

A Fuzzy Multi-Attribute Group Decision Making Method Based on Particle Swarm Optimization algorithm

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Abstract

This paper proposes a group decision making method based on particle swarm optimization for fuzzy multi-attribute group decision making (FMGDM) problem with attribute values as linguistic variables. For a given linguistic evaluation set, the particle swarm optimization algorithm is used to adjust the main value points of the corresponding triangular fuzzy numbers of the linguistic variables under a uniform distribution to obtain a well-consistent individual decision matrix. Then the gray correlation analysis method based on triangular fuzzy numbers is used to sort the fault modes of the A-frame and the boom group in the main structure system of the LIUHUA10-1CEP crane.

Keywords

Fuzzy multi-attribute group decision making; particle swarm optimization algorithm; linguistic evaluation set; triangular fuzzy grey relational analysis.

1. Introduction

Multi-attribute group decision making problem is generally based on multiple experts comparing multiple attributes to a set of programs, establishing an individual decision matrix ^[1], and then rationally assembling individual decision matrices according to certain criteria to form a consistent or compromised group decision matrix. Finally, through a certain way, the attribute evaluation information of each scheme in the group decision matrix is assembled and the scheme is ranked optimally ^[2]. In the actual multi-attribute group decision-making process, due to the limitations of experts from different professional fields on the decision-making problem, the research on multi-attribute group decision making under uncertain environment becomes a hot topic. At present, there are mainly fuzzy, random, rough and multiple uncertainty multi-attribute group decision theory and methods ^[3]. Among them, fuzzy multi-attribute group decision making is based on Zadeh fuzzy set, which mainly includes multi-attribute group decision based on language evaluation information and multi-attribute group decision based on various types of fuzzy numbers ^[4]. For complex group decision-making problems, experts tend to give qualitative variables such as “very good”, “very good”, “poor”, and “very poor”. Experts may use different linguistic variables for different decision-making problems. This difference includes the number of linguistic variables (dimensions of the evaluation set) and their corresponding membership functions. Experimental psychology proves that the level of attribute that humans can correctly distinguish is between 5 and 9, that is, the range of language evaluation sets ranges from 5 to 9 ^[5]. The qualitative linguistic variables usually given by the expert group are ambiguous and uncertain. In the existing literature, one method of dealing with attribute values as linguistic variables is based on the extended principle method ^[6]. The linguistic variable is corresponding to the fuzzy number, and the membership function of the fuzzy number is used to perform information aggregation and scheme ordering. The research based on the evaluation information of each type of fuzzy numbers mainly focuses on interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers and so on. As a typical fuzzy form, the triangular fuzzy number is compared with other fuzzy forms. It quantitatively evaluates the decision object and considers the membership degree. The most important thing is that the number of membership degrees of each real number in a concept is unique^[7], which improves the accuracy of the results while simplifying the group decision process. The conversion of linguistic variables and triangular fuzzy numbers in the existing literature has yielded some results. Chiou and Tzeng used the concept of linguistic variables

proposed by Zadeh to set their correspondence with triangular fuzzy numbers in the evaluation process. Wang Jun and Fan Zhiping elaborated the correspondence between triangular fuzzy numbers and linguistic variables. Experts only need to use linguistic variables in the evaluation process. Chen Xiaohong and Yang shuo also discussed the conversion of linguistic information and triangular fuzzy numbers, and used to solve multi-attribute group decision making. Liu faced mixed multi-attribute decision making, and used triangular fuzzy numbers and linguistic variables to unify decision information. Yang Jing and Qiu Yuhua discuss a triangular fuzzy number multi-attribute sorting method based on projection technology. During the research process, Sari was asked to use linguistic variables in the range of 1-9 scale when giving judgments, and the corresponding triangular fuzzy number researchers were set in advance ^[4]. This paper selects the triangular fuzzy number as the transformation form of the linguistic variable for calculation. In the actual group decision-making process, the decision matrices given by the experts based on the information they have mastered may have large differences, and even conflicts may arise. If these individual decision matrices with large differences are directly assembled, the final decision may be affected rationality of the results ^[8]. Therefore, a core issue of group decision-making is the consistency study of group decision results and the opinions of individual decision makers, that is, the consistency analysis of group decision making. Consistency includes the process of individual deviation (inconsistency) and consistency from group decision results, and is a dynamic, iterative group decision process ^[9]. Herrera-Viedma studied the problem of uniformity in decision-making problems of different preference structure groups, and proposed to measure the degree of consistency by the distance between individual decision matrix and group decision matrix ^[10]. The smaller the average deviation of multiple experts, the better the group decision consistency. Most of the methods of consistency in the literature have been to adjust the individual decision-making preference information to achieve a predetermined level of consensus. In the literature [7], Witold Pedrycz and Mingli Song proposed the decision matrix of the linguistic variables for the attribute values given by the experts, and defined the mapping relationship between the linguistic variables and the interval numbers under the uniform distribution, and established the adjustment linguistic variables in 1-9. The corresponding position on the scale thus minimizes the optimization model of the group inconsistency, and the group inconsistency is used as the fitness function to solve by the particle swarm optimization algorithm. Unlike most of the existing results, Pedrycz et al. proposed a predetermined level of consistency by adjusting the corresponding positions of linguistic variables on the 1-9 scale, in contrast to avoiding artificial consensus.

