# Research on Recovery Model of Closed-Loop Supply Chain for Electronic Products in O2O Model

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## Abstract

**Compared with the traditional closed-loop supply chain recycling model, the selection strategy of the manufacturing enterprises and consumers has changed in the O2O model. The steady-state equilibrium values of two models are given by constructing the closed-loop supply chain recovery model of consumer electronics products under the dominant model of manufacturing enterprises or O2O platform. The results show that the government subsidies, the recycling price, quantity and instantaneous profits gradually increased over time, and which always less than the steady-state equilibrium value. Compared with the recycling mode dominated by O2O platform, the recycling mode led by manufacturing enterprises has high recovery price, large amount of recycling and high government subsidies. Recovery costs, except the recovery prices, will affect the recycling model choice of manufacturing enterprises. The manufacturing enterprise are guiding the recycling model when other recycling costs are low, otherwise the O2O platform which leading recovery model is selected.**

## Keywords

**The O2O model; consumer electronics recycling; government subsidies; channel profit.**

## 1. Introduction

With the increasing speed of the replacement of consumer electronic products such as mobile phones and computers, the disposal of used electronic products has become a major problem that plagues enterprises and consumers. Taking the disposal of used mobile phones as an example, consumers will choose to trade in old-fashioned discounts, or use waste mobile phones to exchange daily necessities, and some consumers will throw away waste mobile phones directly. If the waste electronic products are not properly handled, it will pollute the soil and water sources. The traditional recycling channels for consumer electronic products mainly include municipal solid waste system recycling, regular fixed-point recycling, point-of-sale recycling, and maintenance or junk market treatment. Electronic product recycling in O2O mode which through online valuation, offline manual detection and recycling is a new way of recycling.

Research on reverse logistics and closed-loop supply chains focuses on the following aspects. First, the analysis of closed-loop supply chain models led by different companies. Savaskan[1] established the Stackelberg master-slave game model dominated by manufacturing companies, sellers and third-party reverse logistics as followers, and discussed the joint optimization strategy of supply chain. Xiong[2] studied the expected benefits of different stakeholders when manufacturer or upstream supplier re-manufacturing in a decentralized closed-loop supply chain. Zhao[3] considered the demand for products to be affected by emission reduction rate and low carbon promotion, then constructed a two-level supply chain composed of two manufacturers and one retailer. Ma[4] compared the advantages and disadvantages of channel profit among different cooperation modes of manufacturing enterprises, retailers and recyclers, and pointed out that the cooperation strategy can increase the profit of the alliance and maximize the overall profit of the supply chain.

Second, the government plays an important role in the formation and operation of the closed-loop supply chain. Based on a comparative analysis of formal and informal channel recycling, Liu[5] found that subsidies can effectively support the development of formal enterprises. Studied the competitive relationship between participants in remanufacturing activities and the impact of government subsidies, Mitra[6], Wang[7] and Heydari[8] pointed out that the government provides different reward and penalties can improve and coordinate supply chain efficiency. Xie[9] focused on the duopoly market for color TV recycling. The government reduces the number of consumables and increases subsidies for home appliances, which is conducive to environmental improvement and social welfare. Zhao[10] pointed out that remanufacturers share a certain percentage of subsidies with consumers can obtain more profits. Ma[11] explored the impact of government consumption subsidies on the revenue of e-commerce retailers. Peng[12] studied the closed-loop supply chain decision-making and coordination mechanism for differential pricing of new products and re-products under government subsidies.

Third, the closed-loop supply chain research based on the O2O model. Du[13] proposed the process design of the reverse processing chain recovery processing subsystem for O2O mode mobile phone, and used the system dynamics theory to explore the relationship between recycling price, recycling amount and profit. Tang[14] gives the income function of manufacturers, network distributors and third parties under the condition of complete information static. The research finds that the vendor's independent O2O recovery mode is optimal. Wu[15] analyzes the coordination effect of the two pricing contracts on the O2O supply chain when the demand in the supply chain and the service replacement coefficient are disturbed.

