# Identification of Leakage Inductances of Power Transformer Based on Stochastic Gradient Algorithm

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## Abstract

Transformer protection principle based on models needs accurately identify the parameters of the transformer winding, Parameter identification equations of three-phase three-winding transforms are deduced. Multi-innovation stochastic gradient algorithm with forgetting factor is used in the identification of the leakage inductance of transformer and there is no need to change the TA configuration of the triangle side. The method can identify the leakage inductance of the transformer winding by using the voltage and current of the transformer directly. The algorithm overcomes the limitation of the large amount of calculation of the least square method, Simulation experiments results shows the effectiveness of the algorithm.

## **Keywords**

#### Transformer; parameter identification; leakage inductance; stochastic gradient algorithm.

### **1.** Introduction

Transformer is an important part of power system. It is mainly used to realize the connection and power exchange of different voltage levels of power grid. Its operation state is directly related to the security and stability of power system. For a long time, transformer protection mainly adopts differential protection <sup>[1,2]</sup>. This requires correct identification of inrush current and fault current to determine whether there is a fault, of which the second harmonic braking is the most widely used. However, the waveform characteristics of transformer inrush current are affected by many factors, such as system voltage and equivalent impedance, initial phase angle of closing, magnitude and direction of remanence, winding connection mode and neutral grounding mode, three-phase core structure, core material and assembly technology, hysteresis loop and local hysteresis loop, which will lead to maloperation of protection device and reduce its reliability.

In order to overcome the shortcomings of differential protection, many scholars at home and abroad applied parameter identification to transformer protection, abandoned the traditional differential protection idea, avoided the identification of fault current and inrush current, and had good application prospects.Neural network<sup>[3,4]</sup>, particle swarm algorithm<sup>[5]</sup> and least squares method<sup>[6-9]</sup> are the commonly used methods of transformer parameter identification. The least square method has been widely used because of its simplicity and easy implementation, but it needs to calculate the covariance matrix in the process of parameter estimation, so it has a large amount of calculation, and also has certain requirements for hardware system. In addition to the commonly used methods mentioned above, Stochastic Gradient algorithm (SG), which has the characteristics of simple algorithm and low computational complexity, has been widely studied and applied. Based on the theory of multi-innovation, Ding deduced the Multi-Innovation Stochastic Gradient algorithm (MISG) and analyzed its performance <sup>[10-12]</sup>.

Based on the theory of multi-innovation identification, the leakage inductance and resistance of transformer are identified by using the principle of transformer equivalent circuit balance equation and transformer model. According to the voltage and current signals collected, the leakage inductance and resistance of transformer are identified by using MISG. By introducing forgetting factor, the problem of data saturation is eliminated, and the performance of the algorithm is improved, so that it has faster convergence speed.

#### 2. Multi-Innovation Stochastic Gradient algorithm

In order to improve the convergence speed of the stochastic gradient identification algorithm, the innovation length is introduced. The MISG is a tradeoff between the convergence speed and computational complexity of least squares algorithm and stochastic gradient algorithm (SG)<sup>[13]</sup>.

Let the system be a controlled autoregressive model:

$$A(z)y(t) = B(z)u(t) + v(t)$$
(1)

Where  $A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n_a}$ ,  $B(z) = 1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-b_a}$ ,  $z^{-1}$  is the Unit Backward Shift Operator [i.e.,  $z^{-1} y(t) = y(t-1)$ ], u(t) is the input of the system, y(t) is the output of the system, v(t) is a white noise sequence with zero mean and variance  $\sigma$ .

