Comparative Analysis of Phase Extraction Based on Fourier Transform and Wavelet Transform

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Abstract

Based on wavelet transform and Fourier transform theory, the problems in practical application are considered. The difference between Fourier transform and wavelet transform in phase extraction is studied. The experimental results of matlab simulation show that wavelet transform has great advantages in local analysis, and Fourier transform has more advantages in global transform. For local steep parts, wavelet transform and Fourier transform error are not far from each other; When there is some noise pollution, the Fourier transform can't recover the original image very well, but the wavelet transform is not affected by the noise;When the fundamental frequency in the spectrum has a certain degree of aliasing, the Fourier transform cannot correctly recover the three-dimensional shape of the measured object, and the spectral aliasing has little effect on the wavelet transform.

Keywords

Three-dimensional shape measurement; Fourier transform profilometry; wavelet transform; spectrum aliasing; noise pollution.

1. Introduction

In recent years, the rapid development of precision and accuracy of optical instruments has greatly promoted the development of traditional optical measurement technology, and new three-dimensional sensing and measurement methods have emerged[1]. The Fourier transform in early 3D sensing technology received great attention. However, Fourier transform profilometry has poor recovery capabilities under steep planes and noise interference[2]. To overcome this drawback, window Fourier transform analysis has also been applied. However, when using the window Fourier transform analysis, the window size and area are a constant, so the high and low frequencies cannot be satisfied at the same time. Subsequently, wavelet transform theory and technology research continues to deepen and continue to innovate, so wavelet transform has gradually introduced three-dimensional sensor measurement technology. However, the program that implements the wavelet transform is more difficult than the program that implements the Fourier transform, the calculation time is longer, there is a gap between the global information and the transform and the Fourier transform, and the surface of the object is also the same as the Fourier transform[3-6]. The influence of the shape, if the surface of the object is too complicated or simple, the conversion time is long and the error is large. According to the advantages and difficulties of Fourier transform and wavelet transform in the advantages and practical applications of three-dimensional sensing, this paper takes wavelet transform and Fourier transform theory as the basis, and considers the problems in its practical application. The difference between the Fourier transform and the wavelet transform in phase extraction[7].

2. Fourier transform profilometry

Fourier transform profilometry extracts the corresponding phase distribution of the three-dimensional surface shape of the object from the deformed fringe image, which makes it easy to collect and process the dynamic contour of the object, and has high data acquisition speed and high precision, which is suitable for computer processing[8]. When the projector projects a sinusoidal grating to the reference plane, the CCD acquires the deformed fringes (deformed grating), and the information of the sine grating and the deformed grating are as follows:

$$g(x,y) = a(x,y) + b(x,y)\cos(2\pi f_0 x) \tag{1}$$

$$g_1(x,y) = a(x,y) + b(x,y)\cos(2\pi f_0 x + m(x,y)) \tag{2}$$

Among them： $a(x,y)$ is the light intensity of the background; $b(x,y)$ is reflex rate; $m(x,y)$ is the phase modulation caused by the height of the object; f_0 represent the fundamental frequency of the projected grating, and the Fourier transform of the deformed fringes:

$$G(f,y) = A(f,y) + Q(f - f_0, y) + Q^*(f - f_0, y) \tag{3}$$

Then, by looking at the frequency spectrum of the frequency shift spectrum, the filter window is obtained, and then the fundamental frequency can be filtered out, and then the inverse Fourier transform is performed, and the sinusoidal part can be extracted, and then the phase distribution is obtained . Similarly, the same transformation, filtering, inverse transformation, and phase distribution can be obtained for the sinusoidal grating, and the phase of the sinusoidal grating can be obtained. From then on, we can get the phase difference. From the above we can get the phase difference $m(x,y)$， But this phase difference is not the original function, but a truncated phase distribution. The truncated phase is the result of the actual phase addition and subtraction integer multiple period. The actual purpose of phase unwrapping is to restore the actual simulated object by the truncated phase. This process is exactly the opposite of the phase distribution, but the principle must be satisfied that the phase difference between any two adjacent pixels cannot exceed one cycle, which makes When the surface of the object is complex or steep, it cannot be recovered properly. Because even if the object is too high, it will cause shadows on the deformed stripes, and some two adjacent points will have a phase difference of more than one cycle.

