \subseteq [0,1]\},$$

in which $A = \{((x,u_A), \mu_A(x,u_A)) \mid x \in X, u_A \in j_x \subseteq [0,1]\}$. Another expression for A is

$$A = \int_{x\in X} \mu_A(x)/x = \int_{x\in X} [\int_{u_A \in j_x} f_x(u_A)/u_A]/x,$$

where $\mu_A(x)=\int_{u_A\in j_x} f_x(u_A)/u_A$ is the grade of the membership, $f_x(u_A)=\mu_A(x,u_A)$ is named as a secondary membership function (SMF) where u_A denotes the primary membership function (PMF) of A and j_x is named as the (PMF) of X.

Definition 2.2 [8, 10] A type-2 intuitionistic fuzzy set (T2IFS) A in the finite universe of discourse X is defined as

$$A=\{<(x,u_A,v_A),f_x(u_A),t_x(v_A)>|\,x\in X,u_A\in j_x^u,v_A\in j_x^v\},$$

in which $\mu_A(x)=\int\limits_{u_A\in j_x^u} f_x(u_A)/u_A$ and $\nu_A(x)=\int\limits_{v_A\in j_x^v} t_x(v_A)/v_A$ meet the following conditions as

$$\max_{u_A\in j_x^u}(f_x(u_A)*u_A)+\max_{v_A\in j_x^v}(t_x(v_A)*v_A)\le 1,\forall x\in X.$$

where $u_A\in j_x^u\subseteq[0,1]$, $v_A\in j_x^v\subseteq[0,1]$. u_A and v_A are the primary membership function (PMF) of the membership and primary non-membership functions (PNMF) respectively. In addition, $f_x(u_A)$ and $t_x(v_A)$ are named as secondary membership function (SMF) and secondary non-membership functions (SNMF), respectively.

3. Distance measures between T2IFSs

For convenience, a T2IFS A in the finite universe of discourse X defined as $A=<x(u_A,f_x(u_A),v_A,t_x(v_A))\,|\,x\in X>$. Let $F_2^I(X)$ be the set of T2IFSs in the finite universe of discourse X.

Definition 3.1 Let d be a mapping $d:F_2^I(X)\times F_2^I(X)\to[0,1]$. $d(A,B)$ is said to be a distance measure between A and B. For any $A,B,C\in F_2^I(X)$, d satisfies the following properties:

(P1) $0\le d(A,B)\le 1$;

(P2) $d(A,B)=0$ if and only if $A=B$;

(P3) $d(A,B)=d(B,A)$;

(P4) If $d(A,B)=0$ and $d(A,C)=0$, then $d(B,C)=0$;

(P5) $d(A,C)\le d(A,B)+d(B,C)$ for any $A,B,C\in F_2^I(X)$.

Remark. In Definition 3, non-negativity ($d(A,B)\ge 0$), symmetry (P3) and triangular inequality (P5) are the properties of classical distance measures.

Then, we propose a new distance measure between T2IFSs in the finite universe of discourse X.

Definition 3.2 Let $A=<x(u_A,f_x(u_A),v_A,t_x(v_A))\,|\,x\in X>$ and $B=<x(u_B,f_x(u_B),v_B,t_x(v_B))\,|\,x\in X>$ be two T2IFSs in the finite universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$, then the distance measure between A and B is defined as follows:

$$d(A,B)=\frac{1}{4n}\sum_{i=1}^{n}w_i\Big(\frac{|u_A(x_i)-u_B(x_i)|}{2}+\frac{|f_{x_i}(u_A)-f_{x_i}(u_B)|}{2}+\frac{|v_A(x_i)-v_B(x_i)|}{2}+\frac{|t_{x_i}(v_A)-t_{x_i}(v_B)|}{2}+\Big|\frac{u_A(x_i)+1-v_A(x_i)}{2}-\frac{u_B(x_i)+1-v_B(x_i)}{2}\Big|$$
$$+\Big|\frac{f_{x_i}(u_A)+1-t_{x_i}(v_A)}{2}-\frac{f_{x_i}(u_B)+1-t_{x_i}(v_B)}{2}\Big|\Big),\text{ where }\sum_{i=1}^{n}w_i=1,0\le w_i\le 1,i=1,\ldots,n.$$

Proposition 3.1 $d(A,B)$ is a new distance measure between T2IFSs A and B.