The A-frame and boom components in the main structural system of the LIUHUA10-1CEP crane of a platform in the oilfield flow operation area are the core components of the crane. Therefore, if the component fails, there will be serious safety hazards and damage will occur huge economic loss. In order to ensure the accuracy of multiple failure mode risk analysis, in this paper, under the three criteria of frequency of occurrence, difficulty of detection and severity (taking safety impact as an example), quantitative risk analysis of structural deformation, weld cracking, corrosion and other failure modes is carried out. The expert group's qualitative risk assessment of each evaluation criterion for each failure mode can be regarded as a fuzzy multi-attribute group decision problem with attribute values as linguistic variables.

Based on the above analysis, the first part of this paper briefly reviews the basic concepts of some triangular fuzzy numbers. The second part converts the linguistic decision matrix given by experts into a more consistent decision matrix based on particle swarm optimization. The third part is for the structural deformation of the A-frame and boom components in the main structural system of the LIUHUA10-1CEP crane, weld cracking, corrosion and other failure modes based on the triangular fuzzy number-based gray correlation analysis method for quantitative risk analysis.

2. Preliminary

Definition 1 If a triangular fuzzy number is represented as $M = (l, m, u)$, its membership function is $\mu(x): X \rightarrow [0,1]$, expressed as follows:

$$\mu(x) = \begin{cases} 0, & x \leq l \\ \frac{x-l}{m-l}, & l < x < m \\ 1, & x = m \\ \frac{u-x}{u-m}, & m < x < u \\ 0, & x \geq u \end{cases}$$

Among them, x is real number, and l and u is the lower and upper bounds, respectively. m is the main value point.

To set a triangular fuzzy number $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, then the algorithm is:

$$\begin{aligned} M_1 + M_2 &= (l_1, m_1, u_1) + (l_2, m_2, u_2) \\ &= (l_1 + l_2, m_1 + m_2, u_1 + u_2) \end{aligned}$$

$$\begin{aligned} M_1 \times M_2 &= (l_1, m_1, u_1) \times (l_2, m_2, u_2) \\ &= (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2) \end{aligned}$$

$$\begin{aligned} M_1 - M_2 &= (l_1, m_1, u_1) - (l_2, m_2, u_2) \\ &= (l_1 - l_2, m_1 - m_2, u_1 - u_2) \end{aligned}$$

$$\lambda \times M_2 = \begin{cases} (\lambda \times l_2, \lambda \times m_2, \lambda \times u_2), & \lambda \geq 0 \\ (\lambda \times u_2, \lambda \times m_2, \lambda \times l_2), & \lambda \leq 0 \end{cases}$$

There are many defuzzification mathematical reasoning methods in fuzzy set theory, among which the center of gravity defuzzification method is recognized as the relatively good logic and rigor in mathematics^[11]. For triangular fuzzy numbers $M = (l, m, u)$, after the center of gravity is blurred:

$$Y = \frac{l + m + u}{3}$$

Five-dimensional linguistic evaluation set:

$$S = \{\text{very high(VH), high(H), medium(M), low(L), very low(VL)}\}$$

The linguistic variables are evenly distributed on the scale of 1-9, and the corresponding standard triangular fuzzy numbers are shown in Fig. 1.