FTP transforms a wide (or even infinite) range of signals over the airspace into the frequency domain for filtering, because the infinitely extended sinusoidal signal in the spatial domain is the impact on both sides of the zero frequency in the frequency domain. The fundamental frequency contains the height information of the object, and the fundamental frequency component can be extracted after filtering. Therefore, the filtering becomes the key to extracting the fundamental frequency, and it is also the key of Fourier transform profilometry. However, when complex surface measurements are encountered, the boundary of the object is uneven, and the spectrum is widely distributed in the spectrum, and the energy is relatively divergent. At this time, the "tail" of the fundamental component will coincide with the DC component or the high-frequency component, causesing spectral aliasing. As shown in Figure 3.3 below. If the fundamental frequency and the zero frequency are aliased, the lower frequency limit can be determined. If the fundamental frequency and the high frequency are aliased, the upper limit frequency is difficult to determine, and the fundamental frequency cannot be separated well in either case[9]. Therefore, there is a limit measurable gradient at the height of the surface of the object. If the height changes beyond this gradient, the fundamental frequency cannot be completely extracted, resulting in a large FTP error and loss of meaning.

To avoid FTP failure, the measurement range of FTP is as follows:

$$\left|\frac{\partial m(x,y)}{\partial x}\right|_{\max} < \frac{2\pi f_0}{3} \tag{4}$$

Ideally, a standard sinusoidal grating projection is used, and a π phase shift technique is used. At this time, the lower limit of the fundamental frequency can be zero, and the upper limit can be reached $2f_0$, so that a new measurement range can be obtained as follows:

$$\left|\frac{\partial m(x,y)}{\partial x}\right|_{\max} < 2\pi f_0 \tag{5}$$

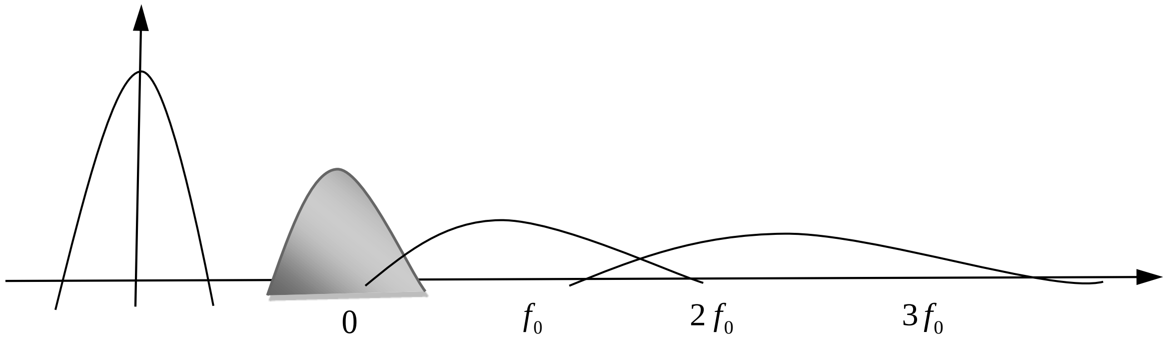


Fig1 A spectrum of overlapping lines

3. Wavelet transform profilometry

In order to facilitate understanding, we use the one-dimensional wavelet transform as an example to illustrate the principle of phase extraction[10]. Set the base wavelet to $\psi(x)$, after the base function is stretched and translated, you can get:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\psi(\frac{x-b}{a}) \tag{6}$$

