

Joint optimization of Maintenance Force and Decision-making Time of Spare Provision for k-out-of-n System in Wartime

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Abstract

To deal with the uncertain response delay problem of the spare part provision for equipments in wartime, a maintenance strategy is proposed with considering the properties of k-out-of-n system and finite maintenance capacity. The adjustment factor is introduced for the uncertain delay on transforming the spare parts. A mathematical model for the joint optimization of maintenance decision-making and spare parts provision is developed to maximize the system availability and minimize the maintenance cost simultaneously, and maintenance strategy parameters, spare part inventory levels and spare part repair capacity are used as variables. The marginal benefit method is adopted to solve the proposed model, and the optimal declaration time of spare parts, as well as the optimal maintenance force configuration scheme is obtained. Finally, the feasibility of this method is verified through the instance, and the sensitivity analysis of the parameters is conducted.

Keywords

k-out-of-n equipment system; uncertain delay; adjustment factor; decision-making time; joint optimization; marginal benefit method.

1. Introduction

This paper studies the dynamic characteristics of the task completion capability of the k/N(G) equipment system when given the maintenance support strategy in wartime. The equipment system is characterized in that in the N units that make up the equipment system, the number of units that must work normally must be no less than k, otherwise the equipment system is shut down. This is similar to the wartime equipment integrity rate requirement, so it is of great practical significance to study the spare parts guarantee of the system. For the mission requirements of the wartime equipment group, the spare parts guarantee process focuses on the determination of the spare parts supply decision timing and the amount of spare parts carried, and the maintenance personnel's equipment under the given maintenance strategy. In this paper, based on the above three aspects, a joint optimization model is established for spare parts supply and spare parts maintenance, and the improved marginal benefit method is used to finally obtain the best system cost-effectiveness ratio.

Relevant research at abroad is mainly divided into the following two categories. In the study of the relationship between system reliability and spare parts carrying capacity in wartime, the literature [1] considers the four kinds of index constraints of spare parts for the ship spare parts carrying scheme, and uses Lagrang The daily constraint factor converts the indicator into a resource constraint and establishes an availability model for the equipment system. The literature [2] considers the cold reserve and heat reserve of the equipment system based on the METRIC model principle, establishes the mission success model of the two-level guarantee system, and uses the marginal analysis method to derive the optimal spare parts configuration plan, but the literature does not consider the situation of spare parts maintenance. In [3], the k/N(G) equipment system for repairable parts is studied. The influence of spare parts response time on task completion rate is mainly studied. The task completion rate and spare parts response time, spare parts carrying capacity and The relationship between spare parts repairs. However, the research hypothesis is that the maintenance team will start repairing the faulty parts when there are no spare parts available. It is relatively simple to consider the repair of

spare parts. The literature [4] considers the preventive maintenance requirements, and the system establishes spare parts under the premise of implementing the maintenance and repair strategy. The demand quantity calculation model is proposed and the model is solved based on the discrete method. Literature [5] studied the relationship between equipment availability and system spare parts satisfaction rate, established the system's spare parts satisfaction rate model, and used marginal analysis to determine the optimal spare parts configuration plan.

For the joint optimization research of equipment maintenance strategy and spare parts, it is mainly to jointly optimize the post-fault maintenance or preventive maintenance strategy and the spare parts supply problem. In [6-9], for the $k/N(G)$ equipment system of a given maintenance support strategy, the reparability of repairable parts and the carrying capacity of spare parts are jointly optimized, and the marginal analysis method is used to find the optimal availability of the system. Configuration scheme. However, in the calculation of spare parts repair time model, the relationship between maintenance capacity and maintenance man-hour is not further refined, so that the calculated value has deviation from the actual. In [10], the $k/N(G)$ system was replaced by the service age under the condition-based maintenance strategy. The analysis of the maintenance strategy, the spare parts inventory level, and the maintenance capability and the priority of the maintenance work on the system availability were established. The availability model is validated using simulation methods. In [11], the conditional maintenance of the single-component $k/N(G)$ system and the grouped preventive replacement of the multi-component $k/N(G)$ system were studied, and the marginal analysis method was used to determine the optimal configuration scheme. In [12], the $k/N(G)$ system under the preventive replacement of service age and the economic inventory strategy of spare parts was studied. The multi-objective programming equation was established with the constraint of preventive replacement time interval and ordering parameters, so that the system satisfies the availability. In the case, the long-term average cost is the smallest. In [13], for the $k/N(G)$ system in series mode, considering the reliability of components and the availability of spare parts, the system uses the availability model, and uses the continuous time Markov chain analysis system spare parts guarantee process, and draws The system with the best availability.