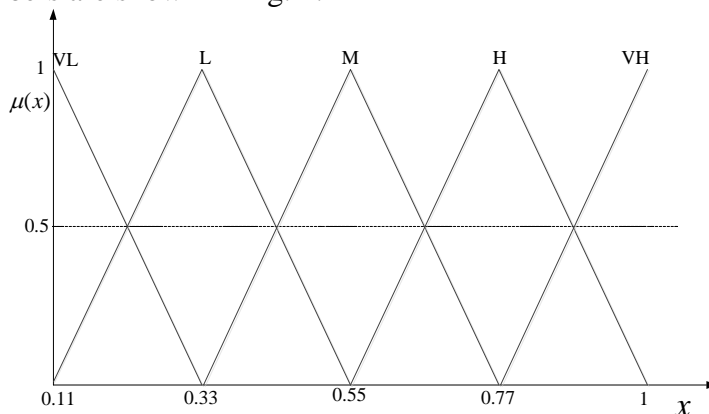


Figure 1 triangular fuzzy numbers

For the convenience of calculation, if the main value point of the triangular fuzzy number corresponding to each linguistic variable is $L=(l_1,l_2,l_3,l_4,l_5)$, the triangular fuzzy number corresponding to the linguistic variable may also be expressed in the form of Table 1. In particular, triangular fuzzy numbers can be represented as table 1 when $L=(0.11,0.33,0.55,0.77,1)$.

Table 1 triangular fuzzy numbers

linguistic variable	triangular fuzzy numbers	
Very high (VH)	$(l_4,l_5,1)$	$(0.77,1,1)$
high (H)	(l_3,l_4,l_5)	$(0.55,0.77,1)$
medium (M)	(l_2,l_3,l_4)	$(0.33,0.55,0.77)$
low (L)	(l_1,l_2,l_3)	$(0.11,0.33,0.55)$
Very low (VL)	$(0,l_1,l_2)$	$(0,0.11,0.33)$

In a group decision problem, the expert set $E = \{e_1, e_2 \dots e_n\}$, the decision object set $O = \{o_1, o_2 \dots o_p\}$, the decision criteria set $A = \{a_1, a_2 \dots a_m\}$, and the linguistic variable set $S = \{s_l | l = 0, \dots, t\}$, t are even numbers. Each expert corresponds to a set of weights $W = \{w_1, w_2 \dots w_n\}$, $\sum_{k=1}^n w_k = 1, w_k \geq 0$. The decision matrix of the linguistic variable given by the expert e_k is transformed into a decision matrix (triangular fuzzy decision matrix) $D_k = (M_{ij}^k)_{p \times m}$, $M_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$.Which is represented by the expert e_k evaluation of the language of the decision object o_i for the decision criterion a_j , $1 \leq k \leq n, 1 \leq i \leq p, 1 \leq j \leq m$. According to the literature [12], the group aggregation decision matrix (referred to as the group decision matrix) $G = (M_{ij})_{p \times m}$, $M_{ij} = (l_{ij}, m_{ij}, u_{ij})$:

$$M_{ij} = \sum_{k=1}^n w_k M_{ij}^k \tag{1}$$

The distance between the individual decision matrix e_k and the group decision matrix G , that is the individual deviation of the expert e_k from the group decision, can be expressed as:

$$d(e_k, G) = \frac{1}{p \times m} \sum_{i=1}^p \sum_{j=1}^m d(M_{ij}^k, M_{ij}) \tag{2}$$

$d(M_{ij}^k, M_{ij})$ is the distance between the triangular fuzzy numbers M_{ij}^k and M_{ij} .

Definition 2 Set any two triangular fuzzy numbers $M_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$, $M_{ij} = (l_{ij}, m_{ij}, u_{ij})$, and the distance between M_{ij}^k and M_{ij} is:

$$d(M_{ij}^k, M_{ij}) = \left(\frac{1}{3} T_0^2 + \frac{1}{12} T_1^2 + \frac{1}{36} T_2^2 + \frac{1}{6} T_0 T_1 \right)^{\frac{1}{2}} \tag{3}$$

$$T_0 = m_{ij}^k - m_{ij} , \quad T_1 = (u_{ij}^k + u_{ij}) + (l_{ij}^k + l_{ij}) , \quad T_2 = (u_{ij}^k - u_{ij}) - (l_{ij}^k - l_{ij}) .$$