$\psi_{a,b}(x)$ is a wavelet sequence obtained by stretching and translating the base wavelet, b is a translation factor and a is a scale factor. When a is small (equivalent to high frequency), the spatial resolution is higher, the frequency domain resolution is lower. when a is changing, the frequency domain and the airspace will change accordingly. This is the "zoom" characteristic of the wavelet. You can also say multi-resolution features. The definition of continuous wavelet transform is:

$$W_f(a,b) = \int_{-\infty}^{\infty} f(x)\psi_{a,b}^{*}(x)dx = \left\langle f(x),\psi_{a,b}(x)\right\rangle \tag{7}$$

Where * indicates conjugate. In this paper, we choose the Morlet complex wavelet which is closest to the sine wave and has good localization performance as the mother wavelet.

$$\psi(x) = \frac{1}{\sqrt{\pi f_b}}\exp(j2\pi f_c x - \frac{x^2}{f_b}) \tag{8}$$

f_c is center frequency (the size is equal to the fundamental frequency); f_b is envelope width.

For the real function, using the complex wavelet for continuous wavelet transform, we can get the wavelet transform coefficient $W_f(a,b)$ .The real part is $real(W_f(a,b))$ .The imaginary part is $imag(W_f(a,b))$.Amplitude is $A(a,b)$, phase is $\varphi(a,b)$ and their relationship is as follows:

$$A(a,b) = \sqrt{\left[imag(W_f(a,b))\right]^2 + \left[real(W_f(a,b))\right]^2} \tag{9}$$

$$\varphi(a,b) = \arctan[imag(W_f(a,b)) / real(W_f(a,b))] \tag{10}$$

Wavelet transform describes the inner product relationship between wavelet sequence and one-dimensional function, which reflects the similarity between wavelet sequence and one-dimensional function. Therefore, the result of continuous wavelet transform can be understood as the degree of similarity, and the magnitude and phase reflect the degree of similarity between them from two different aspects[11]. When the phase of the function is close to or the same as the phase of the wavelet function under a certain scale factor, the wavelet transform coefficient is larger. The wavelet transform is performed in columns. The most valued function is used to select the position where the transform coefficient has the largest amplitude. The connection is a ridge of wavelet transform $ridge(b)$:

$$ridge(b) = \max\left[A(a_i, b)\right] \tag{11}$$

a_i in the formula is a different scale factor value . In the three-dimensional shape measurement, we determine the ridge of the wavelet transform by the above formula, and extract the phase value corresponding to the wavelet ridge, thereby obtaining the height information modulation phase containing the object.

4. Computer simulation and experiment

In order to compare the advantages and disadvantages of Fourier transform profilometry and wavelet transform in local and global, fundamental frequency aliasing and noise pollution, the author used five models to compare and analyze[12]. In the experiment, the standard grating we used is a sinusoidal grating. By adding the object function directly into the phase to obtain the deformed grating, we use the peaks function to demonstrate. The image of the object model (3*peaks) is shown in Fig 2 (a). The FTP deformed grating is shown in Fig 2(b). The CWT deformed grating is shown in Fig 2(c). The wavelet ridge is shown in Fig 2(d).