From the above analysis, most of the current literature is based on the premise of unrepairable parts, to study the number of spare parts needed for the update process; or to study the repairable spare parts system, the conditions for consideration are simple, and the actual maintenance of equipment maintenance is large, resulting in spare parts maintenance. The efficiency is low and the spare parts are carried in large quantities. For wartime $k/N(G)$ equipment systems are less involved, and the timeliness of spare parts support is high during wartime, the maintenance capacity of spare parts and the carrying capacity of spare parts are limited. Therefore, this paper mainly focuses on the characteristics of wartime equipment systems. The following joint optimization model.

2. Joint Optimization Model

2.1 Problem Description and Assumptions

This paper regards the wartime equipment group spare parts guarantee problem as the $k/N(G)$ system and its spare parts maintenance guarantee process. The system contains N identical equipment, and the wartime equipment group at least guarantees $k(k>0)$ The equipment works normally, and its fault time is subject to the exponential distribution of parameters λ and independent of each other. The wartime equipment maintenance adopts replacement parts repair, and the spare parts are repairable parts. Considering that the spare parts response process requires delays caused by equipment and spare parts transportation, the $k/N(G)$ equipment system adopts a pre-repair strategy, that is, when $m(m \leq N - k + 1)$ equipment failures, the equipment commander makes spare parts support decisions, and the equipment maintenance unit dispatches The maintenance personnel carry out the replacement repair, the spare part response time is T_i , during which the equipment system continues to work normally until the remaining good equipment is less than k , the system stops. When the spare parts are ready, the system will immediately stop, and the maintenance personnel will carry out

replacement repairs over T_r time, and the equipment system will enter the next work cycle. At the same time, the maintenance personnel will repair the replaced defective spare parts until all the defective parts are repaired or the maintenance start conditions of the next cycle are reached. The repair time is subject to the exponential distribution with the parameter μ , and the spare parts repair is put into the spare parts library if it is new. When the spare parts available in the spare parts library are insufficient, the maintenance personnel need to repair the faulty spare parts to meet the replacement of this stage, and then carry out replacement parts repair of the equipment system.

