Research on Image Enhancement Technology Based on Fuzzy Mathematics

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Abstract

All images in nature are continuously changing analog images. With the development of computer technology, the images we process are digital. In the process of image generation, transmission and storage, it often receives various noises such as optical noise and quantization noise, which seriously affects the visual effect of the image [1]. In order to reduce noise while maintaining image detail as much as possible, image enhancement technology was born. Due to the uncertainty of noise pollution, the noise source cannot be derived backwards, and the theory of fuzzy mathematics can solve the problem of ambiguity well. The typical fuzzy enhancement method is based on the statistics of all pixels in the global, but ignores the uneven distribution of gray. Therefore, this paper introduces the theory of fuzzy mathematics in the field of digital image processing, and provides theoretical background and method for solving the deterioration of digital image quality caused by uncertain factors.

Keywords

Image enhancement fuzzy mathematics.

1. Introduction

Fuzzy mathematics is a mathematical method for studying and dealing with the concept of ambiguity. As we all know, classical mathematics is characterized by accuracy, while fuzzy mathematics uses precise mathematical methods to deal with fuzzy things that could not be described mathematically in the past [2-4].

Some things can be clearly defined by a precise standard to determine whether they are clear assertions. Such things are called clear things, and some foods cannot find their precise classification criteria. The generics of this food are Gradual transition, that is, subordinate to a certain kind of thing to something that does not belong to a certain type of thing is gradually changing, there is no clear boundary between different categories, such things are called fuzzy things. This kind of unclear generic property is called ambiguity. The probability function that measures which kind of thing this kind of thing belongs to is called the membership function.

In the natural sciences, there are fuzzy mathematics in the fields of computer image, automatic recognition of handwritten words, cancer cell recognition, identification and classification of white blood cells, robot control, classification and evaluation of various types of information, and weather forecasting.

2. Blurring of digital images

The digital image is composed of one pixel. For a gray image, if 8-bit encoding is used, there are 256 kinds of quantized values, namely 0-255, a total of 256, white is 255, black is 0, middle The values are all gray, and the measure of gray is fuzzy, so it can be described by fuzzy mathematics [5].

According to the definition and representation method of the fuzzy subset given by Professor Chad, the fuzzy set representation of the digital image is given.

For an image of any sub-pixel $n = M \times \mathrm{N}$, the gray vector $X = [x_1, x_2, x_3 ... x_n]$ is set to be the image normalized pixel, that is $U(x) = normal(x)$, the function value $U(x)$ is the membership degree of the pixel x.

A membership function of a fuzzy set $F(A)$, if the function satisfies the following conditions:

(1) For any x, there are $F(x) \in [0,1]$

(2) If x is monotonous, $F(x)$ is also monotonous

(3) $F(x)$ is an inverse function

Such a membership function $F(x)$ can enhance the image by blurring. The new domain transformed by the membership function is called the fuzzy domain, and the frequency domain of the Fourier transform has the same effect.

3. Transform Domain Image Enhancement Based on Fuzzy Theory

The transform domain image enhancement based on the fuzzy theory is similar to the image enhancement step in the frequency domain. The method transforms the image from the spatial domain to the fuzzy domain through a specific membership function, and enhances the image in the blurred domain with the corresponding enhancement function. The inverse of the membership function converts the image to the spatial domain, which results in an enhanced image.

According to the principle of image enhancement, the image contrast is increased, and the histogram equalization is performed as much as possible to prevent the grayscale pixels from being too dense. Therefore, the choice of membership function needs to consider the above principles.

3.1 Evaluation of fuzzy enhancement effect.

3.1.1 Subjective evaluation of human visual.

Because the results of various image filtering and image processing are ultimately observed by humans, the human eye supervisory feeling can be said to be the most direct and simplest criterion. But this method also has certain limitations. The same image has different feelings in different people's eyes, so it is not enough to rely solely on the human eye.