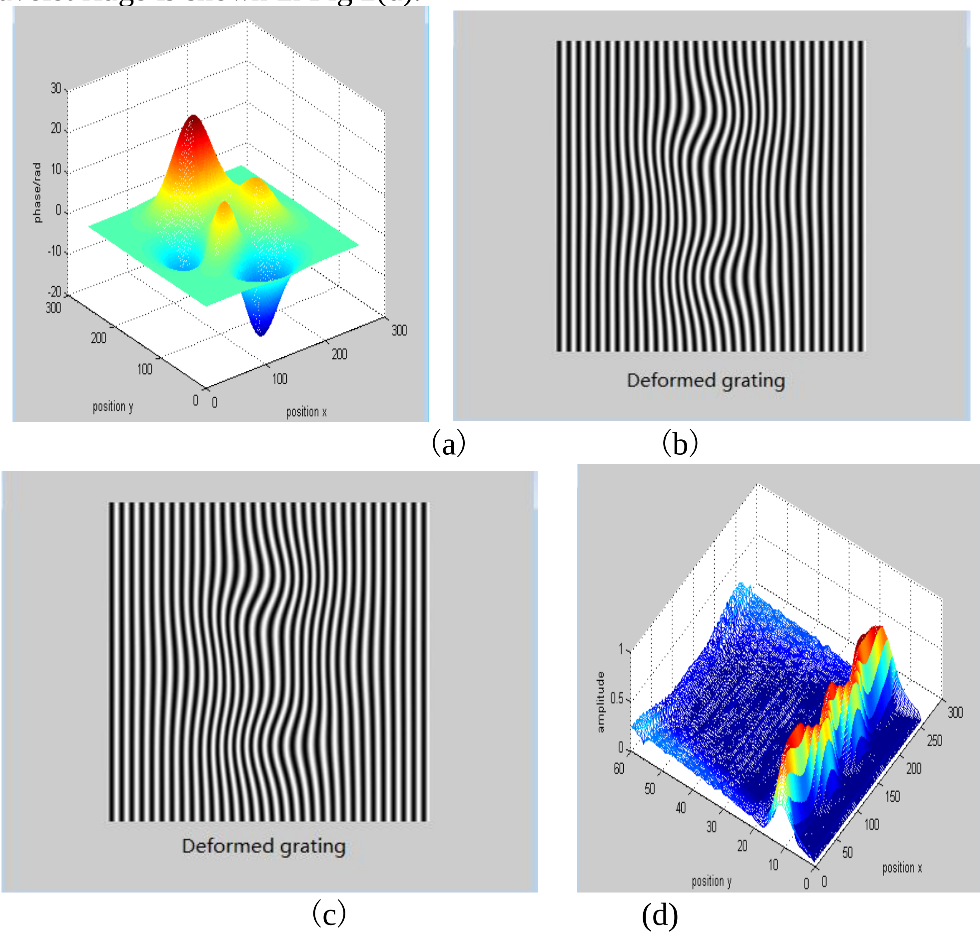


Fig 2 (a) peaks function model, (b) FTP deformed grating, (c) CWT deformed grating, (d) wavelet ridge.