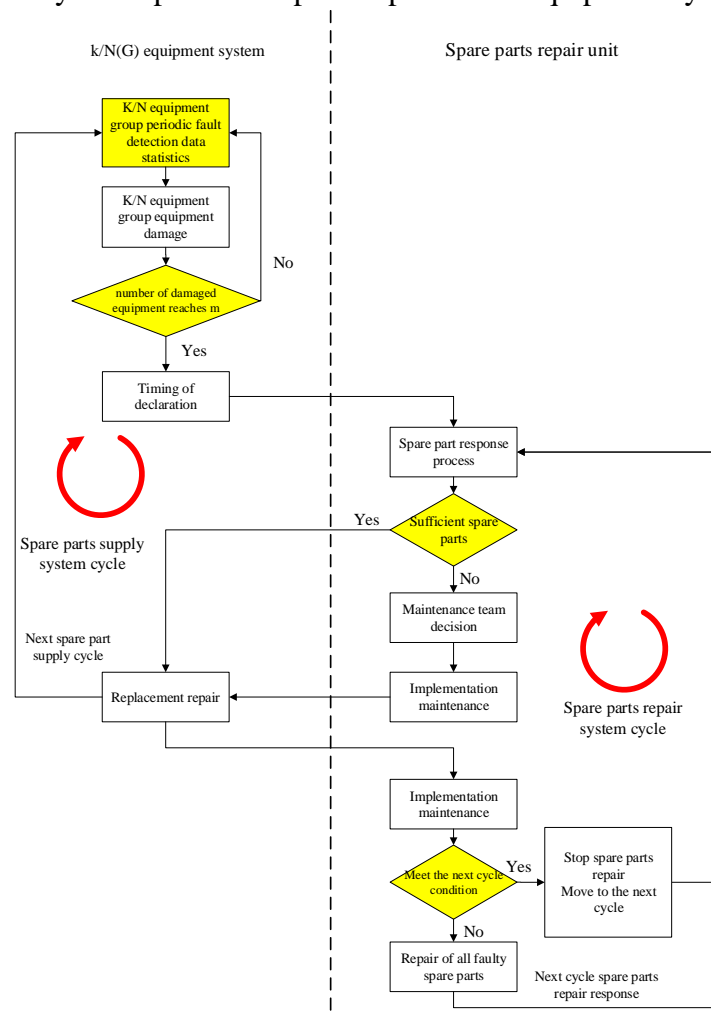


Fig. 1 k/N (G) equipment system equipment maintenance flow chart

As can be seen from the above figure, the spare parts supply process and the spare parts repair process have crossovers in the response and replacement of spare parts, but the two belong to different system cycle processes. Therefore, the dynamic relationship of the spare parts maintenance system when the equipment is replaced should be analyzed.

As shown in Fig. 2, the time allocation of the two systems in the running process has crossover. For the k/N(G) equipment system, one operating cycle includes normal working time T_w , spare parts response time T_l , supplementary fault spare parts repair time t . Replacement repair time T_r ; spare parts repair system includes spare parts repair waiting time T_s , spare parts response time T_l , supplementary fault spare parts repair time t , equipment replacement repair time T_r , fault spare parts repair time T_m .

Taking the above-mentioned spare parts application supply strategy and maintenance strategy, it can be known that for the k/N (G) equipment system, if the number of faulty equipment is a state variable,

the process from state 0 to state m is X, from system m to state 0. This process is Y, then the operation of the k/N(G) equipment system during the mission is a process in which X and Y alternate.

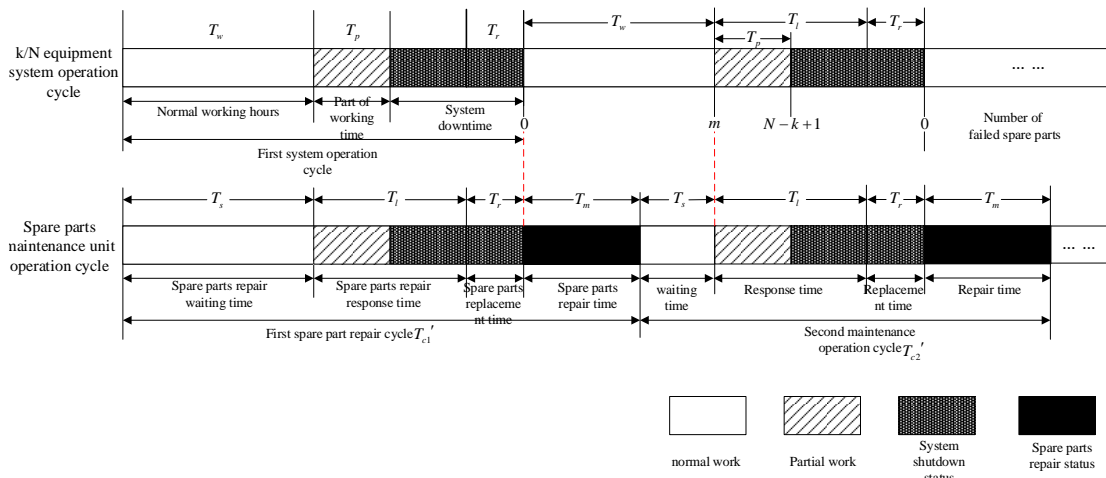


Fig. 2 Analysis of k/N(G) equipment system and spare parts maintenance process

2.2 Model Establishment

2.2.1 Use Availability Calculation

Assuming that the wartime equipment task time is T , and the k/N(G) equipment system has a single operating period of T_c , the equipment system availability A_o can be expressed as:

$$A_o = \frac{E(T_w) + E(T_p)}{E(T_c)} \tag{1}$$

In the formula, T_w is the time between the normal operation of the equipment system and the spare parts supply decision; T_p is the time from the supply of spare parts to the time of system shutdown.

