A Design Idea of American Highway Toll Station

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Abstract

With the increment of the traffic flow and the using of the self-driving car, the design of original highway toll station has been unable to meet the new requirements. This paper studies the mathematical modeling of barrier toll station design.Using queuing theory to analysis the shape, area of toll exit area and the influence of the intersection on toll station capacity. Analysis from the macro and micro aspects: 1)Firstly, the traffic flow model is used to analyze the shape and size of the toll-out area of the toll station from the macroscopic point of view, and give the macroscopic theoretical relational expression. 2) and then from a microscopic point of view: firstly, divide the toll station into Recovery and Departure zones, and use Car-following model to simulate the driving state of vehicles on the same lane. Then consider the lane change habits of vehicles, probability of collision, and the distance between vehicles and other key factors. Use the cellular automata model to analyze the relationship between the throughout and the shape, area of the toll, distribution characteristics in fan-out area. Then use the simulation data to fit the macroscopic theoretical formula to determine the specific parameters.Based on the above theoretical analysis. The paper then presents a new design of the barrier toll station, including the shape of the exit, the area, the ratio of the three types of charging equipment.

Keywords

Highway toll station, queuing theory, cellular automata model.

1. The Background of the Research

1.1 The Reason for Establishing Toll Station on Highway.

The toll station charges costs to repay the loans for the construction of expressway.

1.2 The Traditional and Nowadays Toll Station

Firstly, mainline toll station, the starting point and the end of the highway, and the provincial boundary.

Secondly, ramp toll stations, which are established at where there are important cities, counties, towns and tourist attractions and so on.

1.3 The Obvious Disadvantages of the Traditional Setting of the Toll Station

Nowadays the setting of the toll station result in an increasingly serious traffic congestion. And the wait-time of drivers is longer and longer.

2. The Structure of the Toll Booth

Toll stations are facilities used to collect tolls for passing vehicles placed across the highway. The number of the tollbooths usually more than the number of the incoming lanes of traffic. Vehicles must “fan in” the tollbooths in a barrier toll from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier. There are three different kinds of channels named E-ZPass channel, MTC channel and ATC channel in the toll booth.

3. Notations

Table 1. Notations

Symbol	Meaning
D	Car-following distance between the vehicle and the vehicle in front
D_S	Safety distance between two vehicle
D_l	The minimum distance between two vehicles
RD	The recovery zone and the departure zone
Q	The traffic flow in the RD zone
u	Flow velocity of traffic flow
S	The total area of the RD zone
u_f	Not congested normal speed in RD zone
K	Traffic density of RD zone
C	The cost per square meter in RD zone
K_j	The maximum density of traffic flow in RD zone
β_1	The coefficient of cost in comprehensive evaluation index
β_2	Coefficient of Mean Time to Traffic Flow in Comprehensive Evaluation
BRD	Overall evaluation index of RD zone

4. Model for Recovery Zone and Departure Zone

First, we discuss the model without considering the process before the vehicle enters the tollbooths. By this model, we can find the reasonable construction program of the recovery zone and the departure zone. We build two different models to simulate the macroscopic state and the microscopic state of the vehicles-cellular automata model and traffic-flow model to solve the RD Model (Recovery zone and Departure Zone Model).

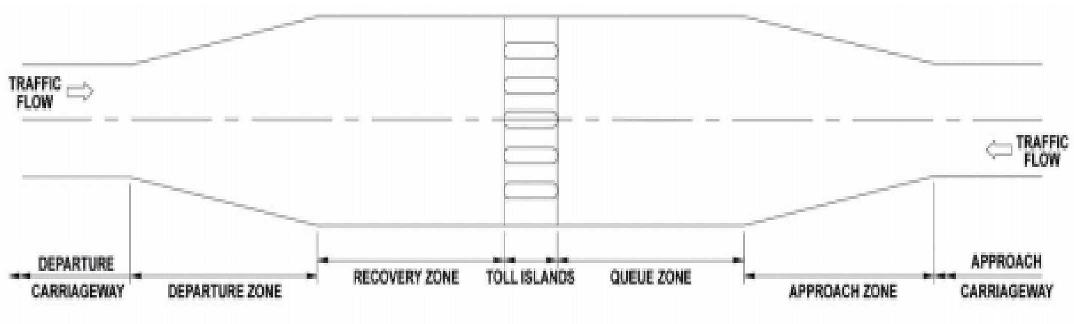

Figure 1. The structure of the toll plaza

4.1 Cellular Automata Model

We see the traffic movement is a discrete phenomenon. In the cellular automata model, each cell is designated as a vehicle. Position is specified as a cell or location in a matrix, while time is incremented using a convenient time step.[1] By observing individual components of the traffic, the cellular automata model may help find the reasonable construction program of the RD Model.

