

## Study on the Micro-lecture of Linear Algebra in Instructional Design—A Case of Matrix's Similarity Diagonalization

Chunxiao Sun

College of Science, Northwest A & F University, Yangling, Shaanxi 712100, China

sunchunxiaowky@nwsuaf.edu.cn

### Abstract

Micro-lecture, a new type of educational information resources, has been widely employed in courses construction in colleges and universities, where teachers tend to make excellent micro-lecture designs in their instructions. By analyzing and studying micro-lecture's instructional design in the case of matrix's similarity diagonalization in linear algebra, the study explores micro-lecture's application to the instruction of linear algebra for the improvement of the teaching effects.

### Keywords

Micro-lecture, Instructional design, Linear algebra.

### 1. Introduction

It is a key part in domestic educational and teaching reform to make a deep combination of information technology and discipline construction, in which the application of micro-lecture to courses instruction is one of the best breakthrough points [1]. With video as its main carrier, micro-lecture is used in the instruction of single knowledge point and the solution to certain teaching problem [2-3]. Its short time, concise content and high availability enable students to make full use of their odd moments in unlimited repetitive learning whenever and wherever possible, which better satisfies the individual needs of students. Linear algebra, a compulsory course of public mathematics for science and engineering students in colleges and universities, has a broad application in many fields, like natural science, engineering technology and life science. Meanwhile, the basic concepts, theories and methodologies in linear algebra are of high logicity as well as abstractness and hard for students to understand [4-5]. As a result, independent classroom teaching cannot help them to completely acquire the knowledge in linear algebra. However, the application of micro-lecture to the instruction of linear algebra helps the visualization and materialization of abstract theories and problems, which effectively stimulates students' motivation in learning. Also, it gives consideration to their multiple requirements with a refined analysis and detailed interpretation of knowledge points. Furthermore, instructional design plays a vital role in making a good micro-lecture to draw students' attention. Next, the author makes a study on the instructional design of micro-lecture in the case of matrix's similarity diagonalization in linear algebra.

### 2. The instructional design of micro-lecture in linear algebra

It is required to make an organized and clear instruction of the courses content, which is lively and interesting during a short time in micro-lecture. For the realization of teaching goal and the assurance of teaching quality, a scientific design of instruction is necessary. Details are as follows.

The knowledge of teaching objects

Micro-lecture mainly aims to help the self-learning of students. For a well-focused micro-lecture's design to realize personalized education, teachers should have an accurate master of students' academic bases and a full understanding of their learning ability.

The determination of teaching goals

Teaching goal is an important link in the design of micro-lecture, and its structure, levels and depth have a direct influence on the learning effects of students. After finishing the study of each micro-lecture, teachers should have a clear knowledge of what and how much students need to master. The teaching goals should be set in accordance with the knowledge points and be concise as well as to the point, easier to realize.

The establishment of teaching contents

The micro-lecture of linear algebra should focus on one single knowledge point each time, ensuring the content concise and full of logic. At same time, the knowledge points in linear algebra are highly dependents on each other. Therefore, teachers should attach great importance to the logical relation among different knowledge points in organizing the teaching content to assistant students in building a knowledge tree of linear algebra in micro-lecture. And then they will have a complete understanding of what they learn in mathematics.

The selection of teaching methods

Micro-lecture is based on video teaching, and its instructional design should focus on students and raise their conscious activity in study. Teachers should be clear about using various teaching methods, like task-driving, elicitation and inquiry method in different situations. However, regardless of which method is adopted, it should draw students' attention to stipulate their motivation in learning.

### 3. The instructional design of micro-lecture in linear algebra: the case of matrix's similarity diagonalization

#### 3.1 A brief introduction of the instructional design

Before the design, teachers should have a basic understanding of students' acquirement of mathematics and their learning ability. In the case of the author's university, those who take the course of linear algebra are science and engineering undergraduates in their second year, most of whom have acquired the knowledge of matrix. The teaching goal is set that a similar relation of N-order matrix and diagonal matrix will be established. Focusing on the knowledge point, the teaching content is well designed with a practical example. According to the character of the knowledge point, the teaching method adopted is to divide the instruction into three steps, namely the introduction of the problem, the solution to the problem as well as the generalization and summary.

#### 3.2 Three steps of the instructional design

Step 1: The introduction of the problem

Example: In 1202, Fibonacci raised a problem, the well-known Fibonacci sequence in his book, Liber Abacci. It is assumed that every month a pair of rabbits produces another pair, and that rabbits begin to bear young one month after their own birth. Let it be required to find out how many rabbits will be there in the beginning of each month supposing that rabbits constantly give birth to new ones without dying.

Analysis: From the perspective of reproducing, rabbits can be grouped into newborn rabbits (small rabbits) that haven't gotten fertile-activity and those (big rabbits) which can produce offspring. Suppose the number of pairs for small rabbits and big rabbits are  $F_0(k)$  and  $F_1(k)$  (unit being the pair)

respectively. Let the vector  $F(k) = \begin{pmatrix} F_0(k) \\ F_1(k) \end{pmatrix}$ , where  $F(1) = \begin{pmatrix} F_0(1) \\ F_1(1) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  represent the number of pairs for small rabbits and big rabbits.