The above literatures provide important references for the research in this paper, but these papers mainly discuss different dominant parties, such as channel revenues under the game of manufacturing enterprises, retailers, and third-party recyclers. However, there are few studies on the closed-loop supply chain led by the O2O platform. At the same time, it ignores the role of government subsidies in the closed-loop supply chain of consumer electronics. In addition, we should further analyze the amount of recycling in the closed-loop supply chain and the change in revenue of the recycling company under the dynamic changes of subsidies. In this paper, using the differential game theory, we constructed the two types of recycling models dominated by manufacturing enterprises and O2O platform. We considered the constraints of dynamic changes of government subsidies and compared the recovery price, quantity and subsidy amount of the two modes. Finally, based on the recovery cost, we compared the recycling strategy which the manufacturing enterprise independent recycling or entrusted the O2O platform recycling.

## 2. Problem Description

Hypothesis 1: Consider a closed-loop supply chain consisting of manufacturing companies, an O2O platform, and consumers. Both the manufacturing enterprise and the O2O platform are independent decision makers. They are all rational people and pursue their own profit maximization. They only consider the single cycle of used electronic products. At the same time, the information of manufacturing enterprises and O2O platforms is shared. Manufacturing companies are leaders in the Stackelberg game.

Hypothesis 2: There is no difference in the products produced from recycled materials and new materials. The unit cost of producing electronic products using recycled materials is $C_r$, The unit cost of producing electronic products using new materials is $C_{\mathrm{m}}, C_{\mathrm{m}} > C_r$, Unit cost savings during remanufacturing, $\Delta = C_{\mathrm{m}} - C_r$.

Hypothesis 3: In the forward supply chain, $w(\mathrm{t})$ is the wholesale price of manufacturing enterprise, $p(\mathrm{t})$ is the retail price of O2O platform. The demand function of the consumers is $D(\mathrm{p}) = \gamma - \mu p$, the potential demand for the market is $\gamma$, and the sensitivity coefficient of consumer demand to price is $\mu$, $\mu > 0$.

Hypothesis 4: In the reverse supply chain, the recycling price paid by the enterprise to the consumer is b(t) when the manufacturing company is responsible for recycling. the recycling quantity of the waste electronic products is $Q(\mathrm{b})=\mathrm{k}+\mathrm{hb}$, k is the consumer's green consumption awareness. h is the sensitivity coefficient of the recovery price and $\mathrm{h}>0$. $C_1(\mathrm{t})$ is the other costs recovered by the manufacturing company (including collection, warehousing, transportation, etc.), and the government subsidizes manufacturing company. When the O2O platform is responsible for recycling, $\mathrm{b}_o(\mathrm{t})$ is the recycling price paid by the O2O platform to consumers, the manufacturing company recycles the waste electronic products from the O2O platform at the transfer price $\mathrm{b}_m(\mathrm{t})$, $b_m(t)\le\Delta$. The amount of recycling is $Q(\mathrm{b}_o)=\mathrm{k}+\mathrm{hb}_o$, the other costs recovered by the O2O platform is $C_2(\mathrm{t})$ and government subsidies O2O platform.

Hypothesis 5: The dynamic constraint of the amount of subsidy given by the government to the unit for recycling goods over time is $\dot{g}(\mathrm{t})=\alpha Q(t)-\beta g(t)$, $\alpha$ is the coefficient of influence of the quantity recovered, $\beta$ Represents the attenuation factor of government subsidies and $\alpha$, $\beta$ is a constant greater than zero. The recycling rate of used electronic products is 100%[16].