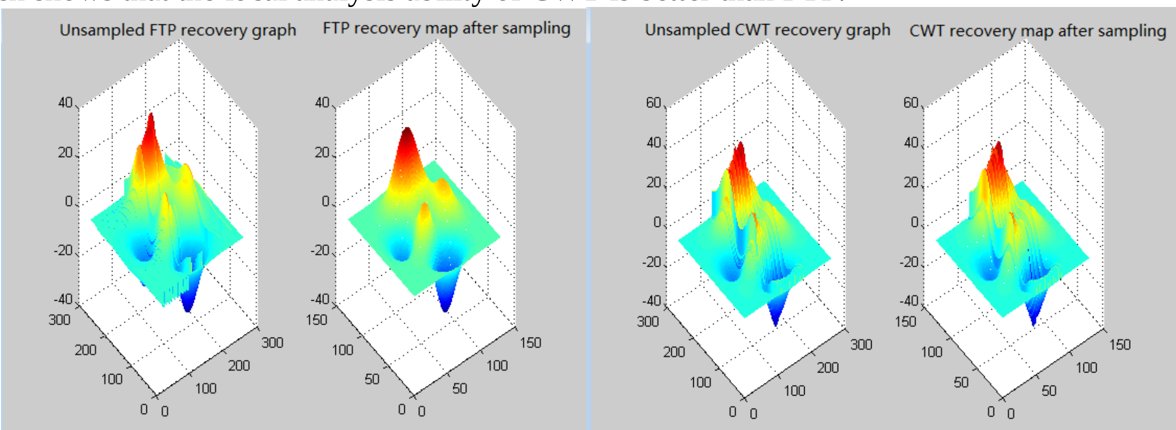
4.1 Global and local

1) Global comparison

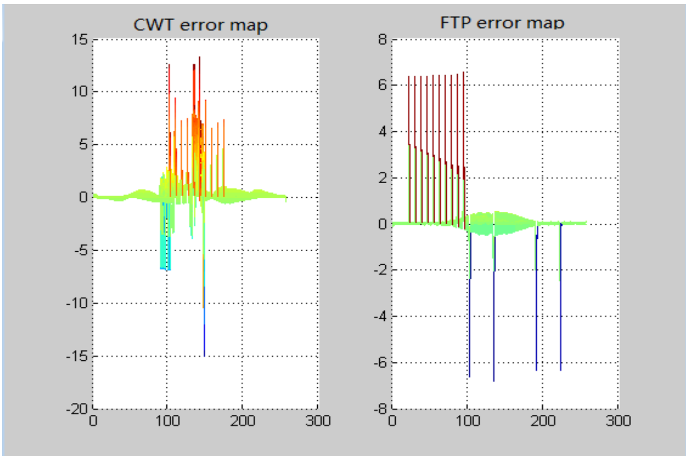
In Figure 3, we can find that the original image can be restored after FTP sampling, and the height is not very serious, and the original image cannot be restored after CWT sampling. From the error graph comparison analysis, we can find the error amplitude. The value of FTP is small, and most of them are mutations at a certain point, and the number of mutation points is also relatively small, but CWT has a small-scale protrusion (the amplitude is not large), which is difficult to improve by sampling. In global analysis, FTP is better than CWT.

2) Local contrast

From the above (a) and (b), it can be seen that the CWT on the top surface is flatter, and FTP is relatively rougher, which is more clearly reflected in (c) and (d). As can be seen from (c), the error in the middle part is not large, but it is not flat. In (d), we can clearly see that the middle part is very flat, which shows that the local analysis ability of CWT is better than FTP.

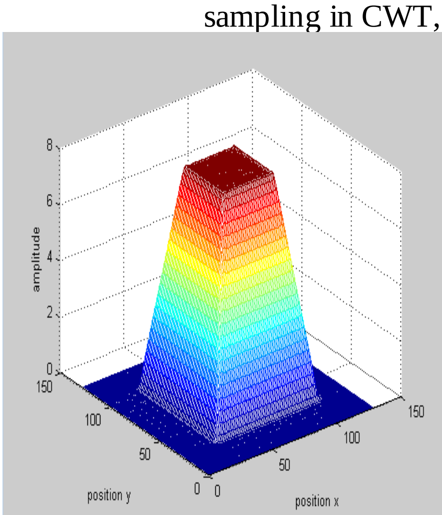


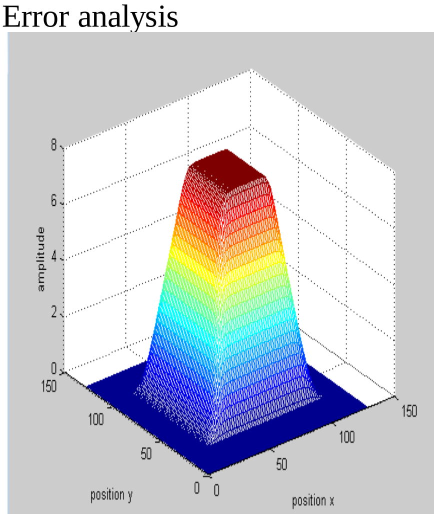
(a) (b)



