Outage Analysis for Two-way Relay Fading Channels in the Low-SNR Regime

Xiaochen Lin

School of Electronic Information Engineering, Shanghai Dianji University, Shanghai 201306, China;

linxc0103@163.com

Abstract

In the slow Ralyleigh-fading scenario, this paper studies the outage performance of two-way relay channels (TWRC) in the low-SNR regime. We derive the outage probabilities for TWRC with amplify-and-forward (AF), decode-and-forward (DF) and bursty amplify-and-forward (BAF) at the low-SNRs. The numerical results show the outage probability of BAF outperforms that of AF, but still is worse than that of DF in the low-SNR region. On the other hand, we further analyze the performance limit of the achievable ∈-outage rates of TWRC with AF, DF and BAF in the low-SNR and low outage probability regime.

Keywords

Two-way relay channels, bursty amplify-and-forward, ∈-outage capacity.

1. Introduction

In the high-SNR regime, the main performance measure of the slow Rayleigh-fading channels is the SNR-asymptotic diversity-multiplexing tradeoff (DMT), which can be considered as an approximate curve of high-SNR outage probability. The high-SNR DMT for two-way relay channels (TWRC) has been sutided in [1] and [2]. For some practical communication systems operating at the finite-SNRs, the finite-SNR DMT evolved from the high-SNR DMT is used as a performance metric. The authors in [3]-[5] present the outage probability and finite-SNR DMT for TWRC in the finite-SNR regime. However, in the low-SNR region, the DMT can not describe the performance of fading channels. Instead, ∈-outage capacity introduced in [6] is main metric for outage performance in the low-SNR regime.

In this paper, we focus on the outage performance of TWRC in the low-SNR regime. There are three reasons why we study the low-SNR regime. First, as shown in [7], the impact of diversity at low SNRs is much more significant than that at high-SNRs in the slow-fading channels. Second, some practical systems (e.g. CDMA) operate at the low-SNRs, also known as the wideband regime. Third, seen from the numerical results in Section III, for TWRC with amplify-and-forward (AF) protocol, the lower bound of outage probability derived at the finite SNRs is loose in the low-SNR regime.

The related work in the low-SNR regime for point-to-point MIMO and one-way relaying can be found in [7]-[10]. The landmark work in [7] shows the standard AF and decode-and-forward (DF) in one-way relaying are suboptimal in the term of ∈-outage capacity, and proposes the new bursty AF (BAF) protocol, which archives the optimal ∈-outage capacity. Renk and Jäkel etc. derive the ∈-outage capacity of one-way incremental relaying with DF protocol in [8] and BAF protocol in [9] at low-SNR ratios. The authors in [10] show the diversity gain is redundant in the low-SNR regime and derive the outage probabilities of different MIMO fading channels at low-SNRs.

In this paper, we study the outage performance of TWRC with standard AF, DF, and BAF protocols in the low-SNR regime. We first derive of the outage probabilities for TWRC with AF, DF and BAF at the low-SNRs. The numerical results show the outage probability of BAF is between that of AF and DF. Moreover, we derive the performance limit of ∈-outage capacity of TWRC with AF, DF and BAF in the low-SNR and low outage probability regime.

2. System Model

As shown in Fig. 1, we consider a single-antenna TWRC, where two source nodes S_1 and S_2 communicate with each other aided by the relay node R. We assume that every node has the same transmit power E and operates in a half-duplex mode. Moreover, the channel state information (CSI) is perfectly known only by receiver not transmitter due to the low-SNR constraint. The links between S_1, S_2 and R are reciprocal, i.e., the channel coefficients satisfy $h_{1R} = h_{R1} = h$, $f_{1R} = f_{R1} = f$, where both h and f are zero-mean complex Gaussian random variables following $h \sim CN(0, \beta_h)$ and $f \sim CN(0, \beta_f)$, respectivly. As a result, the pre-defined X and Y are simplified as $X = |h|^2$ and $Y = |f|^2$ with Exponential distribution $p_x(X) = \frac{1}{\beta_h} e^{-\frac{x}{\beta_h}}$ and $p_y(Y) = \frac{1}{\beta_f} e^{-\frac{x}{\beta_f}}$.