Proof. We need to prove that $d(A,B)$ satisfies (P1)-(P5) of Definition 3. In fact,

(P1) $d(A,B)\ge 0$

$d(A,B)=\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_A(x_i)-u_B(x_i)|+|f_{x_i}(u_A)-f_{x_i}(u_B)|+|v_A(x_i)-v_B(x_i)|+|t_{x_i}(v_A)-t_{x_i}(v_B)|+|(u_A(x_i)+1-v_A(x_i))-(u_B(x_i)+1-v_B(x_i))|+$
$|(f_{x_i}(u_A)+1-t_{x_i}(v_A))-(f_{x_i}(u_B)+1-t_{x_i}(v_B))|)$,
$\leq\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_A(x_i)-u_B(x_i)|+|f_{x_i}(u_A)-f_{x_i}(u_B)|+|v_A(x_i)-v_B(x_i)|+|t_{x_i}(v_A)-t_{x_i}(v_B)|+|(u_A(x_i)-u_B(x_i))-(v_A(x_i)-v_B(x_i))|+$
$|(f_{x_i}(u_A)-f_{x_i}(u_B))-(t_{x_i}(v_A)-t_{x_i}(v_B))|)$,
$\leq\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_A(x_i)-u_B(x_i)|+|f_{x_i}(u_A)-f_{x_i}(u_B)|+|v_A(x_i)-v_B(x_i)|+|t_{x_i}(v_A)-t_{x_i}(v_B)|+|(u_A(x_i)-u_B(x_i))|+|(v_A(x_i)-v_B(x_i))|+$
$|(f_{x_i}(u_A)-f_{x_i}(u_B))|+|(t_{x_i}(v_A)-t_{x_i}(v_B))|)$,
$\leq\frac{1}{8n}\sum_{i=1}^{n}w_i(2|u_A(x_i)-u_B(x_i)|+2|f_{x_i}(u_A)-f_{x_i}(u_B)|+2|v_A(x_i)-v_B(x_i)|+2|t_{x_i}(v_A)-t_{x_i}(v_B)|)$,
$\leq\frac{8}{8}=1$. Therefore, $0\leq d(A,B)\leq 1$.

(P2) If $A=B$, then $u_A(x_i)=u_B(x_i),\ f_{x_i}(u_A)=f_{x_i}(u_B), v_A(x_i)=v_B(x_i), t_{x_i}(v_A)=t_{x_i}(v_B),\ d(A,B)=0$.

(P3) It is easy to see that $d(A,B)=d(B,A)$.

(P4) If $d(A,B)=0$ and $d(A,C)=0$, then $u_A(x_i)=u_B(x_i),\ f_{x_i}(u_A)=f_{x_i}(u_B), v_A(x_i)=v_B(x_i), t_{x_i}(v_A)$ $=t_{x_i}(v_B);\ u_A(x_i)=u_C(x_i),\ f_{x_i}(u_A)=f_{x_i}(u_C), v_A(x_i)=v_C(x_i), t_{x_i}(v_A)=t_{x_i}(v_C)$. Then $u_B(x_i)=u_C(x_i)$, $f_{x_i}(u_B)=f_{x_i}(u_C), v_B(x_i)=v_C(x_i), t_{x_i}(v_B)=t_{x_i}(v_C)$. Thus, $d(B,C)=0$.

