

## Research on Road Adhesion Coefficient Based on Tire Model

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### Abstract

In this paper, established three condition of tire dynamic models using “Magic Formula” in the enviroment of Matlab/Simulink. The relationship among longitudinal force, lateral force and alignment torque, and the relationship between slip rate, longitudinal force and lateral force were obtained by simulation analysis. And introduce improved magic formula to reflect the force coupling characteristics of tires under different road adhesion coefficient.

### Keywords

Tire, Magic formula, side slip rate, road adhesion coefficient.

### 1. Introduction

The tire is the only nonlinear component that maintains direct contact with the road. As the support and transmission unit of the vehicle and the pavement, the mechanical characteristics of the tire directly affect the handling and stability of the vehicle, the safety and the peace and smoothness. The tire model describes the longitudinal slip and sideslip characteristics of the tire during steady-state motion, which can be divided into theoretical model, empirical model and semi-empirical model. The commonly used theoretical models include linear model, UA model, Dugoff model[1], brush model and LuGre model[2]. The commonly used empirical models include Burckhardt model[3], K-D model and LC model[4]. The commonly used semi empirical models include magic formula and Unitire model. The study of tire model describing tire movement is the key problem in the research of tire and automobile control technology, and the tire model is a model describing the tire in the motion state, which can be simplified to the analysis and research of two processes.

In vehicle dynamics simulation, the vertical force, longitudinal force, lateral force and alignment torque of the tire play an important role in the ride comfort, handling stability and safety of the vehicle. Because of the tire structure is complex and the dynamic performance is nonlinear, it is the key to select the tire model which is practical and easy to build the vehicle model and simulate the dynamics. At present, the empirical formula or semi-empirical formula corresponding to the most popular theoretical model is the magic formula. Pacejka put forward the empirical tire model based on Magic Formula (MF), and used the combination formula of trigonometric function to fit tire test data. The general expression of magic formula is as follows:

$$Y(x) = D \sin\{C \arctan[Bx - E(Bx - \arctan(Bx))]\} \quad (1)$$

In the formula,  $B$   $C$   $D$  is determined by the vertical load and the obliquity of the tire;  $Y$  is the output variable, it can be the longitudinal force  $F_x$  or the lateral force  $F_y$  or the alignment torque  $M_c$ ;  $x$  is the input variable which can represent side angle  $\alpha$  or the longitudinal slip rate  $s$  of the tire in different cases. Where  $B$  denotes the stiffness factor;  $C$  indicates the shape factor;  $D$  is the peak factor;  $E$  is the curvature factor. Schematic diagram of magic formula is as follows:

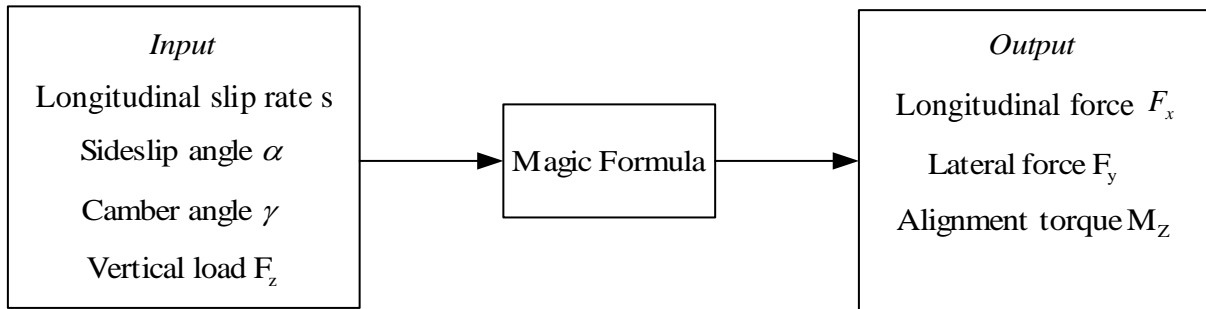


Fig.1 The input and output variables of the magic formula

**2. Establishment of dynamic model of tire**

**2.1 Calculation method of longitudinal force of tire**

Using magic formula to calculate the longitudinal force of the tire is as follows:

$$F_x = D \sin[C \arctan(Bx_1 - E(Bx_1 - \arctan(Bx_1)))] + S_v \tag{2}$$

Where  $x = s + S_h$ ,  $S_h = B_9 F_z + B_{10}$ ,  $S_h$  is horizontal drift of curves and  $S_v$  denotes vertical drift of curves and the value of  $S_v$  is zero in the Eq.(2).  $F_z$  indicates the vertical load of the tire,  $B D E$  are set as:

$$B = (B_3 F_z^2 + B_4 F_z) \times e^{-B_5 F_z} / (C \times D) \tag{3}$$

$$D = B_1 F_z^2 + B_2 F_z \tag{4}$$

$$E = B_6 F_z^2 + B_7 F_z + B_8 \tag{5}$$

Establishing a longitudinal dynamic model through MATLAB and Simulink:

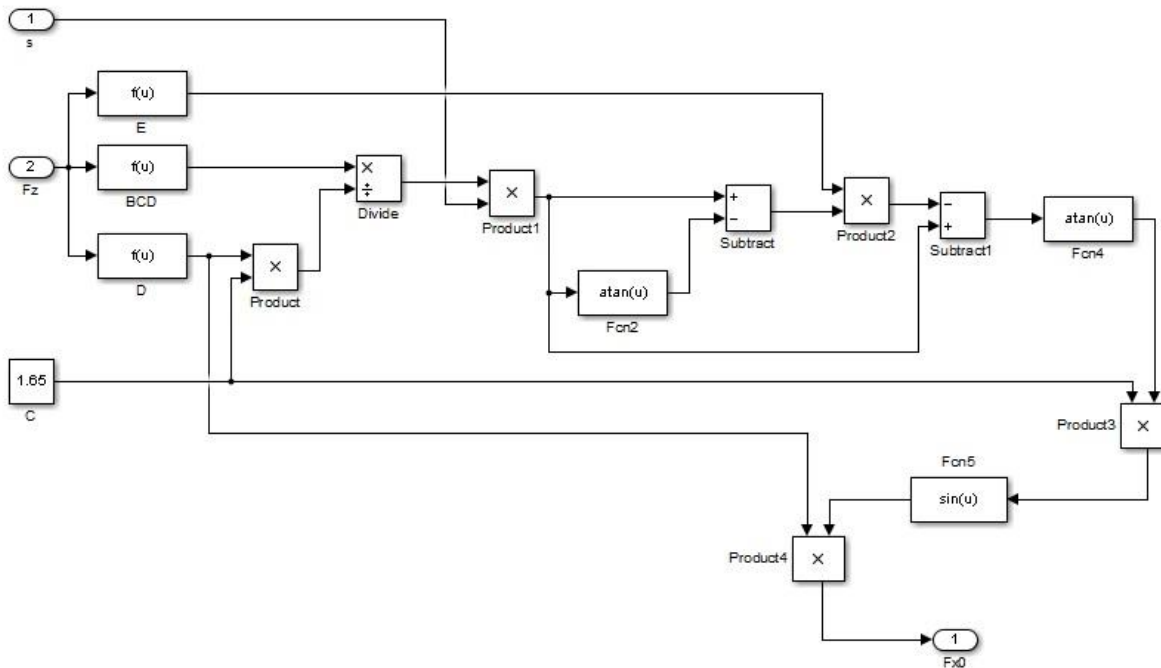


Fig.2 The longitudinal dynamic model

**2.2 Calculation method of lateral force of tire**

Using magic formula to calculate the lateral force of the tire is as follows:

$$F_y = D \sin[C \arctan(Bx - E(Bx - \arctan(Bx)))] + S_v \tag{6}$$

Where  $x = \alpha + S_h$ , the value of  $C$  is  $A_0$  in the Eq. (6).  $S_h$  and  $S_v$  are set as:

$$S_h = A_9 F_z + A_{10} + A_8 \gamma \tag{7}$$

$$S_v = A_{11} F_z \gamma + A_{12} F_z + A_{13} \tag{8}$$

$\gamma$  refers to the camber angle of tire,  $B D E$  are set as:

$$B = A_3 \sin(2 \arctan \frac{F_z}{A_4}) \times (1 - A_5 |\gamma|) / (C \times D) \tag{9}$$

$$D = A_1 F_z^2 + A_2 F_z \tag{10}$$

$$E = A_6 F_z + A_7 \tag{11}$$

Establishing a longitudinal dynamic model through MATLAB and Simulink:

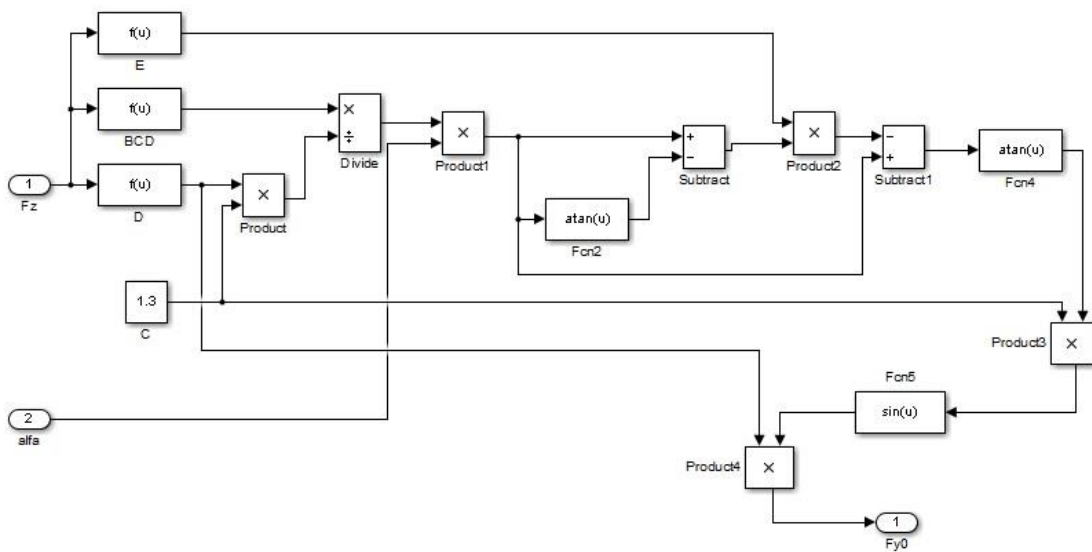


Fig.3 The lateral dynamic model

Establishing a combination of lateral and longitudinal model through MATLAB and Simulink:

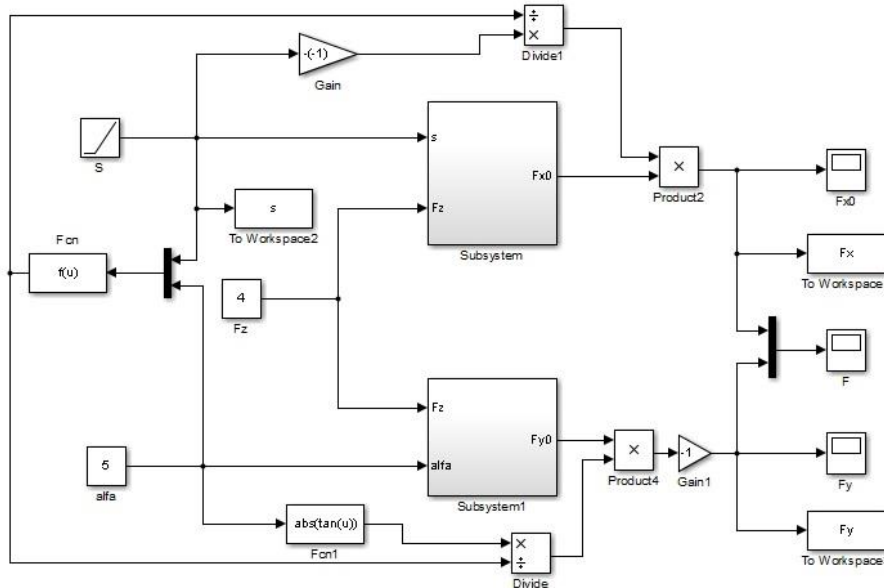


Fig.4 The transverse longitudinal coupling model

**2.3 Calculation method of alignment torque of tire**

Using magic formula to calculate the alignment torque of the tire is as follows:

$$M_z = D \sin[C \arctan(Bx_1 - E(Bx_1 - \arctan(Bx_1)))] + S_v \tag{12}$$

Where  $x_1 = (\alpha + S_h)$ , the value of  $C$  is  $C_0$  in the Eq. (12).  $S_h$  and  $S_v$  are set as:

$$S_h = C_{11}\gamma + C_{12}F_z + C_{13} \tag{13}$$

$$S_v = \gamma(C_{14}F_z^2 + C_{15}F_z) + C_{16}F_z + C_{17} \tag{14}$$

$BCD$  is the torsional stiffness at zero point of alignment torque:

$$BCD = (C_3F_z^2 + C_4F_z) \times (1 - C_6|\gamma|) \times e^{-C_5F_z} \tag{15}$$

The parameter value of magic formula is shown in Table 1:

Table 1 Parameter value of magic formula

Parameter	value	Parameter	value	Parameter	value
				C0	2.34
				C1	1.49
A0	1.65			C2	6.42
A1	-34	B0	2.372	C3	-3.57
A2	1250	B1	-9.46	C4	-0.087
A3	3036	B2	1490	C5	0.098
A4	12.8	B3	130	C6	0.003
A5	0.005	B4	276	C7	-0.001
A6	-0.021	B5	0.088	C8	0.1
A7	0.774	B6	0.004	C9	1.333
A8	0.002	B7	0.061	C10	0.025
A9	0.013	B8	1.2	C11	0.023
A10	0.003	B9	0.03	C12	0.03
A11	19.17	B10	-0.176	C13	0.065
A12	1.213			C14	0.021
A13	6.262			C15	0.895
				C16	-0.099
				C17	-3.337

**3. Simulation graphes and analysis results**

Fig A and B show that the longitudinal force and lateral force are linearly related to the alignment torque in the course of the side angle changing from - 4° to 20° in the case of simple turning. Fig C and D indicate the relationship between vertical force and lateral force and alignment torque when the sides angle is 4°. Fig E and F reveals the conditions of longitudinal force and lateral force when the vehicle is braking and driving.

In order to better reflect the force coupling characteristics of tires under complex working conditions, a improved magic formula tire model was introduced to reflect the road adhesion coefficient:

$$Y(x) = \mu D \sin\{C \arctan[Bx - E(Bx - \arctan(Bx))]\} \tag{16}$$

$$\mu = as + b \tag{17}$$

Where  $a$  is the proportional constant and  $b$  is the bias constant. The slip slope and the tyre road friction coefficient are constant values in ideal conditions with a particular road surface. Based on various simulation experiments, the proportional constant  $a$  and bias constant  $b$  can be found[5]. These constants will not be changed with the vehicle dynamic characteristic or tyre characteristic under the same ideal conditions.