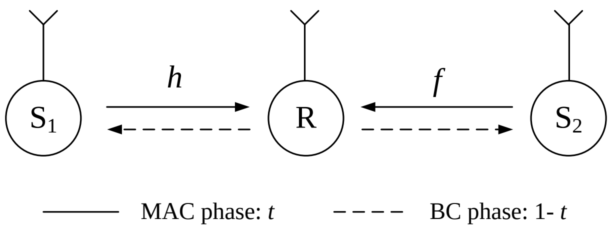

Fig. 1. System model of TWRC

In this paper, we sutdy the two-phase TWRC with AF, DF, and BAF relay protocols. The transmission time of each round of inforamtion exhcange is normalized to one. In specific, in the first phase, also called mutiple-access (MAC) phase, S_1 and S_2 is naturally ignored in the tow-phase scheme.

Let R denote the targe sum-rate of TWRC and α be a rate allocation parameter. Hence, the target rates of S_1 and S_2 are defined as $R_1 = \alpha R$ and $R_2 = (1-\alpha)R$, respectively. The outage probability of TWRC is defined as the probability that the target rate pair (R_1, R_2) lies outside the achivable rate region conditioned on h and f. In this paper, the instantaneous channel capacity is in nats/s/Hz.

3. Outage Probability

3.1 Amplify-and-forward

In this subsection, we consider a simple and popular AF TWRC with the fixed $t = 1/2$. In the AF protocol, S_1 and S_2 simultaneously transmit to R in the MAC phase, and the signal received at R is $y_r = h\sqrt{E}x_1 + f\sqrt{E}x_2 + n_r$, where n_r is an additive white Gaussian noise with $n_r \sim CN(0,1)$. Before sending the received superposed signal, the relay node R amplifies y_r with an amplification coefficient ρ. Then the transmit signal at the relay node is denoted as $x_r = \rho y_r$. To satisfy relay transmit power constraint, we have

$$\rho^2 |h|^2 E + \rho^2 |f|^2 E + \rho^2 = E. \tag{1}$$

So the amplificaiton coefficient ρ is yielded as

$$\rho^2 = \frac{E}{|h|^2 E + |f|^2 E + 1} \approx E. \tag{2}$$

Where the approximate equation holds in the low-SNR regime $E \ll 1$.

Consequently, the relay node broadcasts $\boldsymbol{x_r}$ in the BC phase and signals received at S_1 and S_2 are $y_1 = hx_r + n_1$ and $y_2 = hx_r + n_2$, respectively. After completerly removing the self-interference, the received signals becom to $\hat{y}_1 = \rho hf\sqrt{E}x_2 + \rho hn_r + n_1$ and $\hat{y}_2 = \rho hf\sqrt{E}x_1 + \rho fn_r + n_2$, here n_1 and n_2 are the additive white Gaussian noise with distribution $CN(0,1)$. As a result, the instantaneous SNRs at S_1 and S_2 are

$$\gamma_1 = \frac{|hf|^2 E^2}{2|h|^2 E + |f|^2 E + 1} \approx |hf|^2 E^2 = XYE^2 \tag{3}$$

$$\gamma_2 = \frac{|hf|^2 E^2}{|h|^2 E + 2|f|^2 E + 1} \approx |hf|^2 E^2 = XYE^2 \tag{4}$$

where the approximate equations also hold due to $E \ll 1$. It is interesting that the instantaneous SNRs at S1 and S2 are the same in the low-SNR regime. Then the achievable rate pair (R_1, R_2) can be expressed as