## 3. Model Construction and Analysis

### 3.1 Manufacturing Enterprise-led Recycling Model

When the manufacturer in the supply chain is responsible for product recycling, the manufacturer decides the wholesale price of the product and the recycling price of the waste product. The selling price of the product is determined by the O2O platform. The profit functions of the manufacturer and the O2O platform are as follows,

$$\underset{w,b}{\Pi_m}=\int_0^{\infty}\mathrm{e}^{-rt}\left[(w-C_{\mathrm{m}})D+(\Delta-b+g)Q-C_1\right]dt \tag{1}$$

$$\underset{p}{\Pi_o}=\int_0^{\infty}\mathrm{e}^{-rt}\left[(p-w)D\right]dt \tag{2}$$

The constraints are $\dot{g}(\mathrm{t})=\alpha Q(t)-\beta g(t)$.

According to the optimal control theory, the Hamilton functions of the O2O platform and manufacturing enterprises are obtained as follows:

$$H_o(p,w,b,g,\rho_o)=(p-w)(\gamma-\mu p)+\rho_o\left[\alpha(k+hb)-\beta g\right] \tag{3}$$

$$H_m(p,w,b,g,\rho_m)=(w-C_{\mathrm{m}})(\gamma-\mu p)+(\Delta-b+g)(k+hb)-C_1+\rho_m\left[\alpha(k+hb)-\beta g\right] \tag{4}$$

The co-state variables of the O2O platform and manufacturing enterprises are $\rho_o$ and $\rho_m$. By using the reverse induction method, the first order partial derivatives of formula (3) (4) are obtained and made equal to zero respectively. The optimal decision variables of manufacturing enterprises and the O2O platform are obtained as follows,

$$w=\frac{\gamma+\mu C_{\mathrm{m}}}{2\mu}$$

$$p=\frac{3\gamma+\mu C_{\mathrm{m}}}{4\mu}$$

$$b=\frac{h(\Delta+g+\rho_m\alpha)-k}{2h}$$

In addition,

$$\dot{g}=\frac{\alpha}{2}\left[h(\Delta+g+\rho_m\alpha)+k\right]-\beta g \tag{5}$$

$$\dot{\rho}_m=(r+\beta-\frac{1}{2}h\alpha)\rho_m-\frac{1}{2}\left[k+h(\Delta+g)\right] \tag{6}$$

Proposition 1. When the influence coefficient of recycling quantity is $\alpha < \frac{2\beta(\mathrm{r}+\beta)}{h(\mathrm{r}+2\beta)}$, the closed-loop supply chain dominated by manufacturing enterprises has steady-state equilibrium, which leads to

$$\bar{g}^M = \frac{\alpha(r+\beta)(k+h\Delta)}{2\beta(r+\beta)-\alpha h(r+2\beta)}, \quad \mathrm{b}^M = \frac{1}{2}\left[\Delta + \frac{\alpha(k+h\Delta)}{2(r+\beta)-h\alpha} - \frac{k}{h}\right] + \frac{r+\beta}{2(r+\beta)-h\alpha}\bar{g}^M$$

Proof. Both sides of equation (5) are simultaneously derived from t,

$$\ddot{g} = \left(\frac{\alpha \mathrm{h}}{2} - \beta\right)\dot{g} + \frac{\alpha^2 h}{2}\dot{\rho}_m \tag{7}$$

Substituting (6) into (7),

$$\ddot{g} = \left(\frac{\alpha h}{2} - \beta\right)\dot{g} + \frac{\alpha^2 h}{2}\left[\left(r+\beta-\frac{1}{2}h\alpha\right)\rho_m - \frac{k+h(\Delta+g)}{2}\right] \tag{8}$$

（5）leads to $\rho_m = \frac{1}{\alpha}\left\{\frac{1}{h}\left[\frac{2(\dot{g}+\beta g)}{\alpha} - k\right] - \Delta - g\right\}$ substituting (8), we can get,

$$\ddot{g} - r\dot{g} - \left[\beta(r+\beta) - \alpha h(\frac{r}{2}+\beta)\right]g = -\frac{\alpha(k+h\Delta)}{2}(\beta+r) \tag{9}$$

The above is a second-order differential equation for g. It is easy to know that one eigenvalue is positive. In order to make the solution of the differential equation converge and reach the saddle point equilibrium, there should be another eigenvalue negative. The product of the two eigenvalues is $-\left[\beta(\mathrm{r}+\beta) - \alpha\,\mathrm{h}(\frac{r}{2}+\beta)\right] < 0$. According to $\dot{g} = \dot{\rho}_m = 0$, we can find the value of g and $\rho_m$ at the steady state.