(P5) For any $A,B,C\in \mathrm{T2IFSs}$, then

$d(A,B)=\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_A(x_i)-u_B(x_i)|+|f_{x_i}(u_A)-f_{x_i}(u_B)|+|v_A(x_i)-v_B(x_i)|+|t_{x_i}(v_A)-t_{x_i}(v_B)|+|(u_A(x_i)+1-$
$v_A(x_i))-(u_B(x_i)+1-v_B(x_i))|+|(f_{x_i}(u_A)+1-t_{x_i}(v_A))-(f_{x_i}(u_B)+1-t_{x_i}(v_B))|)$,
$d(B,C)=\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_B(x_i)-u_C(x_i)|+|f_{x_i}(u_B)-f_{x_i}(u_C)|+|v_B(x_i)-v_C(x_i)|+|t_{x_i}(v_B)-t_{x_i}(v_C)|+|(u_B(x_i)+1-$
$v_B(x_i))-(u_C(x_i)+1-v_C(x_i))|+|(f_{x_i}(u_B)+1-t_{x_i}(v_B))-(f_{x_i}(u_C)+1-t_{x_i}(v_C))|)$,
$d(A,C)=\frac{1}{8n}\sum_{i=1}^{n}w_i(|u_A(x_i)-u_C(x_i)|+|f_{x_i}(u_A)-f_{x_i}(u_C)|+|v_A(x_i)-v_C(x_i)|+|t_{x_i}(v_A)-t_{x_i}(v_C)|+|(u_A(x_i)+1-$
$v_A(x_i))-(u_C(x_i)+1-v_C(x_i))|+|(f_{x_i}(u_A)+1-t_{x_i}(v_A))-(f_{x_i}(u_C)+1-t_{x_i}(v_C))|)$,
$|u_A(x_i)-u_B(x_i)|+|u_B(x_i)-u_C(x_i)|\geq|u_A(x_i)-u_B(x_i)+u_B(x_i)-u_C(x_i)|=|u_A(x_i)-u_C(x_i)|$,
Similarly,
$|f_{x_i}(u_A)-f_{x_i}(u_B)|+|f_{x_i}(u_B)-f_{x_i}(u_C)|\geq|f_{x_i}(u_A)-f_{x_i}(u_C)|, |v_A(x_i)-v_B(x_i)|+|v_B(x_i)-v_C(x_i)|\geq|v_A(x_i)-v_C(x_i)|$,
$|t_{x_i}(v_A)-t_{x_i}(v_B)|+|t_{x_i}(v_B)-t_{x_i}(v_C)|\geq|t_{x_i}(v_A)-t_{x_i}(v_C)|$,
$|(u_A(x_i)+1-v_A(x_i))-(u_B(x_i)+1-v_B(x_i))|+|(u_B(x_i)+1-v_B(x_i))-(u_C(x_i)+1-v_C(x_i))|\geq|(u_A(x_i)+1-$
$v_A(x_i))-(u_C(x_i)+1-v_C(x_i))|$,
$|(f_{x_i}(u_A)+1-t_{x_i}(v_A))-(f_{x_i}(u_B)+1-t_{x_i}(v_B))|+|(f_{x_i}(u_B)+1-t_{x_i}(v_B))-(f_{x_i}(u_C)+1-t_{x_i}(v_C))|\geq|(f_{x_i}(u_A)+1-$
$t_{x_i}(v_A))-(f_{x_i}(u_C)+1-t_{x_i}(v_C))|$,
Thus, $d(A,C)\leq d(A,B)+d(B,C)$.

Remark. The distance measure between T2IFSs in this paper is good. On the one hand, it includes differences between PMF, PNMF, SMF and SNMF, as well as the differences between median values of intervals $\frac{u_A(x_i)+1-v_A(x_i)}{2}$ and $\frac{u_B(x_i)+1-v_B(x_i)}{2}$, $\frac{f_{x_i}(u_A)+1-t_{x_i}(v_A)}{2}$ and $\frac{f_{x_i}(u_B)+1-t_{x_i}(v_B)}{2}$. On the other hand, we also consider the importance of the element $x_i \in X$.