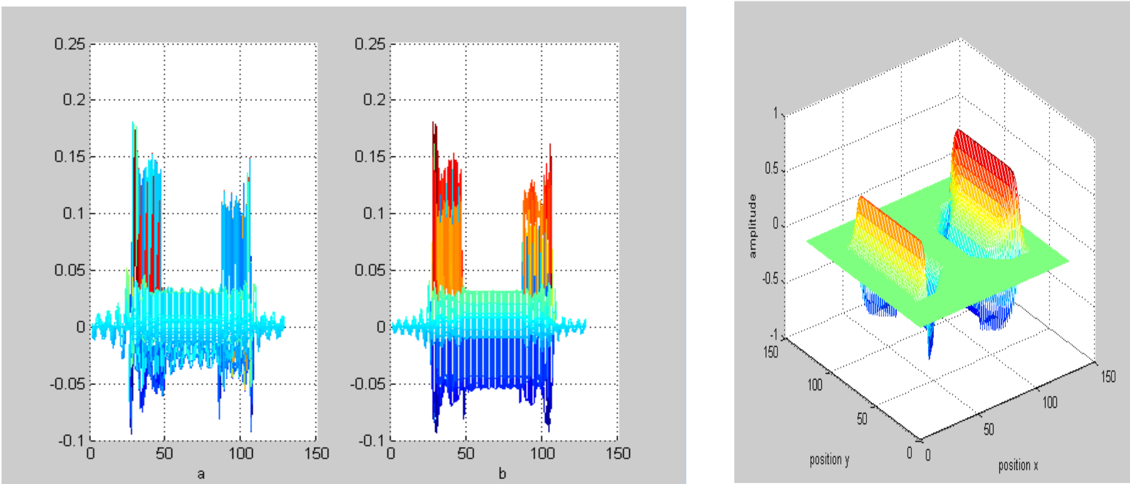
(c)

Figure 3 (a) Comparison of before and after sampling in FTP ,(b) Comparison of before and after sampling in CWT, (c) Error analysis





(a) (b)

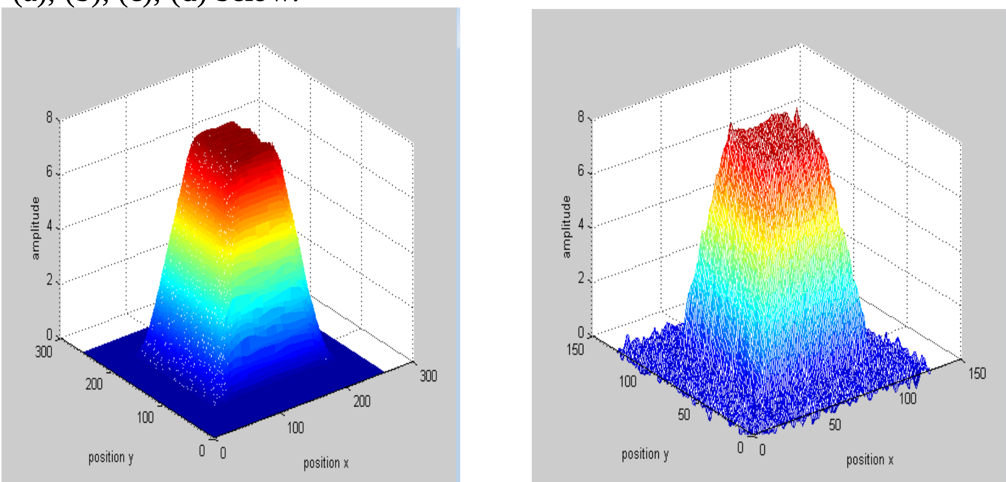


(c) (d)

Figure 4 (a) vertices function FTP recovery graph, (b) prism function CWT recovery image, (c) apex function FTP error map (x-z plot, y-z plot), (d) prism function CWT error map

4.2 Noise pollution

In order to explain the noise suppression ability of CWT, a certain random noise is added, but it cannot be too large, because both methods will fail when the noise is too large[13-14]. Here we use the prism function to add noise. Due to space limitations, we no longer use other The function is proof. See Figure 5 (a), (b), (c), (d) below.