Assuming that the probability of occurrence of vehicles at each tollbooth is in accordance with the Poisson distribution. Then suppose there are four types of vehicles, including small passenger cars, midsize cars, large cars, and trailers, and these vehicles appear with different probabilities.

4.1.1 Car-following Model

The car usually follows the preceding car in the same lane. And this is the "following requirements". Considering driving safety, car-following vehicles should meet two requirements. One is "speed condition" means that the following car's speed can't be higher than the speed of the car ahead all the time. The other is "distance condition" means that two cars should maintain a safe distance. That means the following driver has enough time to respond when the car ahead brakes. And the process of the reaction need some time for the driver. And the time is about one second.

We make the following assumptions for the model:

In order to avoid the occurrence of a crash, the vehicle can only be gradually accelerated or slowed down and there is no emergency braking or sudden acceleration occurring.

The cellular automata programming must ensure that the vehicle is synchronously updated. So, suppose the reaction time of each "man car community" is less than the simulation interval. The following car has to estimate the speed of the car ahead in the next moment, and determine his own speed in the next moment.

To simulate different speed of the vehicles, we assume that each vehicle is decelerated with a probability P. What's more, we set a maximum speed for each vehicle.

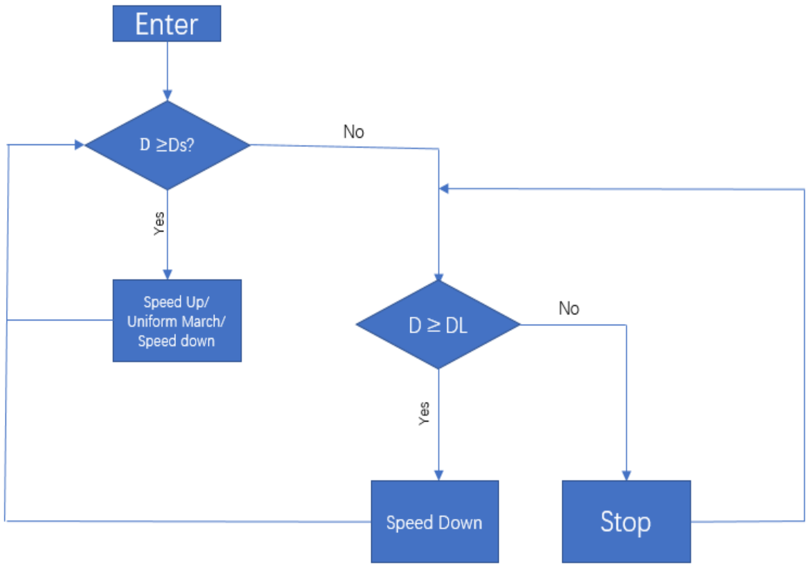

Figure 2. The Car-following model

4.1.2 Lane-changing Model

In the case of confluence, we consider the lane changing model of the vehicle in the departure zone, since the running state of the departure zone is similar to the confluence zone.[2]

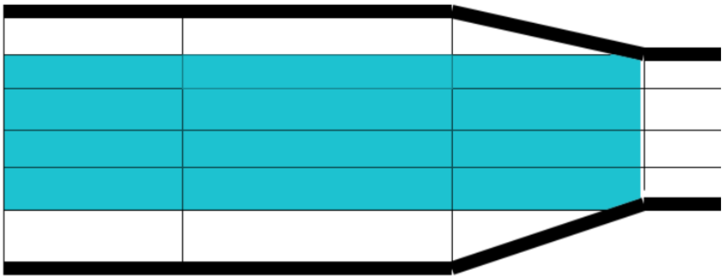

Figure 3. The simple diagram of toll plaza

We make the following assumptions for the model:

The vehicles in the blue area do not change their lane.

The vehicles in the white area of recovery and departure zone have to change their lane if possible.

Active lane-changing model

Some straight-ahead vehicles have to change to the inner lane in order to avoid the impact of weaving vehicles. And it's different with passive lane-changing that the active lane-changing vehicle has the process of generating the lane-changing motive. If the following conditions are met, they will have a certain probability to change to the inner lane.