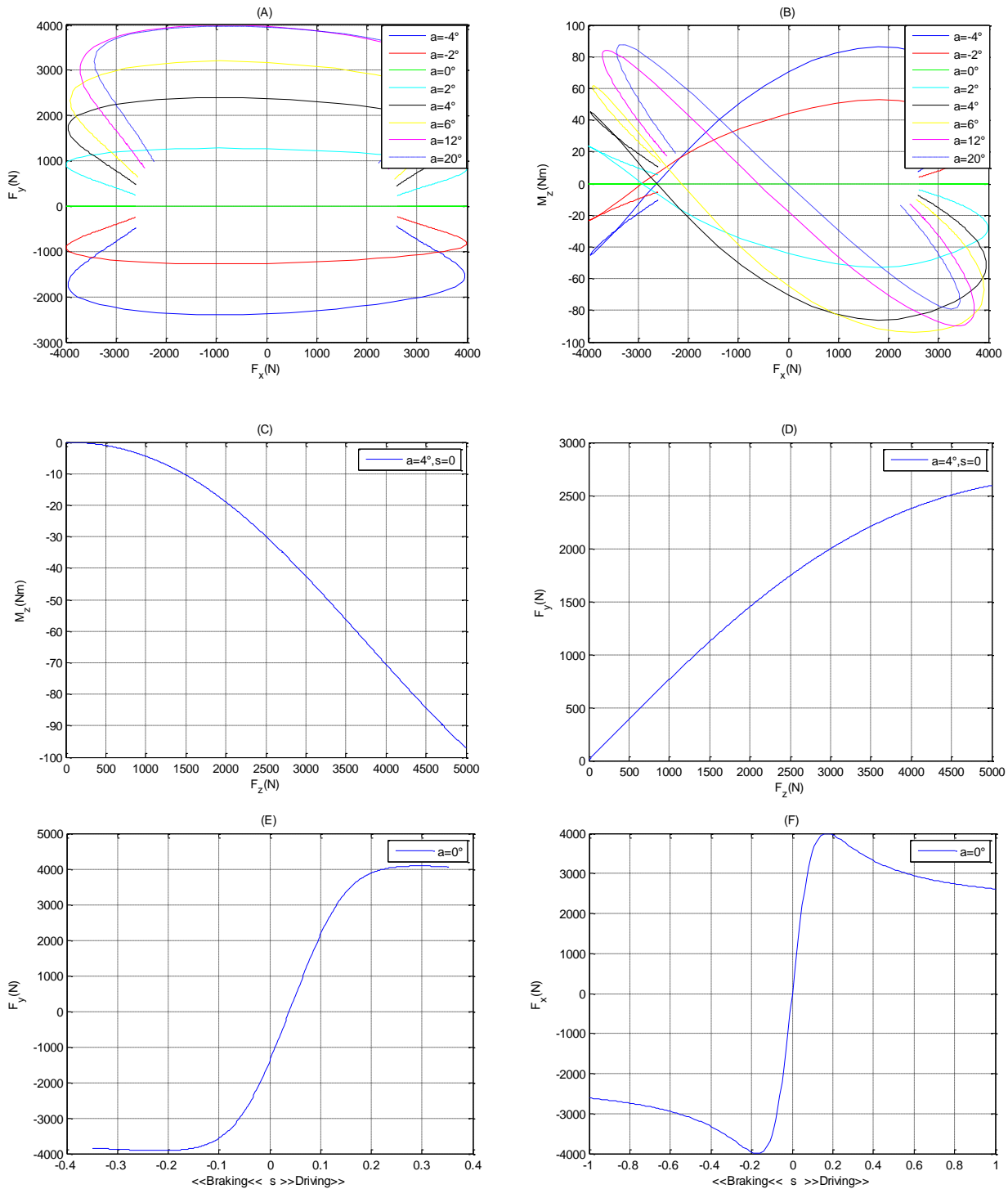


Fig. 5 Simulation graphes

The non-linear characteristics of tires have an important impact on the steering and driving stability of automobiles. The handling stability of automobiles largely depends on the tire cornering characteristics. Improving the tire cornering characteristics can improve the safety of automobiles. Through modeling and simulation under various working conditions, the tire characteristic curves under different working conditions are obtained, and the relationship between the tire state variables is analyzed, which lays a foundation for the study of vehicle dynamics.