Definition:

$$\theta \coloneqq [a_1, a_2, \cdots, a_{n_a}, b_1, b_2, \cdots, b_{n_b}]^T$$
$$\varphi(t) \coloneqq [-y(t-1), -y(t-2), \cdots, -y(t-n_a),$$
$$u(t-1), u(t-2), \cdots, u(t-n_b)]^T$$

Then the system can be written in the form of linear regression model:

$$y(t) = \varphi^{T}(t)\theta + v(t)$$
<sup>(2)</sup>

Where  $y(t) \in \mathbf{R}$ , is the output,  $\varphi(t) \in \mathbf{R}^n$ , is the regression information vector composed of input and output data,  $\theta \in \mathbf{R}^n$ , is the parameter vector to be identified,  $v(t) \in \mathbf{R}^n$ , is zero mean random noise. The stochastic gradient algorithm for parameter vector  $\theta$  in (2) is as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\varphi(t)}{r(t)}e(t)$$
$$e(t) = y(t) - \varphi^{T}(t)\hat{\theta}(t-1)$$
$$r(t) = \sum_{i=0}^{t-1} \left\|\varphi(t-i)\right\|^{2} = r(t-1) + \left\|\varphi(t)\right\|^{2}, r(0) = 1$$

The convergence speed of stochastic gradient algorithm is slow. In order to improve its convergence speed, the innovation scalar  $e(t) \in R$  is extended to the innovation vector, where the positive integer p denotes innovation length, and  $e(t-i) = y(t-i) - \varphi^T (t-i)\hat{\theta}(t-i-1)$ .

$$E(p,t) = \begin{bmatrix} e(t) \\ e(t-1) \\ \vdots \\ e(t-p+1) \end{bmatrix} = \begin{bmatrix} y(t) - \varphi^T(t)\hat{\theta}(t-1) \\ y(t-1) - \varphi^T(t-1)\hat{\theta}(t-1) \\ \vdots \\ y(t-i) - \varphi^T(t-i)\hat{\theta}(t-1) \end{bmatrix}$$

Defining Information Matrix  $\Phi(p,t)$  and stacking output vectors Y(p,t) as

$$\Phi(p,t) = [\varphi(t), \varphi(t-1), \dots, \varphi(t-p+1)] \in \mathbb{R}^{n \times p} \ \mathbf{Y}(p,t) = [\mathbf{y}(t), \mathbf{y}(t-1), \dots, \mathbf{y}(t-p+1)]^T \in \mathbb{R}^{n \times p}$$

Then the innovation vector can be expressed as  $E(p,t) = Y(p,t) - \Phi^T(p,t)\hat{\theta}(t-1)$ . The MISG of parameter vector  $\theta$  in (2) is as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Phi(p,t)}{r(t)} E(p,t)$$
(3)

$$E(p,t) = Y(p,t) - \Phi^{T}(p,t)\hat{\theta}(t-1)$$
(4)

$$r(t) = r(t-1) + \|\varphi(t)\|^2$$
(5)

$$\Phi(p,t) = [\varphi(t), \varphi(t-1), \cdots, \varphi(t-p+1)]$$
(6)

$$Y(p,t) = [y(t), y(t-1), \dots, y(t-p+1)]^{T}$$
(7)

In order to improve the convergence speed of the algorithm, forgetting factor  $\lambda$  is introduced into equation (5):

$$r(t) = \lambda r(t-1) + \|\varphi(t)\|^2, 0 \le \lambda < 1$$
(8)

A multi-innovation forgetting gradient algorithm which can accelerate the convergence speed of the algorithm at the beginning stage is obtained by using equation (8) instead of equation  $(5)^{[14]}$ .

Compared with the stochastic gradient algorithm, the multi-innovation forgetting stochastic gradient algorithm not only uses the new information, but also reuses the previous data. Therefore, it has higher accuracy of parameter estimation and faster convergence speed. By adding the forgetting factor  $\lambda$ , the convergence rate is further improved.