$$R_1 = \alpha R \leq I_1 = \frac{1}{2}\ln(1+\gamma_1), \tag{5}$$

$$R_2 = (1-\alpha)R \leq I_2 = \frac{1}{2}\ln(1+\gamma_2). \tag{6}$$

Theorem 1: The outage probability of TWRC two-phase AF scheme is as

$$P_O^{AF} = 1 - 2\sqrt{\frac{a}{\beta_h\beta_f}}K_1\left(2\sqrt{\frac{a}{\beta_h\beta_f}}\right), \tag{7}$$

where $a = \frac{2\alpha R}{E^2}$ with $\alpha \geq 1/2$, and $\mathrm{K}_1(\cdot)$ is the first order modified Bessel function of the second kind.

Proof: Based on (3)~(6), the outage probability at the low-SNRs is given by

$$\begin{aligned} P_O^{AF} &= P(I_1 < R_1 \; or \; I_2 < R_2) \\ &= P\left(\frac{1}{2}\ln(1+XYE^2) < \alpha R \text{ or } \frac{1}{2}\ln(1+XYE^2) < (1-\alpha)R\right) \\ &= P\left(\frac{1}{2}\ln(1+XYE)\right) < \alpha R \quad \text{——(a)} \\ &= P\left(\frac{1}{2}XYE^2 < \alpha R\right) \quad \text{——(b)} \\ &= P\left(XY < \frac{2\alpha R}{E^2} \triangleq a\right) \\ &= 1 - 2\sqrt{\frac{a}{\beta_h\beta_f}}K_1\left(2\sqrt{\frac{a}{\beta_h\beta_f}}\right). \quad \text{——(c)} \end{aligned} \tag{8}$$

(a) follows from the assumption that $\alpha \geq 1/2$, which is fair for two equivalent source nodes in TWRC. (b) follows from $\ln(1+x) \approx x$ for $x \ll 1$. (c) follows from $\int_0^\infty e^{-\frac{\omega}{4x}-\theta x}dx = \sqrt{\frac{\omega}{\theta}}K_1\left(\sqrt{\omega\theta}\right)$ [11].

3.2 Decode-and-forward

In DF protocol, the relay attempts to jointly decode the two signals. After applying ideal random binning, the relay then transmits thecombined signals to the destinations. Then, the instantanous SNRs of the links from S1 to R and from S2 to R can be denoted as $\gamma_{1R} = E|h|^2 = EX$ and $\gamma_{2R} = E|f|^2 = EY$, respectivley. The instantaneous total received SNR at R is $\gamma_{mac} = E|h|^2 + E|f|^2 = EX + EY$. In the BC phase, we denote the instantaneous SNRs at S1 and S2 as $\gamma_{R1} = E|h|^2 = EX$ and $\gamma_{R2} = E|f|^2 = EY$, respectively. Then, the achievable rate pair (R_1, R_2) of the two sources needs to satisfy

$$R_1 \leq \min\left(I_{1R} \triangleq t\ln(1+\gamma_{1R}), I_{R2} \triangleq (1-t)\ln(1+\gamma_{R2})\right) \tag{9}$$

$$R_2 \leq \min\left(I_{2R} \triangleq t\ln(1+\gamma_{2R}), I_{R1} \triangleq (1-t)\ln(1+\gamma_{R1})\right) \tag{10}$$