3.1.2 Fuzzy entropy.

In order to test and evaluate the enhancement effect, the information entropy corresponding to the two fuzzy sets before and after image enhancement is compared. Let the fuzzy set of n elements X be the information entropy.

$$H(X) = \frac{1}{n \ln 2} \sum_{i=1}^{n} S(x_i) \quad S(x) = -x \ln x - (1-x) \ln(1-x)$$

Extend the unary function to a two-dimensional image, that is

$$H(X) = \frac{1}{MN \ln 2} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} S(x_{ij})$$

When $H(X)$ becomes smaller, it indicates an increase in the direction of entropy reduction, and the image becomes clearer.

3.2 Typical algorithm for fuzzy enhancement S.K. Pal algorithm.

In the Pal algorithm, first determine a membership function, as shown in the following equation:

$$p_{ij} = F(X_{ij}) = \left[1 + \frac{L - 1 - X_{ij}}{F_d}\right]^{-F_e}$$

Where F_d is called the fuzzy influence factor, F_e is the fuzzy index factor, where the value is 2, and when the function value is 0.5, the independent variable x is called the transition point. Performing a fuzzy domain filtering operation on the membership function value p by nonlinear transformation, that is, applying a transformation f , where

$$f(x)=\begin{cases}2x^2, x\in[0,0.5]\\1-2(1-x)^2, x\in[0.5,1]\end{cases}$$

It is easy to see from the above formula that $F(x)$ is a monotonic function for transforming the spatial domain into the fuzzy domain. Because it is a monotonic function, there must be an inverse function. so $x=F^{-1}(p)=L-1+F_d(1-p^{-\frac{1}{F_e}})$,The fuzzy domain can be transformed into a spatial domain. The filter function $f(x)$ is used for image enhancement operations. According to the f function image, each partition is a convex function that moves the edge pixels toward the middle pixels to increase the image contrast.

When the f function is used for filtering multiple times, the gray value of the limit pixel can be greatly reduced. After three image enhancement effects, as shown in Figures 3-1 and 3-2.

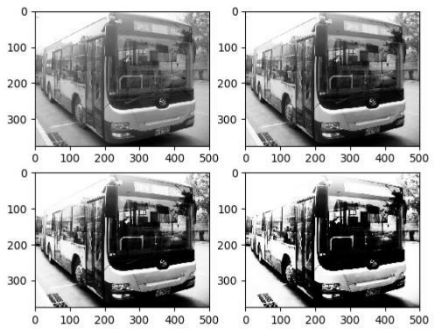

figure 3-1

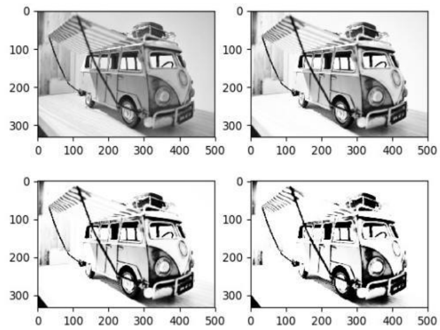

figure 3-2

3.3 Improved algorithm for fuzzy enhancement.

Since the value after the inverse transformation in the Pal algorithm may appear to be less than 0, the gray value of the image may not be negative, so the Pal algorithm sets all the gray values smaller than 0 to 0. This will lose part of the image information, and the image filtering algorithm in the fuzzy domain may be greatly affected.