In addition, the government subsidies $\bar{g} > 0$ at the time of equilibrium, the impact coefficient of the recovered quantity can be obtained is $\alpha < \frac{2\beta(r+\beta)}{h(r+2\beta)}$. The instantaneous profit of the manufacturing company is $\overline{\pi_m}^M = \frac{(\gamma-\mu C_m)^2}{8\mu} + \Delta k - C_1 + \bar{\mathrm{b}}^M(\Delta h - k - h\bar{b}^M + h\bar{g}^M) + k\bar{g}^M$. The instantaneous profit of the O2O platform is $\overline{\pi_o}^M = \frac{(\gamma-\mu C_m)^2}{16\mu}$. The instantaneous profit at equilibrium is positively correlated with the maximum market demand.

Proposition 2. When manufacturing companies recycle, the optimal recycling price for the manufacturing company is:

$$\mathrm{b}^M(\mathrm{t}) = \frac{1}{2}\left[\Delta + \frac{\alpha(k+h\Delta)}{2(r+\beta)-h\alpha} - \frac{k}{h}\right] + \frac{r+\beta}{2(r+\beta)-h\alpha}\left[(g_0 - \bar{g}^M)e^{\frac{\Omega_2}{2}t} + \bar{g}^M\right] \tag{10}$$

$g_0$ is the initial value, $\psi = \sqrt{r^2 + 4\left[\beta(r+\beta) - \alpha h(\frac{r}{2}+\beta)\right]}$, $\Omega_1 = r+\psi$, $\Omega_2 = r-\psi$

Proof. Because the problem is infinite time limit and the system has steady state equilibrium, the boundary conditions of equation (9) is $g(0) = g_0$ and $\lim_{T\to\infty} g(T) = \bar{g}$, solving it we can get that,

$$g^M(t) = \left[g_0 - \frac{\alpha(r+\beta)(k+h\Delta)}{2\beta(r+\beta)-\alpha h(r+2\beta)}\right]e^{\frac{\Omega_2}{2}t} + \frac{\alpha(r+\beta)(k+h\Delta)}{2\beta(r+\beta)-\alpha h(r+2\beta)} \tag{11}$$

$$\rho_m(t) = \frac{k + h[\Delta + g^M(t)]}{2(r+\beta)-h\alpha} \tag{12}$$

Thus, the formula (10) is obtained.

Proposition 3. Under the manufacturing-led recycling model, the state and control strategy of the supply chain system gradually converges to a stable state over time, including: the amount of subsidies granted by the government and the recycling price paid by the manufacturing company to consumers over time. It gradually increases and is always smaller than the steady-state equilibrium value.

Proof. $g_0 < \bar{g}^M$ , $\frac{\partial g^M}{\partial t} = \frac{\Omega_2 (g_0 - \bar{g}^M) e^{\frac{\Omega_2}{2}t}}{2} > 0$

To prove $\frac{\partial b^M}{\partial t} = \frac{r+\beta}{2(r+\beta)-h\alpha} \bullet \frac{\partial g^M}{\partial t} > 0$ , only need prove $2(r+\beta)-h\alpha > 0$

Due to $\beta(r+\beta)-\alpha h(\frac{r}{2}+\beta) > 0$ , $r+\beta-h\alpha > 0$ ,

Therefore $2(r+\beta)-h\alpha > 0$ .