#### 3. Transformer Model Equation

For the three-phase transformer with  $Y/\triangle$  connection method shown in Fig. 1, the primary winding equivalent equation and the secondary winding flux equation are respectively as follows<sup>[15]</sup>:



Fig. 1 Y/ $\triangle$  Transformer wiring diagram

Where  $u_A$ ,  $u_B$  and  $u_C$  are the instantaneous value of Y-side voltage;  $u_a$ ,  $u_b$  and  $u_c$  are the instantaneous value of  $\triangle$ -side voltage;  $i_A$ ,  $i_B$  and  $i_C$  are the instantaneous value of Y-side curren;  $i_a$ ,  $i_b$  and  $i_c$  are the instantaneous value of  $\triangle$ -side curren;  $i_p$  is the circulation in winding  $\triangle$ ; As part of winding current,  $i_{ab}$ ,  $i_{bc}$  and  $i_{ca}$  are the  $\triangle$ -side current that can be obtained by line current  $i_a$ ,  $i_b$  and  $i_c$ .

The winding resistance values of Y and  $\triangle$  are:

$$R_A = R_B = R_C = R_1$$
$$R_{ab} = R_{bc} = R_{ca} = R_2$$

Leakage inductance values are:

$$L_A = L_B = L_C = L_1$$
$$L_{ab} = L_{bc} = L_{ca} = L_2$$

Then the Y-side loop equation is:

$$\begin{cases}
 u_A = R_1 i_A + L_1 \frac{di_A}{dt} + n_1 \frac{d\psi_A}{dt} \\
 u_B = R_1 i_B + L_1 \frac{di_B}{dt} + n_1 \frac{d\psi_B}{dt} \\
 u_C = R_1 i_C + L_1 \frac{di_C}{dt} + n_1 \frac{d\psi_C}{dt}
\end{cases}$$
(9)

Where  $\Psi_A$ ,  $\Psi_B$  and  $\Psi_C$  are common flux links of A/a, B/b and C/c phase windings.  $\psi_A = \psi_a$ ,  $\psi_B = \psi_b$ ,  $\psi_C = \psi_c$ .

The loop equation of  $\triangle$ -side is:

r

$$\begin{cases} u_{a} - u_{b} = R_{2} \left( i_{ab} + i_{p} \right) + L_{2} \frac{d \left( i_{ab} + i_{p} \right)}{dt} + n_{2} \frac{d \psi_{a}}{dt} \\ u_{b} - u_{c} = R_{2} \left( i_{bc} + i_{p} \right) + L_{2} \frac{d \left( i_{bc} + i_{p} \right)}{dt} + n_{2} \frac{d \psi_{b}}{dt} \\ u_{c} - u_{a} = R_{2} \left( i_{ca} + i_{p} \right) + L_{2} \frac{d \left( i_{ca} + i_{p} \right)}{dt} + n_{2} \frac{d \psi_{c}}{dt} \end{cases}$$
(10)

Subtracting from equation (9) leads to:

$$\begin{cases} u_{A} - u_{B} - K(u_{ab} - u_{bc}) = \\ R_{1}(i_{A} - i_{B}) - KR_{2}(i_{ab} - i_{bc}) + L_{1} \frac{d(i_{A} - i_{B})}{dt} - KL_{2} \frac{d(i_{ab} - i_{bc})}{dt} \\ u_{B} - u_{C} - K(u_{bc} - u_{ca}) = \\ R_{1}(i_{B} - i_{C}) - KR_{2}(i_{bc} - i_{ca}) + L_{1} \frac{d(i_{B} - i_{C})}{dt} - KL_{2} \frac{d(i_{bc} - i_{ca})}{dt} \\ u_{C} - u_{A} - K(u_{ca} - u_{ab}) = \\ R_{1}(i_{C} - i_{A}) - KR_{2}(i_{ca} - i_{ab}) + L_{1} \frac{d(i_{C} - i_{A})}{dt} - KL_{2} \frac{d(i_{ca} - i_{ab})}{dt} \end{cases}$$
(11)

At the same time,  $i_{ab}$ ,  $i_{bc}$  and  $i_{ca}$  can be obtained by line current of  $\triangle$ -side:

$$\begin{cases}
 i_{ab} = \frac{(i_a - i_b)}{3} \\
 i_{bc} = \frac{(i_b - i_c)}{3} \\
 i_{ca} = \frac{(i_c - i_a)}{3}
 \end{cases}$$
(12)