$$R \leq I_{2R} \triangleq t\ln(1+\gamma_{mac}). \tag{11}$$

Theorem 2: The outage probability of TWRC two-phase DF scheme is as

$$\mathrm{P_O^{DF}} = \begin{cases} 1 - \mathrm{e}^{\left(-\frac{\mathrm{b}}{\beta_\mathrm{h}} - -\frac{\mathrm{c}}{\beta_\mathrm{f}}\right)\frac{\mathrm{R}}{\mathrm{E}}} & \mathrm{b}+\mathrm{c} \geq \mathrm{d} \\ 1 - \frac{\beta_\mathrm{h}}{\beta_\mathrm{h}-\beta_\mathrm{f}}\mathrm{e}^{\left(-\frac{\mathrm{a}}{\beta_\mathrm{f}} - \frac{\mathrm{d}-\mathrm{c}}{\beta_\mathrm{h}}\right)\frac{\mathrm{R}}{\mathrm{E}}} + \frac{\beta_\mathrm{f}}{\beta_\mathrm{h}-\beta_\mathrm{f}}\mathrm{e}^{\left(-\frac{\mathrm{b}}{\beta_\mathrm{h}} - \frac{\mathrm{d}-\mathrm{c}}{\beta_\mathrm{f}}\right)\frac{\mathrm{R}}{\mathrm{E}}} & \mathrm{b}+\mathrm{c} < d, \beta_\mathrm{h} \neq \beta_\mathrm{f} \\ 1 - \mathrm{e}^{-\frac{\mathrm{dR}}{\beta_\mathrm{h}\mathrm{E}}} - \frac{\mathrm{e}^{-\frac{\mathrm{dR}}{\beta_\mathrm{h}\mathrm{E}}}(\mathrm{d}-\mathrm{c}-\mathrm{b})\mathrm{R}}{\beta_\mathrm{h}\mathrm{E}} & \mathrm{b}+\mathrm{c} < d, \beta_\mathrm{h} = \beta_\mathrm{f} \end{cases} \tag{12}$$

Where $b \triangleq \max\left\{\frac{\alpha}{t}, \frac{1-\alpha}{1-t}\right\}$, $c \triangleq \max\left\{\frac{1-\alpha}{t}, \frac{\alpha}{1-t}\right\}$, and $d \triangleq 1/t$.

Proof: Based on (9)~(11), the outage probability of DF in the low-SNR regime is

$$P_O^{DF} = P(t\ln(1+XE) < \alpha R \text{ or } t\ln(1+YE) < (1-\alpha)R \text{ or} (1-t)\ln(1+YE) < \alpha R$$
$$\text{or}(1-t)\ln(1+XE) < (1-\alpha)R \text{ or } t\ln(1+XE+YE) < R)$$
$$= P(t\,XE < \alpha R \text{ or } tYE < (1-\alpha)R \text{ or } (1-t)YE < \alpha R$$
$$\text{or } (1-t)XE < (1-\alpha)R \text{ or } t(x+Y)E < R) \quad (13)$$

The second eqution follows from $\ln(1+x) \approx x$ for $x \ll 1$. Then, by some basic probability computations, we can obtain the outage probability as in above theorem.

3.3 Bursty Amplify-and-Forward

Since the relay node amplifies the noise in standard AF, the outage performance is limited in low-SNR region. To overcome this weakness, [7] proposed a bursty version of AF, named as bursty amplify-and-forward (BAF) and verified BAF in TWRC and try to anlysis the outage probability and the achievable ∈-outage rate. In BAF, the two source nodes and the relay node transmit in only a fraction of the time τ with high power $\frac{E}{\tau}$, and the remain silent for the rest of the time. Thus the relay node transmit power constraint is as

$$\rho^2|h|^2\frac{E}{\tau} + \rho^2|f|^2\frac{E}{\tau} + \rho^2 = \frac{E}{\tau}, \quad (14)$$

Thus, the instantaneous SNRs at S1 and S2 are

$$\gamma_1^{BAF} = \frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{2|h|^2\frac{E}{\tau}+|f|^2\frac{E}{\tau}+1}, \quad \gamma_2^{BAF} = \frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{|h|^2\frac{E}{\tau}+2|f|^2\frac{E}{\tau}+1} \quad (15)$$

Theorem 3: The outage probability of TWRC two-phase BAF scheme is as

$$P_O^{BAF} = \max\left(1-e^{\left(-\frac{1}{\beta_h}-\frac{2}{\beta_f}\right)a_1}, 1-e^{\left(-\frac{2}{\beta_h}-\frac{1}{\beta_f}\right)b_1}\right), \quad (16)$$

Where $a_1 \triangleq \left(e^{\frac{2\alpha R}{\tau}}-1\right)\frac{\tau}{E}$, $b_1 \triangleq \left(e^{\frac{2(1-\alpha)R}{\tau}}-1\right)\frac{\tau}{E}$.