4. Group decision making with T2IFSs

For a multiple attribute group decision making problem with m alternatives $A_i(i=1,\cdots,m)$, the performance of the alternative A_i concerning the attribute $C_j(j=1,\cdots,n)$ is assessed by a decision organization with several decision makers $D_q(q=1,\cdots,l)$. The corresponding weights of attributes are denoted by $w_j(j=1,\cdots,n),\ 0\le w_j\le 1,\ \sum_{j=1}^{n} w_j=1$ and the weights of decision makers are denoted by $\lambda_q(q=1,\cdots,l), 0\le\lambda_q\le 1, \sum_{q=1}^{l}\lambda_q=1$. A method is given for a multiple attribute group decision making problem with type-2 intuitionistic fuzzy information as follows:

Step 1. Generate assessment information. Based on the decision makers' knowledge and experience, the information of each alternative corresponding to each attribute are described as the linguistic grades. According to the linguistic grades of PMF, SMF and PNMF, SNMF, we can obtained the different alternatives A_i with respect to each attributes C_j from the decision makers D_q. The assessments given by D_q could be expressed as follows:

$$D_q=\{D_qA_iC_j(u_{D_q}, f_{A_iC_j}(u_{D_q}), v_{D_q}, t_{A_iC_j}(v_{D_q}))\mid i=1,\cdots,m;\ j=1,\cdots,n\}.$$

Step 2. Calculate the distance measure between the decision makers D_q and the union decision U, i.e.

$$d(D_qA_iC_j,U), q=1,\cdots,l; i=1,\cdots,m; j=1,\cdots,n.$$

where U is a T2IFSs $U=(1,1,0,0)$, having one PMF and SMF while zero PNMF and SNMF for each alternative with respect to each attribute.

Step 3. Construct type-2 intuitionistic fuzzy alternatives A_i . Find the minimum value of $d(D_{q_0}A_iC_j,U)$ for all the alternatives A_i corresponding to each attribute C_j. The corresponding decision makers denoted as q_0, i.e.

$$d(D_{q_0}A_iC_j,U)=\min\{d(D_qA_iC_j,U)\mid q=1,\cdots,l\}, \forall i=1,\cdots,m;\ j=1,\cdots,n.$$

Therefore, A_i is desicribed as T2IFSs

$$A_i=\{C_j(u, f_{A_i}(u), v, t_{A_i}(v))\mid j=1,\cdots,n\}, \forall i=1,\cdots,m.$$

where $C_j(u, f_{A_i}(u), v, t_{A_i}(v))$ and $D_{q_0}A_iC_j(u_{D_{q_0}}, f_{A_iC_j}(u_{D_{q_0}}), v_{D_{q_0}}, t_{A_iC_j}(v_{D_{q_0}}))$ is same.

Step 4. Calculate the distance measure between alternatives A_i and the union decision U i.e., $d(A_i,U), i=1,\cdots,m.$

Step 5. Rank all the alternatives, $A_{i_1}\succ A_{i_2}\succ\cdots\succ A_{i_m}$. The smaller the distance, the closer the alternative A_{i_1} from the union decision U, and thus, the better the alternative A_{i_1} .

$$d^*=\min\{d(A_i,U)\mid i=1,\cdots,m\}.$$

5. Numerical example

In literature [8], assume that a decision-making problem in which a person invest some money in to the company. Three decision makers of D_1, D_2 and D_3 are employed to evaluate whose weight vector is $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$. Suppose there is a panel with five alternatives such as (i) A_1 is a car company, (ii) A_2 is a food company, (iii) A_3 is a computer company, (iv) A_4 is an arms company, (v) A_5 is a tire company. The investor takes a decision under the four attributes, namely, C_1 is the risk analysis, C_2 is the growth analysis, C_3 is the environmental impact analysis and C_4 is the available space whose weight vector is $(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})$. The linguistic grades of PMF, SMF and PNMF, SNMF are shown in Table 1.