### 3.2 O2O Platform-led Recycling Model

When the O2O platform in the supply chain is responsible for product recycling, the manufacturing company determines $w(t)$ and $b_m(t)$ , which are the wholesale price of the product and the recycling price paid to the O2O platform. The O2O platform determines $p(t)$ and $b_o(t)$ , which are the selling price of the product and the recycling price paid to the consumer. The profit functions of manufacturing companies and O2O platforms are

$$\Pi_{\substack{m \\ w,b_m}} = \int_0^\infty e^{-rt} \left[ (w - C_m) D + (\Delta - b_m) Q \right] dt \tag{13}$$

$$\Pi_{\substack{o \\ p,b_o}} = \int_0^\infty e^{-rt} \left[ (p - w) D + (b_m - b_o + g) Q - C_2 \right] dt \tag{14}$$

The constraints are $\dot{g}(t) = \alpha Q(t) - \beta g(t)$

The Hamilton function of the O2O platform and manufacturing companies are

$$H_o(p,w,b_o,b_m,g,\rho_o) = (p-w)(\gamma-\mu p) + (b_m - b_o + g)(k+hb_o) - C_2 + \rho_o\left[\alpha(k+hb_0) - \beta g\right] \tag{15}$$

$$H_m(p,w,b_o,b_m,g,\rho_m) = (w-C_m)(\gamma-\mu p) + (\Delta - b_m)(k+hb_o) + \rho_m\left[\alpha(k+hb_o) - \beta g\right] \tag{16}$$

The optimal decision variables that can be solved for manufacturing companies and O2O platforms are

$$w = \frac{\gamma + \mu C_m}{2\mu}$$

$$p = \frac{3\gamma + \mu C_m}{4\mu}$$

$$b_m = \frac{1}{2}\left(\Delta - g + \alpha\rho_m - \alpha\rho_o - \frac{k}{h}\right)$$

$$b_o = \frac{1}{4}\left(g - \frac{3k}{h} + \Delta + \alpha\rho_m + \alpha\rho_o\right)$$

In addition $$\dot{g} = \left(\frac{\alpha h}{4} - \beta\right)g + \frac{\alpha}{4}\left[k + h(\alpha\rho_m + \alpha\rho_o + \Delta)\right] \tag{17}$$

$$\dot{\rho}_m = (r + \beta - \frac{\alpha h}{4})\rho_m - \frac{h}{4}\left(\frac{k}{h} + \Delta + g + \alpha\rho_o\right) \tag{18}$$

$$\dot{\rho}_o = (r + \beta - \frac{\alpha h}{4})\rho_o - \frac{1}{4}\left[h(\Delta + g + \alpha\rho_m) + k\right] \tag{19}$$

Proposition 4. When the influence coefficient of the recovered quantity is $\alpha < \frac{4\beta(r+\beta)}{h(r+3\beta)}$, the closed-loop supply chain recovered by the O2O platform has a steady-state equilibrium. When steady state is reached, that $\bar{g}^O = \frac{\alpha(k+h\Delta)(r+\beta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$, $\overline{\rho_m}^O = \overline{\rho_o}^O = \frac{\beta(k+h\Delta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$ is established. The recovery price of the manufacturing enterprise is $\bar{b}_m = \frac{h\Delta - k}{2h} - \frac{1}{2}\bar{g}^O$, and the recovery price of the O2O platform is $\bar{b}_o = \frac{h\Delta - 3k}{4h} + \frac{r+3\beta}{4(r+\beta)}\bar{g}^O$.

Proof. Divide the two sides of equation (17) with respect to t, we can get that $\ddot{g} = \left(\frac{\alpha h}{4} - \beta\right)\dot{g} + \frac{\alpha^2 h}{4}(\dot{\rho}_o + \dot{\rho}_m)$ (20)

Substituting equations (18) and (19) into (20) yields

$$\ddot{g} = \left(\frac{\alpha h}{4} - \beta\right)\dot{g} + \frac{\alpha^2 h}{4}\left[(r+\beta-\frac{\alpha h}{2})(\rho_o+\rho_m) - \frac{h}{2}(\Delta+g) - \frac{k}{2}\right] \quad (21)$$