Substituting equation (12) into equation (11):

Where

$$\begin{cases} u_{A} - u_{B} - K(u_{ab} - u_{bc}) = R_{1}(i_{A} - i_{B}) - KR_{2}i_{abc} + L_{1}\frac{d(i_{A} - i_{B})}{dt} - KL_{2}\frac{di_{abc}}{dt} \\ u_{B} - u_{C} - K(u_{bc} - u_{ca}) = R_{1}(i_{B} - i_{C}) - KR_{2}i_{bca} + L_{1}\frac{d(i_{B} - i_{C})}{dt} - KL_{2}\frac{di_{bca}}{dt} \\ u_{C} - u_{A} - K(u_{ca} - u_{ab}) = R_{1}(i_{C} - i_{A}) - KR_{2}i_{cab} + L_{1}\frac{d(i_{C} - i_{A})}{dt} - KL_{2}\frac{di_{cab}}{dt} \\ i_{abc} = \frac{i_{a} - 2i_{b} + i_{c}}{3}, \quad i_{bca} = \frac{i_{b} - 2i_{c} + i_{a}}{3} \text{ and } i_{cab} = \frac{i_{c} - 2i_{a} + i_{b}}{3}. \end{cases}$$
(13)

Because the coil resistance of the transformer is relatively easy to obtain. Therefore, the parameters of leakage inductance of transformer to be identified can be rewritten as follows:

$$y = u_A - u_B - K(u_{ab} - u_{bc}) - R_1(i_A - i_B) + KR_2i_{abc} = \left\lfloor \frac{d(i_A - i_B)}{dt} - K\frac{di_{abc}}{dt} \right\rfloor \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$
(14)

In order to apply the stochastic gradient algorithm for multi-innovation forgetting, The equation (14) requires a discretization. Take a sufficiently small sampling time T, The equation (14) can be written as:

$$\frac{u_{A}(n+1)+u_{A}(n)}{2} - \frac{u_{C}(n+1)+u_{C}(n)}{2} - K\left[\frac{u_{ab}(n+1)+u_{ab}(n)}{2} - \frac{u_{bc}(n+1)+u_{bc}(n)}{2}\right] - R_{1}\left[\frac{i_{A}(n+1)+i_{A}(n)}{2} - \frac{i_{C}(n+1)+i_{C}(n)}{2}\right] - KR_{2}\frac{i_{abc}(n+1)+i_{abc}(n)}{2} = \left[\frac{i_{A}(n+1)-i_{A}(n)}{T} - \frac{i_{C}(n+1)-i_{C}(n)}{T}\right] \begin{bmatrix}L_{1}\\L_{2}\end{bmatrix}$$
(15)

The regression model equation of transformer can be obtained from equation (15).

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$$y(t) = \mathbf{\varphi}^{T}(t)\mathbf{\theta} + v(t) \tag{16}$$

Where

$$y(n) = \frac{u_A(n+1) + u_A(n)}{2} - \frac{u_C(n+1) + u_C(n)}{2} - K\left[\frac{u_{ab}(n+1) + u_{ab}(n)}{2} - \frac{u_{bc}(n+1) + u_{bc}(n)}{2}\right] - KR_1\left[\frac{i_A(n+1) + i_A(n)}{2} - \frac{i_C(n+1) + i_C(n)}{2}\right] - KR_2\frac{i_{abc}(n+1) + i_{abc}(n)}{2}$$

$$\varphi(n) = \left[\frac{i_A(n+1) - i_A(n)}{T} - \frac{i_C(n+1) - i_C(n)}{T} - K\frac{i_{abc}(n+1) - i_{abc}(n)}{T}\right]^T, \ \mathbf{\theta} = \begin{bmatrix}L_1 & L_2\end{bmatrix}^T \text{ and } v(n) \text{ is a white}$$

noise sequence.