According to the literature [7], the group inconsistency can be expressed as the sum of the individual deviations of multiple experts. The specific expressions are as follows:

$$Q = \frac{\sum_{k=1}^n d(e_k, G)}{n} \tag{4}$$

3. Quantification of language decision table based on particle swarm optimization

For example, the A-frame and boom components in the main structural system of the LIUHUA10-1CEP crane of a platform in an oilfield flow operation area are used to quantitatively analyze the

failure mode. Three teams analyzed failure modes and impacts on important functional products in the A-frame and boom assemblies, resulting in three different analysis reports. Expert set $E = \{e_1, e_2, e_3\}$, using fuzzy language set $S = \{\text{very high(VH), high(H), medium(M), low(L), very low(VL)}\}$, the evaluation frequency of the risk assessment criteria corresponding to 13 failure modes such as F11: structural deformation, F12: weld cracking, the difficulty of detection, and the severity (taking safety impact as an example) , as shown in Tables 2, 3, and 4.

Table 2 Decision table given by experts e_1

Failure mode	occurrence frequency	detection difficulty	safety impact
F11: Structural deformation	M	M	VL
F12: Weld cracking	M	M	VL
F13: Corrosion	M	L	VL
F21: deformation	M	L	VH
F22: wear and tear	M	L	VL
F31: Structural deformation	H	M	VH
F32: weld cracking	H	M	L
F41: Corrosion damage	VL	L	VH
F42: Welding crack	L	VL	VH
F43: Flange deformation	L	L	VH
F51: deformation, bending	L	VL	L
F52: wear and tear	H	L	VL
F53: Break	VL	VL	H

Table3 Decision table given by experts e_2

Failure mode	occurrence frequency	detection difficulty	safety impact
F11: Structural deformation	M	M	VL
F12: Weld cracking	H	M	VL
F13: Corrosion	H	M	VL
F21: deformation	H	L	VH
F22: wear and tear	M	M	VH
F31: Structural deformation	H	L	VH
F32: weld cracking	H	M	VH
F41: Corrosion damage	L	L	VH
F42: Welding crack	M	L	VH
F43: Flange deformation	L	L	VH
F51: deformation, bending	M	L	VH
F52: wear and tear	H	M	VH
F53: Break	VL	L	H

Table 4 Decision table given by experts e_3

Failure mode	occurrence frequency	detection difficulty	safety impact
F11: Structural deformation	M	L	VL
F12: Weld cracking	M	L	VL
F13: Corrosion	H	M	VL
F21: deformation	M	L	VL
F22: wear and tear	H	M	VL
F31: Structural deformation	H	L	VL
F32: weld cracking	H	M	VL
F41: Corrosion damage	VL	VL	VL
F42: Welding crack	L	VL	VL

F43: Flange deformation	L	VL	VL
F51: deformation, bending	L	L	VL
F52: wear and tear	H	M	VL
F53: Break	VL	L	VL

A decision matrix $D_k, k = 1, 2, 3$ that converts it into a triangular fuzzy number of attribute values based on table 1. Therefore, based on equation (4), the following group consistency optimization model is established:

$$\begin{aligned} \min Q(L) &= \frac{1}{n} \sum_{k=1}^n d(D_k, G) \\ &= \frac{1}{n \times p \times m} \sum_{k=1}^n \sum_{i=1}^p \sum_{j=1}^m d(M_{ij}^k, M_{ij}) \\ \text{s.t. } &0 < l_1 < l_2 < l_3 < l_4 < l_5 < 1 \end{aligned}$$

This model is a nonlinear optimization problem. In order to obtain an optimal solution that satisfies the conditions and has good group consistency, this paper uses the particle swarm optimization algorithm to solve. Particle Swarm Optimization (PSO) algorithm is an optimization algorithm based on biological population behavior research [13-16]. The PSO algorithm guides the whole particle swarm to obtain the optimal solution by iterative search by simulating the individual and group behavior during the flight of the flock. It has good group intelligence and memory function, and can solve the multidimensional nonlinear problem faster. Excellent solution. The specific algorithm steps for solving the model using the PSO algorithm are as follows:

Step1 Sets the total number of clusters to be N . The total update algebra is T_{\max} , $L = (l_1, l_2, l_3, l_4, l_5)$ which is regarded as a point on the 5-dimensional space as a particle. In the iteration f , the position of the particle i can be expressed as:

$$X_i(f) = (x_{i,1}(f), x_{i,2}(f), x_{i,3}(f), x_{i,4}(f), x_{i,5}(f)) \quad , \quad x_{i,j}(f) \in [x_{\min}, x_{\max}]$$

Its corresponding velocity vector is:

$$V_i(f) = (v_{i,1}(f), v_{i,2}(f), v_{i,3}(f), v_{i,4}(f), v_{i,5}(f)) \quad , \quad v_{i,j}(f) \in [v_{\min}, v_{\max}]$$

$i = 1, 2, \dots, N, \quad j = 1, 2, \dots, 5, \quad f = 1, 2, \dots, T_{\max}$;

Step2 After each iteration, the particle updates its speed and position according to the following formula:

$$\begin{aligned} V_i(f+1) &= \omega V_i(f) + c_1 r_1 (pbest_i(f) - X_i(f)) + c_2 r_2 (pbest_g(f) - X_i(f)) \\ x_i(f+1) &= X_i(f) + V_i(f+1) \end{aligned}$$

Where, the inertia weight $\omega = 0.9 - (0.9 - 0.4) * f / T_{\max}$, the learning factor $c_1 = c_2 = 2$, r_1, r_2 is a random number generated independently within [0,1];

Step3 Calculates the fitness value Q corresponding to each particle position in the particle group according to the fitness function, and compares and searches the best position $pbest_i$ that the i particle has experienced in the iteration, called the individual optimal solution, and the entire particle group in the iteration. The optimal location $pbest_g$ found is called the population optimal solution. When the maximum number of iterations is reached, $f \leq T_{\max}$, the algorithm terminates. The adjusted interval triangular fuzzy number decision matrix with better consistency can be obtained. Then the pso optimization group inconsistency process is shown in Figure 2.

It can be seen from the figure that within the maximum number of iterations, the algorithm iterates to 5 times, the group inconsistency Q from 0.5456 to 0.4423. The algorithm I optimizes the truncation point of the ambiguous fuzzy number corresponding to the linguistic variable as shown in Fig. 3.

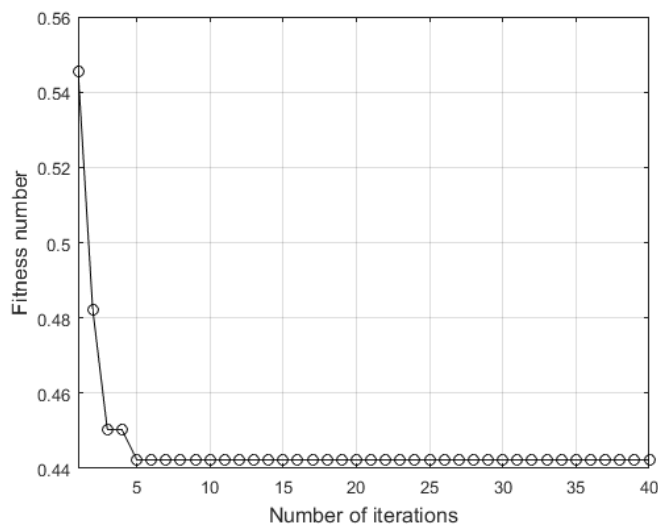


Figure 2 Pso optimization process

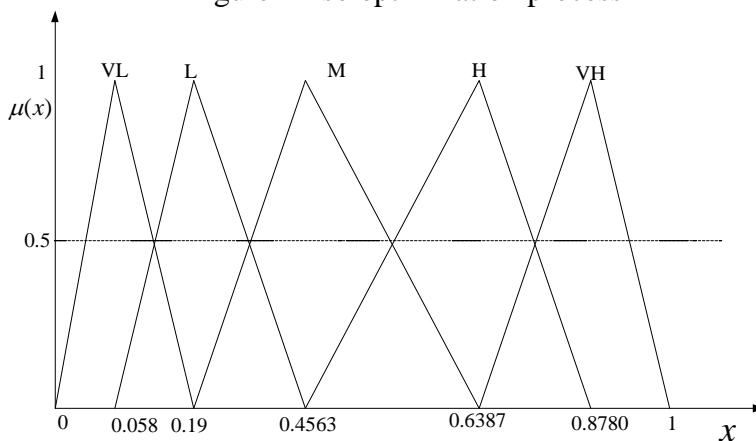


Fig. 3 Optimized triangular fuzzy numbers

The resulting decision matrix that satisfies the conditions and has good group consistency is as follows:

$$D_1 = \begin{bmatrix} (0.19, 0.4526, 0.6387) & (0.19, 0.4526, 0.6387) & (0, 0.058, 0.19) \\ (0.19, 0.4526, 0.6387) & (0.19, 0.4526, 0.6387) & (0, 0.058, 0.19) \\ (0.19, 0.4526, 0.6387) & (0.058, 0.19, 0.4536) & (0, 0.058, 0.19) \\ (0.19, 0.4526, 0.6387) & (0.058, 0.19, 0.4536) & (0.6387, 0.878, 1) \\ (0.19, 0.4526, 0.6387) & (0.058, 0.19, 0.4536) & (0, 0.058, 0.19) \\ (0.4536, 0.6387, 0.878) & (0.19, 0.4526, 0.6387) & (0.6387, 0.878, 1) \\ (0.4536, 0.6387, 0.878) & (0.19, 0.4526, 0.6387) & (0.058, 0.19, 0.4536) \\ (0, 0.058, 0.19) & (0.058, 0.19, 0.4536) & (0.6387, 0.878, 1) \\ (0.058, 0.19, 0.4536) & (0, 0.058, 0.19) & (0.6387, 0.878, 1) \\ (0.058, 0.19, 0.4536) & (0.058, 0.19, 0.4536) & (0.6387, 0.878, 1) \\ (0.058, 0.19, 0.4536) & (0, 0.058, 0.19) & (0.058, 0.19, 0.4536) \\ (0.4536, 0.6387, 0.878) & (0.058, 0.19, 0.4536) & (0, 0.058, 0.19) \\ (0, 0.058, 0.19) & (0, 0.058, 0.19) & (0.4536, 0.6387, 0.878) \end{bmatrix}$$

$$D_2 = \begin{bmatrix} (0.19,0.4526,0.6387) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.19,0.4526,0.6387) & (0.19,0.4526,0.6387) & (0.6387,0.878,1) \\ (0.4536,0.6387,0.878) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0.6387,0.878,1) \\ (0.058,0.19,0.4536) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.19,0.4526,0.6387) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.058,0.19,0.4536) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.19,0.4526,0.6387) & (0.058,0.19,0.4536) & (0.6387,0.878,1) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0.6387,0.878,1) \\ (0,0.058,0.19) & (0.058,0.19,0.4536) & (0.4536,0.6387,0.878) \end{bmatrix}$$

$$D_3 = \begin{bmatrix} (0.19,0.4526,0.6387) & (0.058,0.19,0.4536) & (0,0.058,0.19) \\ (0.19,0.4526,0.6387) & (0.058,0.19,0.4536) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0.19,0.4526,0.6387) & (0.058,0.19,0.4536) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.058,0.19,0.4536) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0,0.058,0.19) & (0,0.058,0.19) & (0,0.058,0.19) \\ (0.058,0.19,0.4536) & (0,0.058,0.19) & (0,0.058,0.19) \\ (0.058,0.19,0.4536) & (0,0.058,0.19) & (0,0.058,0.19) \\ (0.058,0.19,0.4536) & (0.058,0.19,0.4536) & (0,0.058,0.19) \\ (0.4536,0.6387,0.878) & (0.19,0.4526,0.6387) & (0,0.058,0.19) \\ (0,0.058,0.19) & (0.058,0.19,0.4536) & (0,0.058,0.19) \end{bmatrix}$$

4. Triangular fuzzy gray correlation analysis

The grey relational analysis method is used to analyze the mutual influence and interdependence between different schemes. The essence is to measure the similarity or dissimilarity of the development trend between the programs. The higher the similarity, the greater the degree of correlation between the two. The steps can be summarized as:

Step 1: Determine the reference sequence, the elements in the reference sequence are the maximum attribute values for each option of the specification after the specification, namely:

$$\bar{D}_0^k = \{\bar{M}_{01}^k, \bar{M}_{02}^k, \dots, \bar{M}_{0m}^k\}$$

$$\bar{M}_{0j}^k = \max_j \bar{M}_{ij}, j = 1, 2, \dots, m;$$

Step2: Calculate the distance Δ_{ij}^k between each element in the reference sequence and the corresponding element of the attribute value series according to formula (1), namely:

$$\Delta_{ij}^k = d(\bar{M}_{0j}^k, \bar{M}_{ij}^k), i = 1, 2, \dots, p, j = 1, 2, \dots, m$$