Passive lane changing model

If vehicles are not on the L roads and has not entered the L roads yet at the end of the confluence zone, they must stop, and wait into the L roads. This model is passive lane-changing model.

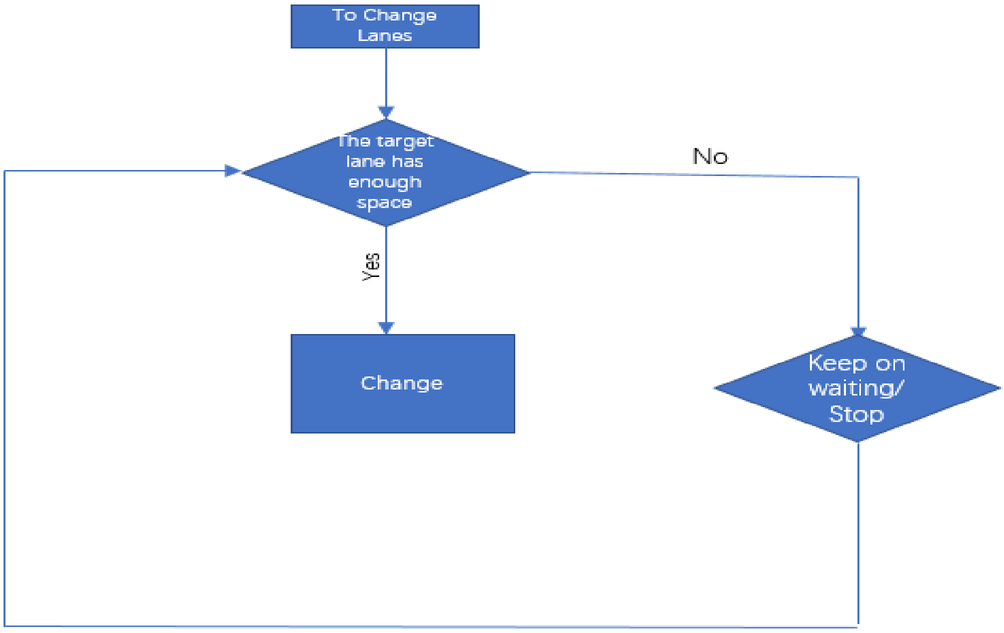

Figure 4. The Lane-changing model

4.2 Traffic-Flow Model

We discuss the microscopic model of the traffic in the previous section. Now we discuss the macroscopic model of the traffic.

According to the relevant literature, relationship between fluid velocity and density is as follows

$$u = \begin{cases} \frac{1}{2}u_f(1+\frac{k_f}{k}) & 0 < k < \frac{1}{4}k_j - k_f \\ \frac{1}{8}u_f k_j & \frac{1}{4}k_j - k_f \le k \le \frac{1}{2}k_j \\ \frac{1}{2}u_f(1-\frac{k}{k_j}) & \frac{1}{2}k_j < k \le k_j \end{cases} \quad (1)$$

We make the following assumptions for the model:

The traffic flow cannot be compressed during congestion

The density of the traffic flow remains essentially constant at the exit when crowded

The traffic flow is uniformly distributed in the recovery and departure zone

We only consider the situation where the traffic flow is high, So we can draw the conclusion that

$$u = \frac{1}{2}u_f(1-\frac{k}{k_j}) \quad (2)$$

4.3 Model Solving

When the traffic flow is low, there will be no traffic jams, so we discuss the situation where the traffic flow is high.

Then we take the cost into account. The area of the recovery and departure zone S should make the get the minimum value.

We simulate the cellular automata model in MATLAB, the following figure is obtained.