2.2.2 Calculation of the Total Cost of the Equipment System

The expected cost per unit time of the k/N(G) equipment system in the specified mission time includes the purchase cost of a single spare part C_b ; the storage and depreciation expense per unit time C_s ; the start-up cost of a single repair C_q ; the replacement cost of a single faulty spare part C_r ; Maintenance staff costs C_p .

$$C = \frac{S \cdot C_b}{T} + S \cdot C_s + \frac{(m + B_p) \cdot C_r + C_q}{T_c} + c \cdot C_p \tag{2}$$

$$= \frac{S \cdot C_b}{T} + S \cdot C_s + \int_{T_l + p \cdot t_l}^{T_l + p \cdot t_l + t} \frac{(m + B_p) \cdot C_r + C_q}{T_w + T_l + T_r + \sum t} \cdot f(T_l) dT_l + c \cdot C_p$$

2.2.3 Establish a Joint Optimization Model

Taking the typical k/N (G) equipment system in wartime as an example, when the number of fault preparations in the equipment system reaches m, the spare parts response starts, and after T_l time, the equipment system is replaced and repaired, and then enters the next cycle, and the maintenance system performs faulty spare parts. Repair until the next cycle of spare parts supply decision timing. According to the mission requirements of the wartime equipment system, under the condition of satisfying the usability of the equipment system, the equipment system has the highest usability and the total maintenance cost is the smallest. In the two cycle systems of spare parts supply and spare parts maintenance, the spare parts supply decision condition m, the maintenance personnel c, and the

initial value of the spare parts library S are the key parameters for different situations, and also the decision variables for the target optimization, then the following optimization equations are available:

$$\begin{cases} \max A_o; \min C \\ \sum (E(T_w) + E(T_p)) \leq T \end{cases} \quad (3)$$

3. Model Solving

The most common solution to equation (15) is the marginal effect method, which is a typical greedy algorithm. Each calculation compares and concludes the maximum benefit from the unit product. The essence of the marginal effect method is the cost-benefit ratio analysis. The constraint equation in the paper is the availability of the equipment system, which belongs to the system's benefit. The objective function of this paper is to make the total downtime and total cost, which belongs to the system cost. The core idea of marginal analysis is to calculate the increase in cost and benefit each time by adding one unit of product, and finally choose the one with the most cost-effective ratio. The main step is to calculate the marginal variation of each decision variable at each logical iteration, and pick out the scheme that uses the greatest increase in availability. Therefore, before the calculation, it is necessary to clarify the logical relationship of the system and the traversal of the decision variables.

Such models can be optimized by the marginal effect method and selected as the marginal efficiency factor. Formula (4) represents the unit cost supply efficiency obtained by appending the unit variable when a decision variable is selected. By comparing the values of each decision variable in each cycle, the decision variable that has the greatest influence on the supply efficiency of the equipment system is determined, and the adjustment weight ratio of the decision variable is considered to be the largest, and the number of other decision variables is increased by one. constant. After several iterations, the spare parts supply system meets the target requirements.

$$\Delta = \max \left\{ \frac{A(m_i) - A(m_i - 1)}{C(m_i) - C(m_i - 1)}, \frac{A(S_i) - A(S_i - 1)}{C(S_i) - C(S_i - 1)}, \frac{A(c_i) - A(c_i - 1)}{C(c_i) - C(c_i - 1)} \right\} \quad (4)$$

4. Case Analysis

In the wartime, a certain type of self-propelled artillery group carried out a total of 200 combat missions, requiring the equipment group to maintain a 90% readiness and integrity rate when performing missions, and the mission was up to $T = 1000$ hour. In addition, the self-propelled gun failure obeys the exponential distribution with the parameter $\lambda = 0.0008$ (h), and the required spare parts repair time obeys the exponential distribution with parameter $\mu = 0.1$, the spare part response time parameter of the required spare parts is $T_o = 150$, $p = 0.5$, $\mu_l = 50$, $\sigma_l = 5$, $t_l \in [0, 50]$, and the unit spare parts replacement time is $s_r = 0.2$ hours/person. The spare parts maintenance unit has a maximum of $S \leq 100$ spare parts and $c \leq 4$ maintenance personnel. The cost parameters required for replacement and repair of spare parts are $C_b = 100$, $C_s = 50$, $C_q = 5000$, $C_r = 100$, $C_p = 200$:

According to formula (3), the scheme with the highest performance factor at each stage is selected. As the optimal scheme for this stage, using the availability and cost as the initial target value of the next stage, conclusions can be drawn, as shown in Table 1.

It can be seen from Table 1 that when $c = 3$ and $c = 4$, the increase of the usability of the equipment system is not large, and the availability of the equipment of the four schemes exceeds 0.90, which can be used as a wartime spare parts guarantee scheme. If the economy is the best if the availability of equipment is met, the best option is $m = 6$, $S = 39$, $c = 1$, $A_o = 0.9054$, $C = 241.7099$. If the cost of military economic benefits is neglected in wartime and the equipment is used as much as possible, the best solution is $m = 1$, $S = 98$, $c = 4$, $A_o = 0.9214$, $C = 865.227$. Under the premise of Safing the equipment usage, this paper considers the availability of economy and equipment, so that the military

benefits are optimal. From the slope calculation formula of the cost availability curve, it can be concluded that when the slope value of the curve the maximum value, the best guarantee program is the spare parts guarantee program $c = 2$.

Table 1. Optimal value of each stage of marginal analysis

	Initial targeting value	m	S	c	A ₀	C
stage 1 (c=1)	m=0, S=0, c=1 A1=0.2280, C1=215.0376	6	39	1	0.9054	241.7099
stage 2 (c=2)	m=6, S=39, c=1 A1=0.8754, C1=241.7099	2	71	2	0.9159	454.8597
stage 3 (c=3)	m=2, S=71, c=2 A1=0.8909, C1=454.8597	2	100	3	0.9207	664.8248
stage 4 (c=4)	m=2, S=100, c=3 A1=0.8917, C1=664.8248	1	98	4	0.9214	865.227

5. Sensitivity Analysis

In order to further analyze the influence of uncertain delay time parameters on the decision-making plan for spare parts supply, the relative error is used to calculate the cost rate. the formula is as follows:

$$RE = \left| \frac{W'^* - W^*}{W^*} \right|, W^* = \frac{A_0}{C} \tag{5}$$

W^* is the cost-effectiveness ratio of the optimal spare parts support scheme for the comprehensive military benefits in this paper, W'^* is the optimal cost-effectiveness ratio after the relevant parameters are changed. The parameters involved in this paper are p, μ_l, σ_l . the influence of the parameter changes on the cost rate is calculated by using the single factor test method. the influence results of RE are shown in table 2.

It can be seen from Table 2 that when $p \geq 0.8, \mu_l \geq 50, \sigma_l \geq 6$ is calculated, the calculated availability of equipment cannot meet the task requirements, and therefore, there is no optimal solution that satisfies the condition. Because the probability of delays in supply and transportation increases, the probability of delay increases, and the delay correction factors μ_l, σ_l determine the amount of delay time. The larger μ_l, σ_l , the longer the delay, the greater the cumulative number of equipment failures during the delay, the equipment will continue to malfunction during the transportation delay, resulting in an increase in the total number of faulty equipment when the spare parts arrive, resulting in an increase in the number of maintenance personnel and spare parts. When the number of equipment failures exceeds the equipment maintenance capability, it is calculated that the equipment availability of the scheme cannot meet the mission requirements.