Step3: Find the maximum difference and the minimum difference:

$$\Delta_{\max}^k = \max_{i,j} \Delta_{ij}^k$$

$$\Delta_{\min}^k = \min_{i,j} \Delta_{ij}^k$$

Step4: Calculating a correlation coefficient matrix $(\xi_{ij}^k)_{p \times m}$ between the reference number column and the attribute value series:

$$\xi_{ij}^k = \frac{\Delta_{\min}^k + \rho \Delta_{\max}^k}{\Delta_{ij}^k + \rho \Delta_{\max}^k}, \quad i = 1, 2, \dots, p, j = 1, 2, \dots, m.$$

For the resolution factor $\rho \in [0,1]$, the smaller the resolution, the larger the resolution. Usually, $\rho=0.5$.

Step5: Calculate the gray correlation degree γ_i between each candidate attribute value sequence and the reference number column :

$$\gamma_i^k = \sum_{j=1}^m \xi_{ij}^k \cdot w_j, \quad i = 1, 2, \dots, p$$

$w_j \in W$, W is the attribute weight set.

Step6: According to the gray correlation degree γ_i^k , it is worth to sort the schemes. The larger the scheme, the better the corresponding scheme.

After the center of gravity is defuzzified, the risk-free fault mode is set as the reference object. The reference object is a zero matrix, and the calculated range of the resolution coefficients are: $0.6568 \leq \xi_1 \leq 0.8758$, $0.8902 \leq \xi_2 \leq 1.187$, $0.6003 \leq \xi_3 \leq 0.8004$. The resolution coefficients selected in this paper are: 0.75, 0.9, 0.75. Factor weights are: $\omega[S_p, S_d, S_s] = [0.2970, 0.1634, 0.5396]$. According to the analysis steps of the quantitative risk analysis model of fuzzy set-grey relevance, the gray correlation degree is calculated from the explicit value of the failure mode risk factor. Similarly, the gray correlation degree and the priority relationship of the security risks of the failure modes in Report II and Report III can be calculated as Table 5. However, in the results of quantitative risk analysis of failure modes based on fuzzy set-grey correlation, there are still a few cases where the risk value and the risk level are the same. Based on the fuzzy Borda ordinal value combination group evaluation model, the failure mode security risk analysis results in the three reports of the failure mode are comprehensively analyzed, and the Borda value of each failure mode security risk is obtained, which is the risk value of each failure mode security risk. And sort the risks according to the risk value as shown in Table 5.

Table 5 Total risk ranking

Failure mode	report I		report II		report III		Comprehensive risk analysis	
	gray correlation degree	rank	gray correlation degree	rank	gray correlation degree	rank	Borda value	rank
F11	0.7376	8	0.7673	8	0.7231	3	25.2689	13
F12:	0.7376	8	0.7448	7	0.7231	3	29.5399	11
F13:	0.8127	12	0.7448	7	0.6721	1	29.9379	10
F21:	0.5184	2	0.5390	3	0.7231	3	58.2131	3
F22:	0.7641	11	0.5365	2	0.6721	1	45.4408	5
F31:	0.4690	1	0.5390	3	0.7002	2	65.6192	1
F32	0.6307	6	0.514	1	0.6721	1	60.5987	2
F41:	0.6128	5	0.6068	5	0.8565	7	30.1066	9
F42:	0.5920	4	0.5615	4	0.8034	5	41.4835	6
F43:	0.5666	3	0.6068	5	0.8034	5	41.4621	7
F51:	0.7537	10	0.5615	4	0.7741	4	30.8828	8
F52:	0.7412	9	0.5140	1	0.6721	1	51.7865	4
F53:	0.6708	7	0.6068	5	0.8273	6	28.4916	12

It can be clearly seen from Table 4 that the results of the failure mode security risk analysis based on the fuzzy set-grey correlation degree risk quantitative analysis method are combined and analyzed

based on the fuzzy Borda sequence method combined group evaluation method. The risk level of all failure modes is completely distinguished, so that the risk ranking is not repeated.

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