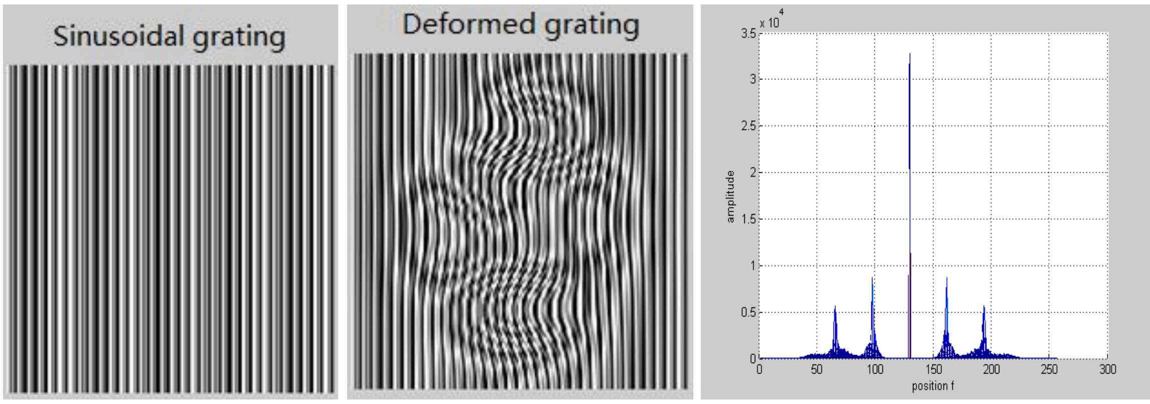
(a) (b)

Fig5 (a) Prism function CWT restores the image (adding noise), (b) FTPT restores the image (adds noise), (c) Error graph comparison analysis (adds noise)

From the image in Figure 5, it is easy to see that FTP has poor recovery ability when it encounters noise, and it can hardly recover. The error is just like noise. Although CWT also has errors, the middle part is still relatively flat and can recover the original image. . This shows that CWT has a certain ability to suppress noise, but it is also very limited. If noise is increased, neither FTP nor CWT can achieve better results.

4.3 Basic frequency aliasing

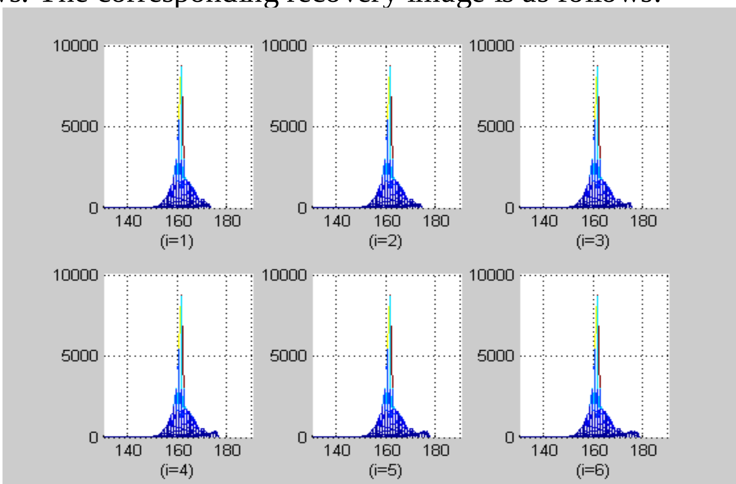
In order to make the fundamental frequency easier to alias, I use a dual-frequency sine grating to select the peaks function to model[15]. As shown in Fig 6.