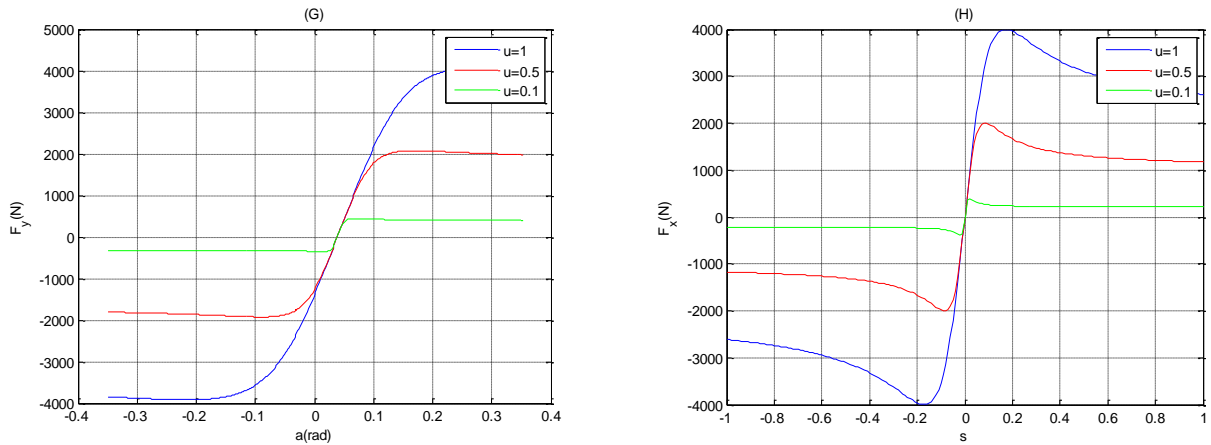


Fig. 5 Simulation graphes

Road adhesion (ratio of tire longitudinal force to vertical load) is a function of tire longitudinal slip rate, so the relationship between pavement adhesion rate and slip rate under direct working condition is set as:

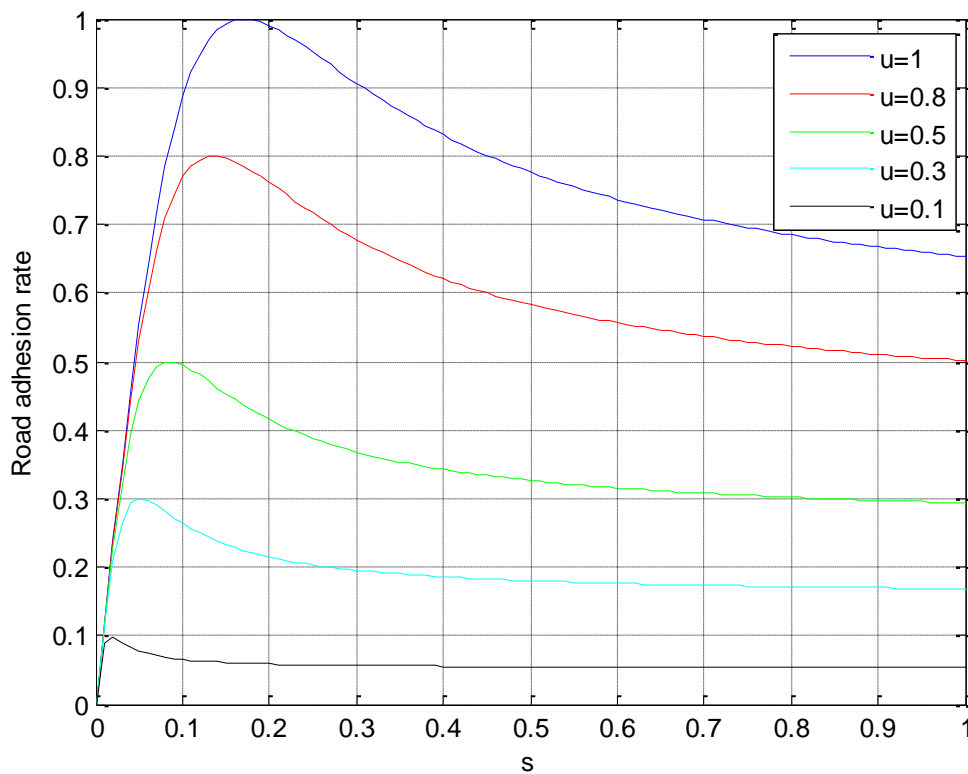


Fig. 6  $\mu-s$  Relation diagram

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