Proof: Based on (15), the outage probability at the low-SNRs is given by

$$P_O^{BAF} = P\left(\frac{\tau}{2}\ln\left(1+\frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{2|h|^2\frac{E}{\tau}+|f|^2\frac{E}{\tau}+1}\right) < \alpha R \text{ or } \frac{\tau}{2}\ln\left(1+\frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{2|h|^2\frac{E}{\tau}+|f|^2\frac{E}{\tau}+1}\right) < (1-\alpha)R\right)$$
$$= P\left(\frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{2|h|^2\frac{E}{\tau}+|f|^2\frac{E}{\tau}+1} < \left(e^{\frac{2\alpha R}{\tau}}-1\right)\frac{\tau}{E} \triangleq a_1 \text{ or } \frac{|hf|^2\left(\frac{E}{\tau}\right)^2}{2|h|^2\frac{E}{\tau}+|f|^2\frac{E}{\tau}+1} < \left(e^{\frac{2(1-\alpha)R}{\tau}}-1\right)\frac{\tau}{E} \triangleq b_1\right) \quad (17)$$

Since the outage event of BAF is an union event of two outage events of two directional flow, we first derive the two outage probability P_1^{BAF} and P_2^{BAF} as

$$P_1^{BAF} = P\left(\frac{|hf|^2}{2|h|^2+|f|^2+\frac{\tau}{E}} < a_1\right)$$
$$= P\left(\frac{|hf|^2}{2|h|^2+|f|^2} < a_1\right) \quad \text{——(a)}$$
$$\geq \mathrm{P}_{1lb}^{BAF} = P\left(\frac{1}{2}\min(2|\mathrm{h}|^2, |\mathrm{f}|^2) < \mathrm{a}_1\right) \quad \text{——(b)}$$
$$= 1-e^{\left(-\frac{1}{\beta_h}-\frac{2}{\beta_f}\right)a_1} \quad (18)$$

where the subscript 'lb' represents the lower bound, (a) follows the suitable fashion τ to satisfy $\frac{\tau}{E} \to 0$ as in [7]. (b) follows from the Harmonic inequality $\frac{xy}{x+y} < \min(x,y)$.

In the similar way, we can get the lower-bound of P_2^{BAF} as follows

$$P_2^{BAF} \geq P_{2b}^{BAF} = 1 - e^{\left(-\frac{2}{\beta_h} - \frac{1}{\beta_f}\right)} b_1 \tag{19}$$

The derivation of outage probability of BAF is not avalible, so we get a simple low bound as the maximum of outage probaiblities of the two union events.

$$P_{Olb}^{BAF} = \max\,(P_{1lb}^{BAF}, P_{2lb}^{BAF}) \tag{20}$$

3.4 Numerical Results

This subsection presents numerical results to valid the outage probabilities in the low-SNR regime in above Theorem 1 and 2. For conciseness, the relay lies between S_1 and S_2. The distance between S_1 and S_2 is fixed at 1. Let D and 1-D denote the distances from S_1 to R and from S_2 to R, respecitvley. We apply the path loss model suitable to urban areas, i.e., β_h and β_f can be expressed as: $\beta_h = D^{-4}$ and $\beta_f = (1-D)^{-4}$.