Table 1 Linguistic grade and corresponding PMF, PNMF, SMF and SNMF value

Grades	PMF value	PNMF value	SMF value	SNMF value
Very poor(VP)	0	0	0	0
Poor(P)	0.2	0.1	0.2	0.1
Medium Poor(MP)	0.4	0.2	0.4	0.2
Fair(F)	0.5	0.4	0.5	0.4
Medium Good(MG)	0.7	0.5	0.7	0.5
Good(G)	0.9	0.7	0.9	0.7
Very Good(VG)	1	1	1	1

Next, we decide the best choice by the T2IFSs method for a group decision making.

Step 1. Each decision maker assesses alternative A_i corresponding to each of the attributes C_j as follows.

Step 2. Calculate the values of $d(D_qA_iC_j,U)$ in Tables 2-6.

$$D_1=\begin{bmatrix}(1.0,0.7,0.0,0.2) & (1.0,0.9,0.0,0.1) & (1.0,0.7,0.0,0.2) & (1.0,0.9,0.0,0.1)\\ (0.7,0.9,0.1,0.1) & (0.7,0.7,0.4,0.4) & (1.0,0.7,0.0,0.2) & (1.0,0.9,0.0,0.1)\\ (0.5,0.4,0.4,0.5) & (0.9,1.0,0.1,0.0) & (1.0,0.9,0.0,0.1) & (1.0,0.4,0.0,0.5)\\ (0.7,0.5,0.2,0.4) & (0.9,0.7,0.1,0.2) & (1.0,0.9,0.0,0.1) & (0.7,0.5,0.2,0.4)\\ (0.7,0.5,0.2,0.4) & (1.0,0.9,0.0,0.1) & (0.9,0.4,0.1,0.5) & (0.9,0.7,0.1,0.2)\end{bmatrix}$$

$$D_2=\begin{bmatrix}(0.9,1.0,0.1,0.0) & (0.9,0.4,0.1,0.5) & (0.9,1.0,0.1,0.0) & (0.9,1.0,0.1,0.0)\\ (0.9,0.7,0.0,0.1) & (0.9,0.7,0.1,0.2) & (0.9,1.0,0.1,0.0) & (0.9,1.0,0.1,0.0)\\ (0.9,0.5,0.1,0.4) & (0.7,0.4,0.2,0.5) & (0.9,0.9,0.1,0.1) & (0.7,0.9,0.2,0.1)\\ (0.9,0.5,0.1,0.4) & (0.7,0.4,0.2,0.5) & (0.9,0.5,0.1,0.4) & (0.9,0.5,0.1,0.4)\\ (0.7,0.4,0.2,0.5) & (0.5,0.5,0.4,0.4) & (0.7,0.4,0.2,0.5) & (0.7,0.4,0.2,0.5)\end{bmatrix}$$

$$D_3=\begin{bmatrix}(0.9,0.4,0.1,0.5) & (0.9,0.4,0.1,0.2) & (0.9,0.9,0.1,0.1) & (0.9,0.5,0.1,0.4)\\ (0.2,0.5,0.7,0.4) & (0.2,0.5,0.7,0.4) & (1.0,0.9,0.0,0.1) & (1.0,0.7,0.0,0.2)\\ (0.2,0.4,0.7,0.5) & (0.4,0.7,0.5,0.2) & (0.4,0.7,0.5,0.2) & (0.4,0.4,0.5,0.5)\\ (0.2,0.0,0.7,1.0) & (0.4,0.5,0.5,0.4) & (0.4,0.7,0.5,0.2) & (0.5,0.7,0.4,0.2)\\ (0.9,0.7,0.0,0.1) & (0.4,0.7,0.5,0.2) & (0.4,0.7,0.5,0.2) & (0.5,0.7,0.4,0.2)\end{bmatrix}$$