When identifying parameter vector  $\theta$  by multi-innovation forgetting stochastic gradient algorithm, t=1 and the initial values of  $\theta$  must be set first. Then the input and output data are collected. According to equation (6) and (7), information matrix and stacked output vector are constructed. Then the innovation vector E(p,t) and r(t) are calculated according to equation (4) and (5). Then the vector  $\theta$ is estimated by refreshing the parameter according to equation (8). Repeat the above steps to refresh the parameter estimation vector  $\theta$  and complete the identification of leakage inductance.

To sum up, the leakage inductance of transformer can be obtained by measuring the voltage and current of Y-side and  $\triangle$ -side <sup>[16]</sup>.

#### 4. Case simulation and analysis

Using MATLAB/Simulink to simulate, the voltage and current of transformer are generated by the model of simulation software. The connection of the simulation system is shown in Fig. 2.



Fig. 2 Simulation experiment system

The three-phase transformer is composed of three single-phase transformers that are connected in  $Y/\Delta$  mode. The rated voltage of the transformer is set to 1000V/500V, the rated capacity is 25 MVA, and the period is 1E-4s. The winding resistance and leakage inductance values at the high and low voltage sides of the transformer are respectively set as:R1=0.9 $\Omega$ ,R2=0.42 $\Omega$ ,L1=3mH,L2=1mH.

Phase A current waveforms of primary side and secondary side of transformer during normal operation are shown in Fig. 3.



Fig. 3 Waveform of primary and secondary side phase current of transformer in normal operation The identification results of transformer primary inductance under different innovation lengths and forgetting factors are shown in Fig. 4 and 5.



Fig. 4 Identification results of primary side leakage inductance with different innovation length As shown in Fig. 4, with the increase of p, the convergence speed of identification of primary side leakage inductance is accelerated. When p = 1, the identification results do not converge to the set values in the simulation time. When p = 6, the identification result can converge to the set value when t = 0.2s. While the identification speed is still accelerated with the increase of p, the increase of identification speed is limited, but the larger p value increases the computational burden of the system.



Fig. 5 Identification results of primary side leakage with different forgetting factors

As shown in Fig. 5, compared with  $\lambda = 1$ , the convergence speed of the algorithm is further accelerated when  $\lambda = 0.99$ , and it can converge to the set value when t = 0.1s. Continuing to reduce  $\lambda$ , the convergence speed of the algorithm is faster when  $\lambda = 0.95$ . When t = 0.05s, it can converge to the set value, and the identification result is 0.027H.

When p=6 and  $\lambda$ =0.95, the identification results of the secondary inductance are shown in Fig. 6.



Fig. 6 Identification results of secondary side leakage inductance

As shown in Fig. 6, the identification speed of the secondary side leakage inductance is less than that of the primary side, but it can still identify the secondary side leakage inductance at t=0.1s. The identification result is 0.0011H.

Phase A current waveforms of primary side and secondary side at closing are shown in Fig. 7. The transformer closes at t=0.1s.



Fig. 7 Current waveforms of primary side and secondary side when transformer closing

When closing, the inductance identification results of primary side and secondary side at p=6 and lambda=0.95 are shown in Fig. 8 and 9.



Fig. 8 Identification results of primary side leakage inductance when transformer closing



Fig. 9 Identification results of secondary side leakage inductance when transformer closing From Fig. 8 and 9, the leakage inductances of primary and secondary sides can be identified quickly by using multi-innovation forgetting stochastic gradient algorithm when transformer closes. The leakage inductances of primary and secondary sides are 0.026H and 0.0093H.

### 5. Conclusion

Based on the theory of multi-innovation identification, combined with the model of transformer, the leakage inductances of primary and secondary sides are identified by the multi-innovation forgetting stochastic gradient identification algorithm according to the voltage and current signals collected by using the voltage and current equation of transformer. The simulation results verify the effectiveness of the algorithm. Compared with the stochastic gradient algorithm, the algorithm has faster convergence speed and higher identification accuracy.

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