It is obviously seen from Fig. (2), the lower bound of finite-SNR outage probability in [5] is tight when SNR≥ -10 dB, but loose in the SNR≤ -10 dB region due to ignoring of the constant 1 in the denominator of SNRs. Oppositely, the closed-form of outage probability in Theorem 1 is tight in the SNR< -10 dB regime but loose when SNR≥ -10 dB because of the ignoring the effection of E in the denominator of SNRs. So the complete outage performance would combine the outage probability in low-SNR regime and finite-SNRs. Moreover, another point worth noting in Fig. (2) is that the outage probability of BAF is better than that of AF due to two source nodes and the relay node apply bursty plus transmit to reduce the noise interferece. However, DF still outperforms BAF in term of outage probability.

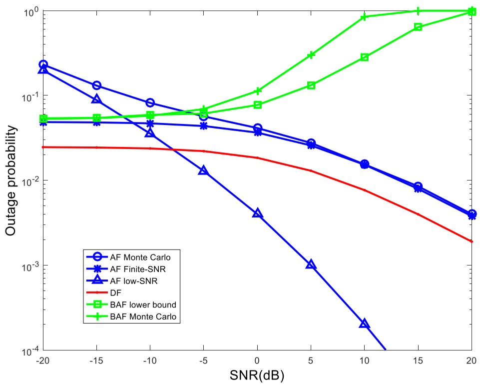

Fig. 2 Outage probability of TWRC AF, BAF and DF Protocols in the SNR regime.

4. ϵ-Outage Capacity

ϵ -outage capacity is an important metric in the low-SNR regime, which is defined as [6]

$$C_\epsilon \triangleq \sup_{R:P_{out}(R,SNR)\leq\epsilon} R \tag{21}$$

for a given, $0 \leq \epsilon \leq 1$. This is the largest rate of transmission R such that the outage probability $P_{out}(R)$ is less than ϵ. In this section, we focus on the low SNR and low outage probability regime, where the adverse impact of fading is much more significate but is the potential gain for the relaying. Based on the outgae probability results of the previous section, we can now analyze the limit behaviour of achivable ϵ -outage rate for TWRC AF, DF and BAF protocols.

4.1 Amplify-and-forward

Theorem 4: For the achievable ϵ -outage rate of TWRC two-phase AF scheme, we have

$$\lim_{\epsilon\to 0, E\to 0} \frac{R_\epsilon^{AF}}{\epsilon E^2} = \frac{hf}{2\alpha} \tag{22}$$

Proof: From (8), we know

$$P_O^{AF}(R) = P\left(XY < \frac{2\alpha R}{E^2}\right) \geq P\left(X < \frac{\sqrt{2\alpha R}}{E}\right) P\left(Y < \frac{\sqrt{2\alpha R}}{E}\right) \tag{23}$$

Since $\epsilon \to 0$ implies that $\frac{R}{E} \to 0$, we have the following expression

$$\lim_{\epsilon\to 0, E\to 0} \frac{P_O^{AF}}{\left(\frac{R}{E}\right)^2} \geq \lim_{E\to 0, \frac{R}{E}\to 0} \frac{\left(1-e^{-\frac{\sqrt{2\alpha R}}{hE}}\right)\left(1-e^{-\frac{\sqrt{2\alpha R}}{fE}}\right)}{\left(\frac{R}{E}\right)^2} \tag{24}$$

Thus, for the achievable ϵ -outage rate of TWRC AF protocol as (22).

This theorem shows that at low-SNRs and for small outage probability, the ahievable ϵ -outage rate fo the TWRC AF protocol is

$$R_\epsilon^{AF} \approx \frac{hf}{2\alpha} \epsilon E^2. \tag{25}$$

The first thing to note is the ϵ -outage rate is optimal with the fair rate allocation coefficient $\alpha = 1/2$ whatever the channel conditions, outage probability and SNR, because the two source nodes are equality in the TWRC. It is interesing to note that in the low SNR and low outage probability region, the achievable rate is proportional to ϵ and square of SNR.