From equation (17) $\rho_o + \rho_m = \frac{1}{\alpha}\left\{\frac{4}{\alpha h}\left[\dot{g} - \left(\frac{\alpha h}{4} - \beta\right)g\right] - \frac{k}{h} - \Delta\right\}$ (22)

Substituting equation (22) into (21),

$$\ddot{g} - \left(r - \frac{\alpha h}{4}\right)\dot{g} - \left[\beta(r+\beta) - \frac{\alpha h}{4}(r+3\beta)\right]g = -\frac{\alpha}{4}(r+\beta)(k+h\Delta) \quad (23)$$

When the system reaches stability, the instantaneous profit of the manufacturing company is $\overline{\pi_m}^O = \frac{1}{8}\left[\frac{(\gamma-\mu C_m)^2}{\mu} + (k+h\Delta)^2 + \frac{2h(r+2\beta)(k+h\Delta)}{(r+\beta)}\bar{g}^O + \frac{h^2(r+3\beta)}{(r+\beta)}\left(\bar{g}^O\right)^2\right]$, the instantaneous profit of the O2O platform is $\overline{\pi_o}^O = \frac{(\gamma-\mu C_m)^2}{16\mu} + \frac{(k+h\Delta)^2}{16h} + \frac{k+h\Delta}{8}\bar{g}^O + \frac{h(r-\beta)(r+3\beta)}{16(r+\beta)^2}\left(\bar{g}^O\right)^2 - C_2$.

Proposition 5. When the O2O platform leads recycling, the optimal recycling price for the manufacturing company is $b_m(t) = \frac{h\Delta - k}{2h} - \frac{1}{2}\left[\left(g_0 - \bar{g}^O\right)e^{\frac{Z_2}{2}t} + \bar{g}^O\right]$. The optimal recycling price for the O2O platform is: $b_o(t) = \frac{h\Delta - 3k}{4h} + \frac{r+3\beta}{4(r+\beta)}\left[\left(g_0 - \bar{g}^O\right)e^{\frac{Z_2}{2}t} + \bar{g}^O\right]$

$$\xi = \sqrt{\left(r - \frac{\alpha h}{4}\right)^2 + 4\left[\beta(r+\beta) - \frac{\alpha h}{4}(r+3\beta)\right]},\ Z_1 = r - \frac{\alpha h}{4} + \xi,\ Z_2 = r - \frac{\alpha h}{4} - \xi$$

Proof. The boundary conditions of the differential equation (23) is $g(0) = g_0$ and $\lim_{T\to\infty} g(T) = \bar{g}$, the solution is

$$g^O(t) = \left[g_0 - \frac{\alpha(k+h\Delta)(r+\beta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}\right]e^{\frac{Z_2}{2}t} + \frac{\alpha(k+h\Delta)(r+\beta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$$

$$\rho_m(t) = \rho_o(t) = \frac{\beta}{\alpha(r+\beta)}\left[g_0 - \frac{\alpha(k+h\Delta)(r+\beta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}\right]e^{\frac{Z_2}{2}t} + \frac{\beta(k+h\Delta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$$

Then we can obtain the optimal recycling price for manufacturing companies and O2O platforms.

Proposition 6. When the O2O platform dominates recycling, the state and control strategy of the supply chain system gradually converges to a stable state over time. The subsidies granted by the government gradually increase over time and are always less than the amount of subsidies at the time of stabilization. The recycling price paid by the manufacturing company to the O2O platform decreases over time, and the recycling price paid by the O2O platform to consumers increases over time.

Proof. $g_0 < \bar{g}^O$, 故 $\frac{\partial g^O}{\partial t} = \frac{Z_2(g_0 - \bar{g}^O)e^{\frac{Z_2}{2}t}}{2} > 0$

$$\frac{\partial b_m}{\partial t} = -\frac{1}{2} \bullet \frac{\partial g^O}{\partial t} < 0, \quad \frac{\partial b_o}{\partial t} = \frac{r+3\beta}{4(r+\beta)} \bullet \frac{\partial g^O}{\partial t} > 0$$

### 3.3 Comparison of Recycling Models

Proposition 7. When the system reaches steady state, there is less recycling quantity under the leadership of the O2O platform. When $\alpha < \frac{\beta(r+\beta)}{h(r+2\beta)}$, it exists that $\bar{b}^M > \bar{b}_m > \bar{b}_o$. That is to say, the recycling price paid by the manufacturing company to the consumer is higher.