Table 2. Optimal value of each stage of marginal analysis

p	A_0^*	RE	μ_l	A_0^*	RE	σ_l	A_0^*	RE
0	0.9343	0.0162	0	0.9256	0.0243	0	0.9113	0.0051
0.1	0.9246	0.0139	10	0.9287	0.0205	1	0.9083	0.0084
0.2	0.9189	0.0099	20	0.9201	0.0173	2	0.9074	0.0118
0.3	0.9117	0.0074	30	0.9165	0.0138	3	0.9069	0.0149
0.4	0.9061	0.0057	40	0.9116	0.0116	4	0.9062	0.0177
0.5	0.9054	0	50	0.9054	0	5	0.9054	0

0.6	0.9032	0.0059	60	-	-	6	0.9046	0.
0.7	0.9006	0.0077	70	-	-	7	-	-
0.8	-	-	80	-	-	8	-	-
0.9	-	-				9	-	-
1	-	-						

6. Conclusion

In this paper, the typical repair process of k/N(G) equipment system in wartime is studied. Considering the delay caused by spare parts response, the pre-maintenance strategy is proposed, and the dynamic law of equipment failure is realized by steady-state Poisson process and update process. The analysis is carried out, and the repair time of the faulty spare parts is calculated by the M/M/C model in the queuing theory. The joint optimization model of spare parts supply and spare parts maintenance strength is established, and the improved marginal benefit method is used to optimize the scheme. It lays a theoretical foundation for solving the spare parts supply and maintenance system optimization of wartime equipment groups, and has certain practical significance.

In addition, based on the research of this paper, we can also conduct in-depth research from the following aspects: (1) consider the situation of imperfect spare parts maintenance; (2) consider the optimization of spare parts in the case of horizontal replenishment between multi-level equipment groups.

References

- [1] Ruan. Optimization method of spare parts support scheme for ship equipment carrying multiple constraints[J]. *Acta Armamentari* ,2013,34(9):1144-1149.
- [2] Wang Rui, Li Qingmin, Yan Zhi. Optimization of repairable spare parts based on the success of combat unit missions[J]. *Journal of Beijing University of Aeronautics and Astronautics*, 2012,38(8):1040-1045.China.
- [3] Dong Yue, Zhang Liu, Meng Mingqiang. The K/N system of Markov process is analyzed using task completion ability [J]. *Firepower and Command and Control*, 2012, 37(3): 72-75.
- [4] Hu Qiwei, Jia Xisheng, Zhao Jianmin. Calculation model of spare parts demand considering preventive maintenance[J]. *Acta Armamentari*, 2016, 37(5): 916-922.
- [5] Zhai Manzhi, Li Qingmin, Peng Yingwu et al. Spare part satisfaction rate model and optimization method for arbitrary structural systems[J]. *Systems Engineering Electronics*, 2011, 8: 1799-1803.
- [6] Zhang Yongqiang, Xu Zongchang, Hu Kaikai. Joint optimization of maintenance time and spare parts carrying capacity of k/N system[J].*Journal of Beijing University of Aeronautics and Astronautics*,2016,42(10):2189-2196.
- [7] Sleptchenko M C, van der Heijden M C, van Harten A. Effects of finite repair capacity in multi-echelon, multi-indenture service part supply systems. *Int J Product Econom* 2002;79:209 – 30.
- [8] Sleptchenko A, Van der Heijden M C, van van Harten A. Trade-off between inventory and repair capacity in spare part networks. *J Oper Res Soc* 2003;54(3):263 – 72.
- [9] Sarkar Jyotirmoy, Biswas Atanu. Availability of a one-unit system supported by several spares and repair facilities. *journal of the Korean statistical society*.2010,39:165-176.
- [10]De Smidt-Destombes K S, Van der Heijden M C, Van Harten A. On the interaction between maintenance,spare part inventories and repair capacity for a k-out-of-N system with wear-out. *Eur J Oper Res* 2006;174:182-200.

- [11]De Smidt-Destombes K S, Van der Heijden M C, Van Harten A. joint optimization of spare part inventory, maintenance frequency and repair capacity for k-out-of-N systems. *Int J Product Econom* 2009;118:260 – 268.
- [12]Chen liangpeng, YE zhi sheng, Xie min. joint maintenance and spare component provisioning policy for k-out-of-N systems. *Asia Pac. J Oper Res.*2013,30:256-277.
- [13]Xie wei, Liao haitao, Jin tongdan. Maximizing system availability through joint decision on component redundancy and spares inventory. *Eur J Oper Res* 2014;237:164-176.