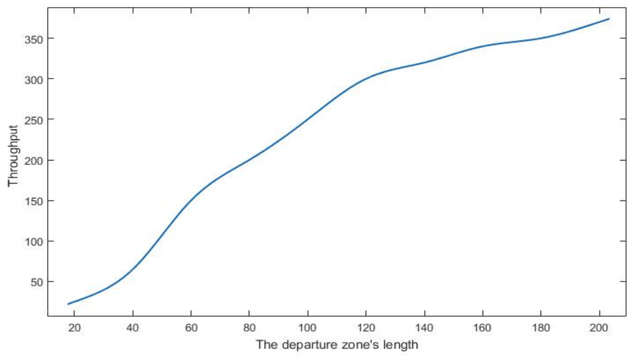

Figure 5. The departure zone’s length and throughput

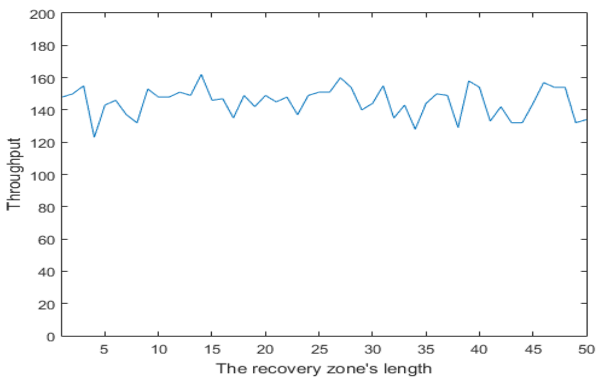

Figure 6. The recovery zone’s length and throughput

Then, the shape and size of toll plaza are determined by considering construction cost, throughput and safety factor. According to the result got from previous sections[3], we can draw the conclusion that the size of the recovery and departure zone is $S_{\min}$.

$$Q = u T \tag{3}$$

$$Q = SK \tag{4}$$

$$u = \frac{1}{2} u_f (1 - \frac{k}{k_j}) \tag{5}$$

We define BRD as the overall evaluation index of the recovery and departure zone (RD Zone).

$$BRD = SC\beta_1 + \beta_2 T \tag{6}$$

$$T = \frac{2Q}{\mu_f (1 - \frac{Q}{Sk_j})} \tag{7}$$

$$BRD = SC\beta_1 + \frac{2\beta_2 Q}{\mu_f (1 - \frac{Q}{SK_j})} \tag{8}$$

β_1 and β_2 are different for different evaluation system. We define $\beta_1 = \beta_2$ =0.5 here.

When BRD gets the minimum value, we get the optimal solution. The results are as follows:

$$S = \frac{Q}{K_j} (\sqrt{\frac{2\beta_2}{\mu_f \beta_1 C}} - 1) \tag{9}$$

$$\frac{(Bl_1 + Ll_2)h_1}{2} + Bl_1 h_2 = \frac{Q}{K_j} (\sqrt{\frac{2\beta_2}{\mu_f \beta_1 C}} - 1) \tag{10}$$

l_1:The width of the tollbooth

l_2:The width of the road

h_1:The length of the recovery zone

h_2:The length of the departure zone

By the Cellular Automata Model we can draw the conclusion that the longer of the length of departure zoon, the better the toll plaza works when the number of tollbooths and lanes is fixed.

Consider the situation where h_2=0, we can regard h_1 as ideal length of the departure zone.

4.4 Safety Factors

It also has influence of safety factors on the models. Define the number of hundred cars conflict(times/100) f for the safety factor evaluation index.

$$f = 100 * TC/Q \tag{11}$$

TC: the sum of the rear-end conflicts' number and lane changes' number

Q: traffic equivalent

Taking the safety factors into account, the length of recovery zone(h_2) has big influence to the variable Q. TC can be got from statistics. We see TC as constant. he length of the recovery zoon has positive negative correlation with safety. Taking cost into account, modeled on the method of the previous section. We can draw the conclusion that the ideal length of recovery zone is BL1.

4.5 Conclusion

According to the previous results we got. We can draw the conclusion that the length of recovery zone has little relevance with traffic equivalent. The length of the recovery zoon has positive negative correlation with safety. And the length of departure zoon has positive correlation with throughput. Taking safety factors into account, the longer the departure zone is, the safer it is.

5. Entrance Model

5.1 The Capacity of Mixed Channel

For E-ZPass and MTC mixed channel, according to the layout of the channel, the mixed channels are designed primarily with E-ZPass channel standards, and they generally meet the fee requirements of the E-ZPass vehicle. When the channel is only used by the E-ZPass vehicle, its capacity is the capacity of the E-ZPass channel. And when the channel is mixed, the capacity of the channel will decrease gradually as the proportion of MTC vehicles in the channel increases.[4] So when the channel is all used by the E-ZPass vehicle, its capacity is the capacity of the E-ZPass channel. And when the proportion of MTC vehicles is lower, the reduced rate in the capacity of the mixed channel will be smaller. So when the proportion of MTC vehicles is higher, the reduced rate in the capacity of the mixed channel will be higher.

Therefore, the capacity of the mixed channel will decrease with the increase of the proportion of MTC vehicles, and the rate of change gradually increased from 0, and then gradually become smaller. The process of that change can be approximated by the cosine function to represent, so the capacity of the mixed channel can be expressed as follows:

$$C_m = \frac{C_e - C_m}{2} COS(B\Pi) + \frac{C_e + C_m}{2} \tag{12}$$

B: The proportion of MTK vehicles in mixed tollbooth

ATC channels

The capacity of the ATC channels is as the MTC channels. Take the serve time of ATC into account, the capacity of the ATC channel.