Proof. $\bar{b}^M = \frac{h(\Delta + \bar{g}^M + \overline{\rho_m}\alpha) - k}{2h}$, $\bar{b}_m = \frac{h\Delta - k}{2h} - \frac{1}{2}\bar{g}^O$, $\bar{b}_o = \frac{h\Delta - 3k}{4h} + \frac{r+3\beta}{4(r+\beta)}\bar{g}^O$

$$\bar{b}^M = \frac{h\Delta - k}{2h} + \frac{\alpha(k+h\Delta)}{4(r+\beta)-2h\alpha} + \frac{(r+\beta)}{2(r+\beta)-h\alpha}\bar{g}^M$$

$2(r+\beta)-h\alpha > 0$ 且 $\bar{g}^M$, $\bar{g}^O > 0$, so $\bar{b}^M > \bar{b}_m$, $\bar{b}^M > \bar{b}_O$ is always established.

$\bar{b}_m - \bar{b}_o = \frac{[\beta(r+\beta)-\alpha h(r+2\beta)](k+h\Delta)}{h[4\beta(r+\beta)-\alpha h(r+3\beta)]}$ so when $\alpha < \frac{\beta(r+\beta)}{h(r+2\beta)}$, it exists $\bar{b}_m > \bar{b}_o$.

$$Q = k + hb, \quad Q^M = \frac{\beta(r+\beta)(k+h\Delta)}{2\beta(r+\beta)-\alpha h(r+2\beta)}, \quad Q^O = \frac{\beta(r+\beta)(k+h\Delta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$$

Known from Proposition 3, so

$$2(r+\beta)-h\alpha > 0, \quad 2\beta(r+\beta)-\alpha h(r+2\beta) < 4\beta(r+\beta)-\alpha h(r+3\beta), \quad Q^M > Q^O$$

It can be seen from Proposition 7 that the recycling price of the manufacturing enterprises is higher and the recycling quantity is higher. On the contrary, the recovery mode dominated by the O2O platform is lower in recycling price and less in recycling. Since the manufacturing company can remanufacture the recycled products, the O2O platform needs to entrust other companies to recycle and reuse the products, so the manufacturing companies can use the recycled products more efficiently, so that consumers can pay higher recycling prices. This in turn leads to a higher amount of recycling.

Proposition 8. When steady state equilibrium is reached, the amounts of subsidies granted by the government under the leading recycling of manufacturing enterprises is higher than the O2O platform leading recovery.

Proof. $\bar{g}^M = \frac{\alpha(r+\beta)(k+h\Delta)}{2\beta(r+\beta)-\alpha h(r+2\beta)}$, $\bar{g}^O = \frac{\alpha(k+h\Delta)(r+\beta)}{4\beta(r+\beta)-\alpha h(r+3\beta)}$,

$2\beta(r+\beta)-\alpha h(r+2\beta) < 4\beta(r+\beta)-\alpha h(r+3\beta)$, so $\bar{g}^M > \bar{g}^O$.

The recycling amount under the manufacturing-led recycling model is larger. The subsidy amount granted by the government is mainly determined by the amount of recycling. Therefore, the subsidy amount granted by the government under the leadership of manufacturing enterprises is higher. Government subsidies for manufacturing companies can encourage companies to green manufacturing, reduce pollution from the source to improve ecological performance, and promote green supply throughout the supply chain.