It is suggested that the relation between the amount of small rabbits and big rabbits in the beginning of  $(k+1)^{\text{th}}$  month can be expressed as

$$\begin{cases} F_0(k+1) = F_1(k) \\ F_1(k+1) = F_0(k) + F_1(k) \end{cases}$$

That means the number of small rabbits equals that of big rabbits last month, and the number of big rabbits each month is that of big rabbits and small rabbits together last month.

The above formula can be written in matrix form

$$F(k+1) = AF(k) \text{ where } A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

A formula can be got with recursive method, that

$$F(k+1) = AF(k) = A^2F(k-1) = \dots A^kF(1)$$

Therefore, the determination of  $A^k$  is necessary for the calculation of the number of rabbits in the beginning of  $(k+1)^{\text{th}}$  month.

Step 2: The solution to the problem

The problem is solved by the similarity diagonalization of matrix.

Definition: Suppose that there is a revertible matrix,  $P$  making  $P^{-1}AP = \Lambda$ , where  $\Lambda$  is a diagonal matrix. And it is considered that  $A$  can be similarly diagonalized, denoted by  $A \sim \Lambda$ .

Theorem: The sufficient and necessary condition that  $n$ -order matrix  $A$  and diagonal matrix  $\Lambda$  are similar is that  $A$  has  $n$  linearly independent eigenvectors.

From the above definition and theorem, the eigenvalue of  $A$ ,  $\lambda_1, \lambda_2$  and corresponding eigenvectors  $\xi_1, \xi_2$  is determined, namely  $\lambda_1 = \frac{1+\sqrt{5}}{2}$ ,  $\lambda_2 = \frac{1-\sqrt{5}}{2}$ ,  $\xi_1 = \begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{pmatrix}$  and  $\xi_2 = \begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \end{pmatrix}$ , where,  $\xi_1$  and  $\xi_2$  are linearly independent.

And matrix  $A$  is similar to diagonal matrix  $\Lambda = \text{diag}(\lambda_1, \lambda_2) = \begin{pmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{pmatrix}$

The similar transformation matrix  $P = (\xi_1, \xi_2) = \begin{pmatrix} 1 & 1 \\ \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \end{pmatrix}$ , and

$$P^{-1} = \begin{pmatrix} \frac{-1+\sqrt{5}}{2\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1+\sqrt{5}}{2\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{pmatrix}$$

Since  $P^{-1}AP = \Lambda$ ,  $A = P\Lambda P^{-1}$ . And,  $A^k = P\Lambda^k P^{-1}$ .

Thus, the sum of rabbits in the beginning of  $(k+1)^{\text{th}}$  month is

$$F(k+1) = F_0(k+1) + F_1(k+1) = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{k+1} - \left( \frac{1-\sqrt{5}}{2} \right)^{k+1} \right].$$

Step 3: The generalization and summary

On the supposing condition, the mathematical model in the law of rabbits' population development is established with matrix and its multiplication. Furthermore, the sum of rabbits in the beginning of each month is determined through the theory of matrix's similarity diagonalization.

#### 4. Summary

A theoretical course as linear algebra is, it enjoys a wide range of application. The micro-lecture's design of many other courses can follow that of linear algebra. In other words, teachers start new lessons with practical problems, give an instruction of new theories and finally solve the problems with those theories. Such micro-lecture's design not only imparts new knowledge to students, but improves their ability in solving practical problems with mathematical knowledge, which can stimulates students' motivation and raise their conscious activity in learning to a great extent. In the meanwhile, a good micro-lecture's design of linear algebra is pivotal to ensuring the success of its teaching, which puts forward higher requirements for mathematics teachers. An excellent

instructional design of micro-lecture demands a comprehensive consideration of the cognitive characteristics of students and the methods as well as the contents of teaching. Teachers should remember their role as the guide and devote themselves to research. By doing so, they can improve their ability of micro-lecture's design and better serve the instruction of linear algebra.

## References

- [1] Deng Shengyue, Tan Jintao, Li Xueyong and Song Riqing. Research on practice of teaching reform of linear algebra course, *Theory and Practice of Contemporary Education*, vol.10 (2018),41-44.
- [2] Zhou Xiaohui, Huang Cui and Gu Guiding. Teaching study on linear algebra of computer major integrated by programming, *College Mathematics*, vol.34(2018),31-35.
- [3] Shan xian. Exploration on the Teaching Reform of Linear Algebra under the Background of "Internet + Education", *Education Teaching Forum*, 10(2018),115-117.
- [4] Huang Yonghui. Deep Thinking on the Application of MOOC to the Teaching of Advanced Mathematics, *Heilongjiang Researches on Higher Education*, 8(2017),172-174.
- [5] Zhao Xiao, Xin Lin and Xiao Peng. Application of teaching mode of micro lesson in flipping classroom of linear algebra. *Journal of Ningde Normal University (Natural Science)*, 29(2017), 323-327.