5.2 The Capacity of the Toll

The capacity of the toll station system is generally obtained by stacking the capacity of multiple channels. Define C to the capacity of the toll.

$$C = C_m + C_e + C_a + C_{mi} \tag{13}$$

Due to the process of MTC charges slower vehicles and the need for parking, the traffic flow of E-ZPass has little influence to the traffic flow of MTC. However, the traffic flow of MTC has big influence to the E-Zpass traffic flow. We introduce correction factor to optimization the model.

1) Ne>L, E-ZPass tollbooths are in the center:

$$C_e = C_s(n_s - 1 + \alpha_e) \tag{14}$$

2) Ne<L, E-ZPass tollbooths are in the center:

$$C_e = C_s(n_s + \sum_{j=1}^{n_R - 1} f_{Nj} + f_{Nn_R} \cdot \alpha_e) \tag{15}$$

3) Ne<L, E-ZPass tollbooths are in the lateral:

$$C_e = C_s(n_s + \sum_{j=1}^{n_g} f_{Nj}) \cdot \alpha_e \tag{16}$$

4) Ne>L, E-ZPass tollbooths are in the lateral:

$$C_e = (C_s + C_s \sum_{j=1}^{n_g - 1} f_{Nj}) \cdot \alpha_e \tag{17}$$

Do the same optimization procedure to other kinds of tollbooths. We can get the most optimization model.

5.3 The New Toll Plaza Model

According to the 5.2 section and the 4 section, we design a new type of toll plaza which is different from the existing toll in the New Jessy.