Table 2 Distance measure $d(D_qA_iC_1,U)$

		D_1	D_2	D_3
	A_1	0.125	0.050	0.325
	A_2	0.150	0.125	0.600
C_1	A_3	0.500	0.275	0.650
	A_4	0.350	0.275	0.875
	A_5	0.350	0.400	0.125

Table 3 Distance measure $d(D_q A_i C_2, U)$

		D_1	D_2	D_3
	A_1	0.050	0.325	0.175
	A_2	0.350	0.175	0.600
C_2	A_3	0.050	0.400	0.400
	A_4	0.175	0.400	0.500
	A_5	0.050	0.450	0.400

Table 4 Distance measure $d(D_q A_i C_3, U)$

		D_1	D_2	D_3
	A_1	0.125	0.050	0.100
	A_2	0.125	0.050	0.050
C_3	A_3	0.050	0.100	0.400
	A_4	0.050	0.275	0.400
	A_5	0.325	0.400	0.400

Table 5 Distance measure $d(D_q A_i C_4, U)$

		D_1	D_2	D_3
	A_1	0.050	0.050	0.275
	A_2	0.050	0.050	0.125
C_4	A_3	0.275	0.175	0.550
	A_4	0.350	0.275	0.350
	A_5	0.175	0.400	0.350

Step 3. Find the minimum value of $\{d(D_q A_i C_j, U) \mid q = 1,2,3\}$ from Tables 2-5 for all alternatives A_i corresponding to each attribute C_j and hence construct the T2IFS alternative A_i as follows:

$$A_1 =< C_1(0.9,1.0,0.1,0.0), C_2(1.0,0.9,0.0,0.1), C_3(0.9,1.0,0.1,0.0), C_4(1.0,0.9,0.0,0.1) >,$$
$$A_2 =< C_1(0.7,0.9,0.1,0.1), C_2(0.9,0.7,0.1,0.2), C_3(0.9,1.0,0.1,0.0), C_4(1.0,0.9,0.0,0.1) >,$$
$$A_3 =< C_1(0.9,0.5,0.1,0.4), C_2(0.9,1.0,0.1,0.0), C_3(1.0,0.9,0.0,0.1), C_4(0.7,0.9,0.2,0.1) >,$$
$$A_4 =< C_1(0.9,0.5,0.1,0.4), C_2(0.9,0.7,0.1,0.2), C_3(1.0,0.9,0.0,0.1), C_4(0.9,0.5,0.1,0.4) >,$$
$$A_5 =< C_1(0.9,0.7,0.0,0.1), C_2(1.0,0.9,0.0,0.1), C_3(0.9,0.4,0.1,0.5), C_4(0.9,0.7,0.1,0.2) >.$$

Step 4. According to the distance measure, we can obtained $d(A_1,U) = 0.0125, d(A_2,U) = 0.0266,$ $d(A_3,U) = 0.0344, d(A_4,U) = 0.0485$ and $d(A_5,U) = 0.0422$.

Step 5. We can rank the alternatives as, $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$. Therefore, A_1 is the best alternative to invest money than others.

Compare the method of the proposed method with some existing methods [8,11,12,13], and their corresponding results are summarized in Table 6. From this table, we can see that the proposed method is suitable and valid.

Table 6 The compromise values by existing methods and the proposed method

Method	Ranking	The best alternative
[8]	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$	A_1
[11]	$A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$	A_1

[12]	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$	A_1
[13]	$A_1 \succ A_2 \succ A_5 \succ A_3 \succ A_4$	A_1
This paper	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$	A_1

6. Conclusion

In this paper, a new distance measure is given under the type-2 intuitionistic fuzzy environment. Due to the high uncertainty of the information, a group decision making method is proposed based on T2IFSs in this paper. And the feasibility and validity of this method are verified by example analysis. This method can also be applied to market investment, economic management and other fields.

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