4.2 Decode-and-forward

Theorem 5: For the achievable ϵ -outage rate of TWRC two-phase DF scheme, we have

$$\lim_{\epsilon\to 0, E\to 0} \frac{R_\epsilon^{DF}}{\epsilon E} = \frac{hf}{bf+ch} \tag{26}$$

Proof: For convenient analysis, we rewrite the outage probability of TWRC DF based on (15) as following

$$\begin{aligned} P_O^{DF}(R) &= P\left(X < b\frac{R}{E}\right) + P\left(X > b\frac{R}{E}\right) P\left(Y < c\frac{R}{E}\right) + P\left(X > b\frac{R}{E}\right) P\left(Y < c\frac{R}{E}\right) P\left(X+Y < d\frac{R}{E}\right) \\ &= 1 - e^{-\frac{bR}{hE}} + e^{-\frac{bR}{hE}}\left(1 - e^{-\frac{cR}{fE}}\right) + e^{-\frac{bR}{hE}} e^{-\frac{cR}{fE}}\left(P\left(X+Y < d\frac{R}{E}\right)\right) \end{aligned} \tag{27}$$

As $\epsilon \to 0, \frac{R}{E} \to 0$. Furthermore, $\lim_{\frac{R}{E}\to 0} \frac{P\left(X+Y<d\frac{R}{E}\right)}{\frac{R}{E}} = 0$. We can get

$$\begin{aligned} \lim_{\epsilon\to 0, E\to 0} \frac{P_O^{DF}}{\frac{R}{E}} &= \lim_{E\to 0, \frac{R}{E}\to 0} \frac{1-e^{-\frac{bR}{hE}}}{\frac{R}{E}} + e^{-\frac{bR}{hE}} \frac{1-e^{-\frac{cR}{fE}}}{\frac{R}{E}} + e^{-\frac{bR}{hE}} e^{-\frac{cR}{fE}} \frac{P\left(X+Y<d\frac{R}{E}\right)}{\frac{R}{E}} \\ &= \frac{b}{h} + \frac{c}{f} \end{aligned} \tag{28}$$

So, for the achievable **ϵ**-outage rate of TWRC two-phase DF scheme as (26).

This result shows that the achievable **ϵ**-outage rate in the low SNR and low outage proability regime is

$$R_\epsilon^{DF} \approx \frac{hf}{bf+ch} \epsilon E \tag{29}$$

Note that the achievable **ϵ**-outage rate of TWRC DF protocol is proportional to **ϵ** and SNR. This SNR is more significate for AF than DF protocol.

4.3 Bursty Amplify-and-forward

Theorem 6: For the achievable **ϵ**-outage rage of TWRC two-phase BAF scheme, we have

$$\lim_{\epsilon\to 0, E\to 0, \tau\to 0} \frac{R_\epsilon^{AF}}{\epsilon E} = \frac{hf}{2\alpha(h+2f)}, \ or \ \frac{hf}{2(1-\alpha)(2h+f)} \ . \tag{30}$$

Proof: From (20), we know

$$\lim P_{Olb}^{BAF} = \max(\lim P_{1lb}^{BAF}, \lim P_{2lb}^{BAF}) \ . \tag{31}$$

Since we focus on $\epsilon \to 0$, we choose a available fashion $\tau = \sqrt{RE}$ as in [7]. So $\left(e^{\frac{2\alpha R}{\tau}} - 1\right)\frac{\tau}{E} \geq \frac{2\alpha R}{E}, \frac{2\alpha R}{E} \to 0$. Thus, by applying Lema (1.2) in [7] with $\delta \triangleq \frac{\tau}{E}$, $g_1(\delta) = \left(e^{\frac{2\alpha R}{\tau}} - 1\right)\frac{\tau}{E}$, we have the following expression

$$\lim_{E\to 0, \frac{R}{E}\to 0, \tau\to 0} \frac{P_{1lb}^{BAF}}{\frac{2\alpha R}{E}} = \frac{h+2f}{hf}. \tag{32}$$

In the similar way with $g_2(\delta) = \left(e^{\frac{2(1-\alpha R)}{\tau}} - 1\right)\frac{\tau}{E}$.