Proposition 9. Manufacturing companies will choose their own recycling channels when $\frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \left\{ \frac{r+\beta-\alpha h}{[2\beta(r+\beta)-\alpha h(r+2\beta)]^2} - \frac{2(r+\beta)-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2} \right\} > C_1$. On the contrary, they will choose the O2O platform for recycling. The O2O platform dominates recycling and makes a big profit when $\frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \frac{r+\beta-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2} > C_2$.

Proof. When manufacturing companies and O2O platforms dominate recycling, the instantaneous profits of the manufacturing company that can be obtained.

$$\overline{\pi_m}^M = \frac{(\gamma-\mu C_m)^2}{8\mu} + \frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \frac{r+\beta-\alpha h}{[2\beta(r+\beta)-\alpha h(r+2\beta)]^2} - C_1$$

$$\overline{\pi_m}^O = \frac{(\gamma-\mu C_m)^2}{8\mu} + \frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \frac{2(r+\beta)-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2}$$

Therefore, when $\frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \left\{ \frac{r+\beta-\alpha h}{[2\beta(r+\beta)-\alpha h(r+2\beta)]^2} - \frac{2(r+\beta)-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2} \right\} > C_1$, it exists $\overline{\pi_m}^M > \overline{\pi_m}^O$, manufacturers will choose their own recycling channels. Instead, they will choose O2O platform recycling.

When manufacturing companies and O2O platforms dominate recycling, the instantaneous profits of the O2O platforms that can be obtained.

$$\overline{\pi_o}^M = \frac{(\gamma-\mu C_m)^2}{16\mu}$$

$$\overline{\pi_o}^O = \frac{(\gamma-\mu C_m)^2}{16\mu} + \frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \frac{r+\beta-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2} - C_2$$

When $\frac{\beta^2(k+h\Delta)^2(r+\beta)}{h} \bullet \frac{r+\beta-\alpha h}{[4\beta(r+\beta)-\alpha h(r+3\beta)]^2} > C_2$, it exists $\overline{\pi_o}^M < \overline{\pi_o}^O$. The O2O platform is more profitable when it leads recycling.

Proposition 9 illustrates that the recycling strategy of a manufacturing company or O2O platform depends on its own recycling costs. When the recycling cost is low, the manufacturing company will choose to recycle itself. If the manufacturing company cannot effectively control the recycling cost, it can only be entrusted to the O2O platform for recycling. If the O2O platform cannot control the cost of recycling, it will lose the meaning of the existence of the platform. Therefore, after a period of time, the large-scale O2O platform is difficult to control costs, the economic profit margin will become smaller and smaller, and some platforms have to withdraw the recycling market.

## 4. Conclusion

Based on the closed-loop supply chain of consumer electronics remanufacturing in O2O mode, this paper establishes the recovery decision model of two dominant modes of manufacturing enterprise and O2O platform, and uses the differential game theory to give the steady-state equilibrium solution of different modes. The main conclusions of the study are as follows: (1) Under the two recycling modes, the recovery price, the amount of recycling, the amount of government subsidies, and the instantaneous profit of the participants all showed an upward trend, and finally stabilized. (2) Compared with the recycling mode dominated by the O2O platform, the recovery mode of the manufacturing-led recycling mode is higher, and the recycling quantity is larger. (3) Compared with the O2O platform recovery, the government subsidies are higher when the manufacturing enterprises lead recycling in the steady-state equilibrium. (4) The recycling channel selection strategy of the manufacturing enterprise or O2O platform depends on its own recycling cost. When the recycling cost is small, it prefers its own leading recycling. When the cost increases, the profit margin becomes smaller, and the enthusiasm for participating in recycling will decrease. Numerical examples show that if the O2O platform can effectively control the cost of the recycling process, the manufacturing company can make the O2O company responsible for recycling, and the manufacturing company is responsible for the post-recycling treatment and remanufacturing, which can increase the mutual benefits.

This paper also has certain limitations. The data in the numerical examples are based on the empirical values of the relevant literature. In the future, the first-hand data of the enterprise can be collected through interviews for calculation.

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