In order to improve the negative value of the Pal algorithm, it is necessary to reconstruct the membership function. In the classical Pal algorithm, the transition point is directly selected by 0.5, but in different images, there are different image features, so the selection of 0.5 is inappropriate. The algorithm membership function is as follows:

$$F(x)=\begin{cases}s_1\tan^2(\frac{\pi x}{4(L-1)}), x\in[0,x_T]\\1-s_2(1-\tan^2(\frac{\pi x}{4(L-1)}))^2, x\in[x_T, L-1]\end{cases}$$

In order for the above function to guarantee continuity, it needs to be continuous at the boundary point x_T , so

$$s_1=\frac{x_T}{(L-1)\tan^2(\frac{\pi x_T}{4(L-1)})}\qquad s_2=\frac{L-1-x_T}{(L-1)(1-\tan(\frac{\pi x_T}{4(L-1)}))^2}$$

Where x_T is the critical transition point determined by the OTSU Otsu method. According to the transition point, the image divides all the pixels into two parts, one part is the area with lower gray value and the other part is higher gray value. region. Then, the image filtering operation is performed on the blurring domain, so that the region with a lower gray value becomes lower, and the region with a higher gray value becomes higher, thereby enhancing the contrast of the image. Similarly, for the filter function, select

$$f(x)=\begin{cases}\frac{1}{x_T}x^2, x\in[0,x_T]\\ 1-\frac{(1-x)^2}{1-x_T}, x\in[x_T,1]\end{cases}$$

As an image filter operator, according to the nature of the function, the functional inverse S-type function is edged on both sides of the critical point x_T, which can achieve the purpose of enhancing contrast.

For $F(x)$, the inverse image can be used to obtain the processed image, that is

$$F^{-1}(x)=\begin{cases}\frac{4(L-1)\arctan\sqrt{\frac{x}{s_1}}}{\pi}, x\in[0,x_T]\\ \frac{4(L-1)\arctan(1-\sqrt{\frac{1-x}{s_1}})}{\pi}, x\in[x_T,1]\end{cases}$$

If the image is encoded in the uint8 format, the function has a range of exactly $[0,255]$, which ensures that the image grayscale information is not lost. The experimental results are shown in Figure 3-3 and Table 3-1.

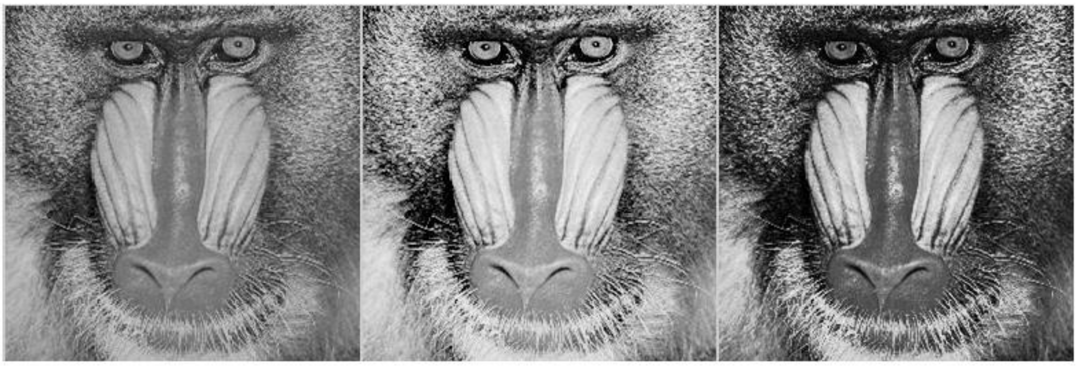

Figure 3-3 Original image, Pal algorithm enhancement, improved Pal algorithm enhancement effect diagram

Table 3-1 Experimental result

Image algorithm	Information entropy
Pal algorithm	7.1795
Improved Pal algorithm	7.1329

As can be seen from the above chart, the improved algorithm is more effective than the original Pal algorithm, and the image details are better protected under the premise of enhancing the image.

4. Conclusion

In summary, the fuzzy mathematical method can be used to provide a scientific data model, and data processing can also be performed to improve the overall effect and integrity of the image. In the field of image enhancement, which belongs to the transform domain, the image enhancement algorithm based on fuzzy mathematics needs to further grasp the membership function and the fuzzy domain filter function to achieve a wider range of applications.

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