$$\lim_{E\to 0, \frac{R}{E}\to 0, \tau\to 0} \frac{P_{2lb}^{BAF}}{g_2(\delta)} = \lim_{E\to 0, \frac{R}{E}\to 0, \tau\to 0} \frac{P_{2lb}^{BAF}}{\frac{2(1-\alpha)R}{E}} = \frac{2h+f}{hf}. \tag{33}$$

This theorem shows that at low-SNRs and for small outage probability, the achievable **ϵ**-outage rate of TWRC BAF protocol

$$R_\epsilon^{BAF} = \frac{hf}{h+2f}\frac{\epsilon E}{2\alpha} \text{ or } \frac{hf}{2h+f}\frac{\epsilon E}{2(1-\alpha)}. \tag{34}$$

5. Conclusion

In this paper, we study the outage performance of TWRC in the low-SNR regime in the slow Rayleigh-fading scenario. First, we derive the closed-forms of the outage probabilities for TWRC with AF, DF and BAF at the low-SNRs. The numerical results valid the outage performance of BAF is better than that of AF, but is worse than that of DF. Finally, we analyze the performance limit of the achievable **ϵ**-outage rates of TWRC with AF, DF and BAF in the low-SNR and low outage probability regime.

Acknowledgements

This work was supported by Research and Innovation Project of Shanghai Municipal Education Commission (No.15ZZ105) .

References

[1] D. Gündüz, M.A. Khojastepour, A. Goldsmith, and H.V. Poor, Multi-hop MIMO relay networks: Diveristy-Multiplexing Trade-off Analysis. IEEE Trans. On Wireless Commun., Vol. 9(2010), no. 5, p. 1738-1747.

[2] R. Vaze and R.W. H. Jr., On the Capacity and Diversity-Multiplexing Tradeoff of the Two-Way Relay Channel, IEEE Trans. Inf. Theory, Vol. 57 (2011), no. 7, p. 4219-4234.

[3] P. Liu and I. –M. Kim, Performance analysis of bidirectional communication protocols based on decode-and-forward relaying, IEEE Trans. on Commun., Vol. 58 (2010), no.9, p.2683-2696.

[4] Z. Yi and I. –M. Kim, Outage probability and optimum power allocation for analog network coding, IEEE Trans. Wireless Commun., Vol.10(2011), no.2, p.407-412.

[5] X. Lin, M. Tao, Y. Xu and R. Wang, Outage Probability and finite-SNR Diversity-Multiplexing tradeoff for two-way relay fading channels., IEEE Trans. on Vehicular Technology,Vol.62(2013), no. 7, p.3123-3136.

[6] L. H. Ozarow, S. Shamai, and A. D. Wyner, Information Theoretic Considerations for Cellular Mobile Radio, IEEE Trans. on Vehicular Technology, Vol. 43(1994), no. 2, p.359-377.

[7] A. S. Avestimehr and D. N. C. Tse, Outage Capacity of the Fading Relay Channel in the Low-SNR regime, IEEE trans. on Inforamtaion Tehory, Vol. 53(2007), no. 4, p. 1401-1415.

[8] T. Renk, H. Jäkel, F. K. Jondral, D. Gündüz, and A. Goldsmith, Outage capacity of incremental relaying at low signal-to-noise ratios, Sep. 2009, Anchorage, Alaska.

[9] T. Renk, H. Jäkel, F. K. Jondral, D. Gündüz, and A. Goldsmith, Outage capacity of bursty amplify-and-forward with incremental Relaying, *International Symposium On Information Theory & Its Applications*(Taichuang, Taiwan,2010), Vol.1, P.1007.

[10] S Loyka and G Levin, Diversity-Multiplexing Tradeoff in the low-SNR regime, IEEE Comm. Letter, Vol.15(2011), no. 5, p.542-544.

[11] I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrads, *Series and Products*(San Diego. Academic Press, 2000).