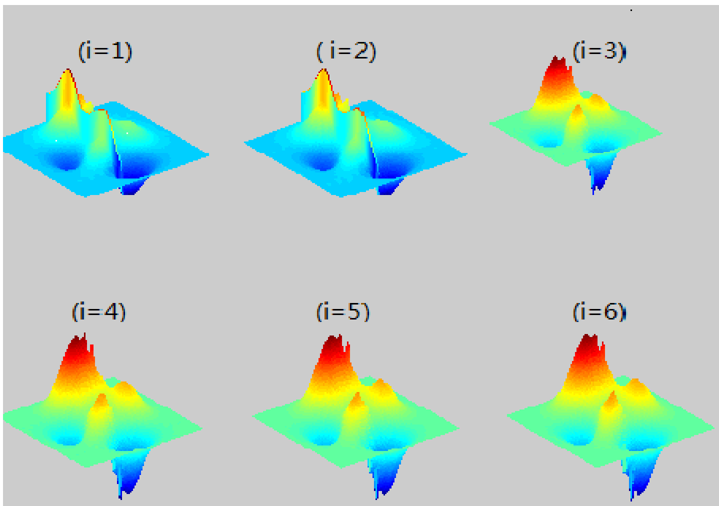
(b) (c)

Fig 6 (a) sinusoidal grating, (b) anamorphic grating, (c) spectrum of the 128th line

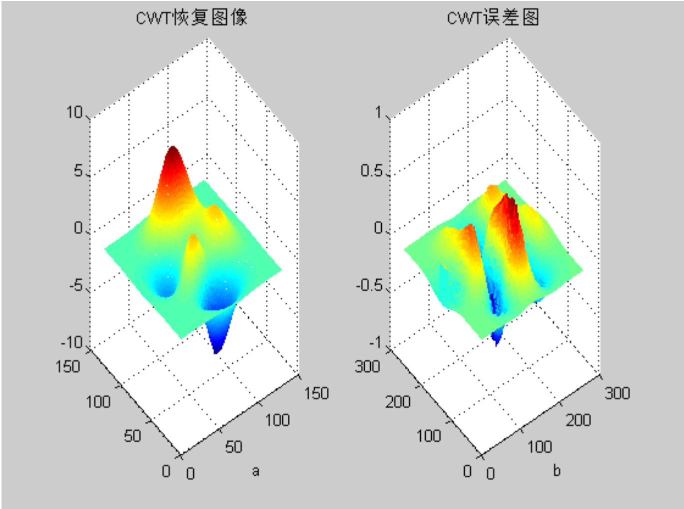
It can be seen from Fig 6(c) that aliasing occurs at the fundamental frequency and the high frequency. To find the exact filter position, we sweep x from 172 to 177 (corresponding to i from 1 to 6), and the image is as follows. The corresponding recovery image is as follows:



(a)



(b)



(c)

Fig 7 (a) filtered fundamental frequency diagram (x-z plot), (b) FTP recovery image, (c) CWT recovery image and error map.

It can be seen from Figure 7(b) that FTP can't restore the original image when the fundamental frequency is aliased. The restored image and error in Figure (c) can meet our requirements. Again, when the aliasing occurs at the fundamental frequency, the wavelet is explained. Change is more advantageous.

5. Conclusion

(1) When Fourier transform is applied to three-dimensional shape measurement, the local analysis ability is insufficient, the noise suppression ability is poor, the object structure is affected, and the fundamental frequency aliasing cannot accurately recover the object, but the speed is fast, the calculation time is short, and the global analysis is performed. strong ability.

(2) Wavelet transform In the application, the large amount of data computation leads to long operation time, insufficient global analysis ability, and great influence on the structure of the object, but the wavelet transform has strong local analysis ability, certain noise suppression capability, and is suitable for aliasing of the fundamental frequency. The recovery results are less affected.

(3) The limitation of Fourier transform accuracy is the completeness of the fundamental frequency information. The more complete the fundamental frequency information, the smaller the recovery error. The more the fundamental frequency information is lost, the larger the recovery error, resulting in insufficient fundamental frequency information. There are many reasons. For example, the structure of an object may cause the fundamental frequency to extend far away from both sides, and widening the high frequency range results in aliasing of the fundamental frequency.

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