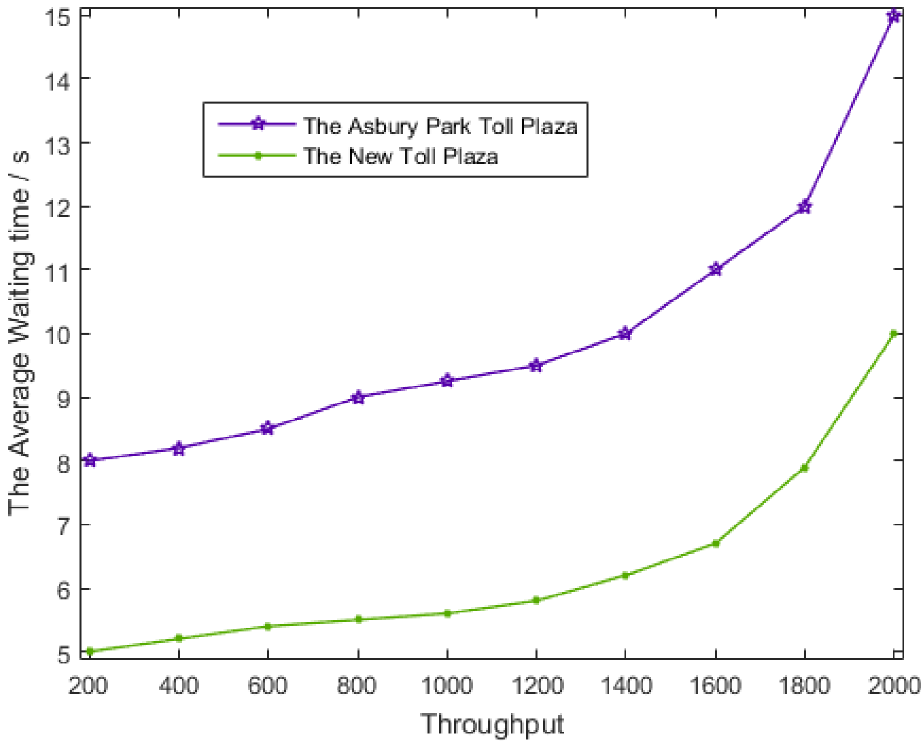

Figure 7. The comparison of two plazas

5.3.1 The Schematic of the Toll Plaza

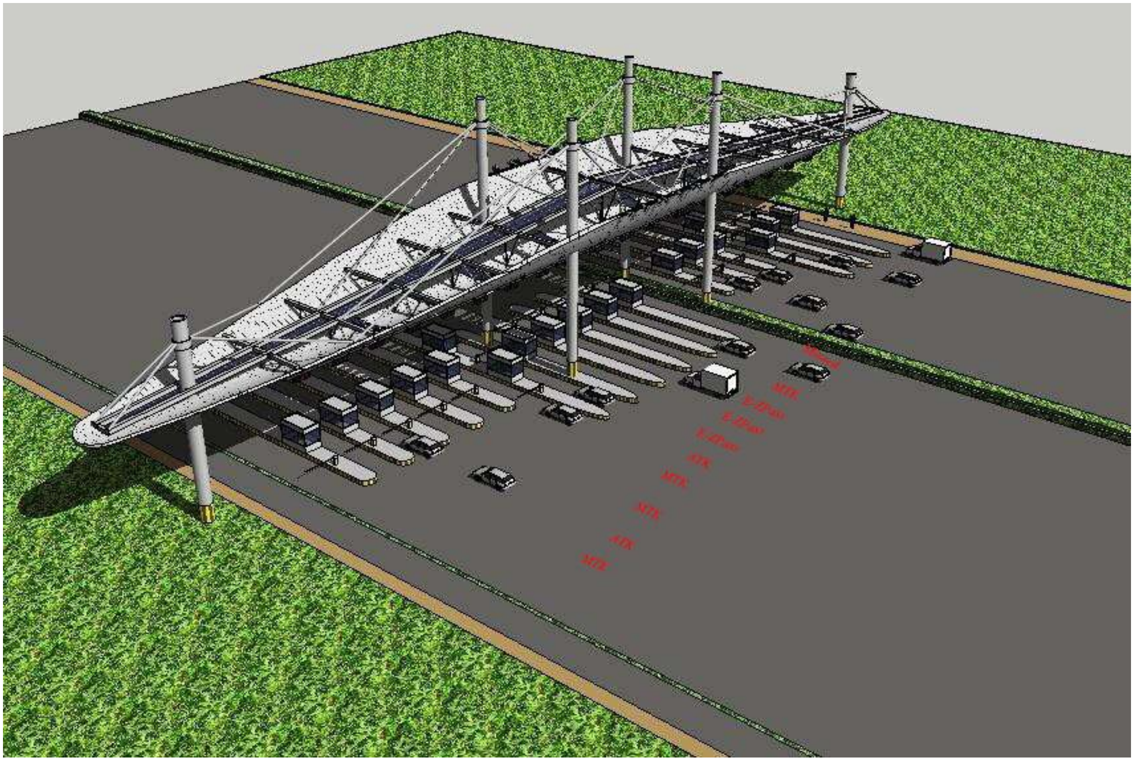

Figure 8. the schematic of the toll plaza

5.3.2 The Design Principles of the New Toll Plaza

1)the number of different tollbooths

Define β_1 β_2 β_3 β_4 as the average capacity of different tollbooths, A B C D as different number of different tollbooths.

$$A+B+C+D=BN \tag{18}$$

BN: the number of the tollbooths

Then we through linear programming get the maximum of the following equation

$$CM=\beta_1 A+\beta_2 B+\beta_3 C+\beta_4 D \quad (19)$$

CM: the biggest capacity of the toll

2)the distribution of different tollbooths

According to the 5.2 section, we should minimize the correction factor. So we design the E-ZPass tollbooths locate in the center area to reduce the interference rate of vehicles to maximize the capacity.

6. Conclusion

According to the material, and combined it with actual examples from life. We thought of a novel idea about the toll station.

Vehicle congestion at the toll station is mainly due to the long service time. However, the cash payments always lead to the time of change longer, and result in the longer service time. And nowadays, more and more people are accustomed to paying online. Hence, we think it is convenient to put the electronic payment applied into toll system. If so, the service time will be reduced greatly, and the vehicle congestion will be improved.

References

[1] Mallet D G, De Pillis L G. A cellular automata model of tumor–immune system interactions[J]. Journal of Theoretical Biology, 2006, 239(3): 334-350.

[2] Ben-Akiva M E, Choudhury C, Toledo T. Lane changing models[C]//Proceedings of the International Symposium of Transport Simulation. 2006.

[3] Gifford J, Yermack L, Owens C. E-ZPass: Case study of institutional and organizational issues in technology standards development[J]. Transportation Research Record: Journal of the Transportation Research Board, 1996 (1537): 10-14.

[4] Wu D, Yin Y, Yang H. The independence of volume–capacity ratio of private toll roads in general networks[J]. Transportation Research Part B: Methodological, 2011, 45(1): 96-101.