A Quantum Genetic Algorithm with Bidirectional Decoding Strategy for Solving Optimization Problems

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Abstract

Compared with genetic algorithms, quantum genetic algorithm has good global search ability. Its unique quantum bit coding method and high efficiency performance brought by quantum gate update operation are one of the reasons why this algorithm has been widely used in recent years. However, quantum genetic algorithm may still have premature problem, and it's search stability needs to be strengthened. In view of the above situation, we proposes a quantum genetic algorithm with bidirectional decoding strategy. Through the flexible decoding of the qubit, the algorithm performs multiple local searches in the operation process, thereby enhancing the optimization stability of the algorithm and improving the convergence speed. In addition, the application of full interference crossover operation and quantum variation operation enables the algorithm to better overcome the premature problem and avoid falling into the local optimal solution. Finally, the improved quantum genetic algorithm based on bidirectional decoding strategy is tested by test function.

Keywords

Quantum genetic algorithm, Bidirectional decoding strategy, Full interference cross.

1. Introduction

Quantum Genetic Algorithm is a kind of evolutionary algorithm that introduces several concepts in quantum computing theory. In recent years, various evolutionary algorithms have been used in many fields [1, 2]. The combination optimization problem [3] is undoubtedly a frequent label in the above applications. Whether it's engineering applications or project management, a series of modeling abstractions are used to turn problems into optimization problems. This type of problem is often seen as an NP-hard problem. Compared with traditional genetic algorithms, quantum genetic algorithms have new features and new architectures [4]. Therefore, when dealing with the above problems, they can bring new perspectives and new methods for solving problems.

The idea of quantum genetic algorithm [5] was first proposed by Narayanan in his paper in 1996 and applied to solve the TSP problem. However, the algorithm implemented in this paper does not use Quantum Bit coding or quantum gates. With the operation of distinct quantum computing characteristics, it cannot be regarded as a true quantum genetic algorithm. The true quantum genetic algorithm was implemented by Han et al. in 2003 and successfully applied to solve the knapsack problem.

In order to improve the efficiency of quantum genetic algorithm and the reliability of the algorithm, this paper changes the binary decoding method after simulating quantum collapse. By means of bidirectional decoding, the binary state after observation is used as much as possible to select better individuals. At the same time, it is proved that when the observed binary digits tend to infinity, the bidirectional decoding mechanism can be used to reduce the number of observations by half. In view of the fact that quantum genetic algorithm has no practical significance for the intersection of probability amplitude, this paper applies the full interference cross operation to enable the observed population to exchange their information, so that all individuals inside the population can participate in this crossover operation. Quantum non-gate [6] is used to make the chromosome probability amplitude mutate with a certain probability. The full interference crossover enhances the intra-domain search performance of the algorithm, and the quantum variation enhances the inter-domain search performance of the algorithm.

2. Bidirectional Decoding Strategy

2.1 Quantum bit coding

In the coding method, the quantum genetic algorithm uses quantum bit coding. A quantum bit is defined by introducing the probability amplitude in the quantum computing theory [7], and its value can express any superposition state of two eigenstates of the traditional bit 0 and 1. And in the process of operating the qubit, the two probability amplitudes interact and interfere with each other. The quantum bit can be converged to a state by simulating the process of quantum collapse.

2.2 Decoding process

The process of quantum collapse is simulated by observation, and the diversity of quantum bit systems expressed by probability amplitude disappears. The observed qubits are generally in binary form. On this basis, subsequent decoding and fitness calculation and evaluation can be performed. The existence of the Bidirectional Decoding mechanism extends the diversity of population decoding in binary form. In the case of equivalent coding, two mutually different decoded values participate in the fitness calculation of the algorithm. As the dimension of optimization problems increases, the efficiency of the bidirectional decoding strategy continues to increase. When the dimension of the optimization problem tends to infinite dimension, the bidirectional decoding mechanism can make the algorithm only need to use half of the binary form expression to explore the entire search space, thereby saving space resources and improving search efficiency.

For example, there are two individuals who are observed in the quantum ground state, and their binary forms are: 101011 110011. Bidirectional decoding from left to right and from right to left is performed separately. It can be obtained that in the binary coding mode, the values of the first individual after bidirectional decoding are: 43 and 53 respectively. The values of the second individual after bidirectional decoding are: 51 and 51 respectively. It can be seen from the above example that when the ground state individual is an asymmetric individual, the values after bidirectional decoding are different. When the ground state individual is a symmetric individual, the values after bidirectional decoding are the same. Therefore, the performance loss of the bidirectional decoding mechanism in the multidimensional qubit binary form is as shown in equation (1):

$$E = 2^{-\frac{n}{2}}. \tag{1}$$

The performance loss of the bidirectional decoding mechanism with the number of variables is shown in Figure (1).

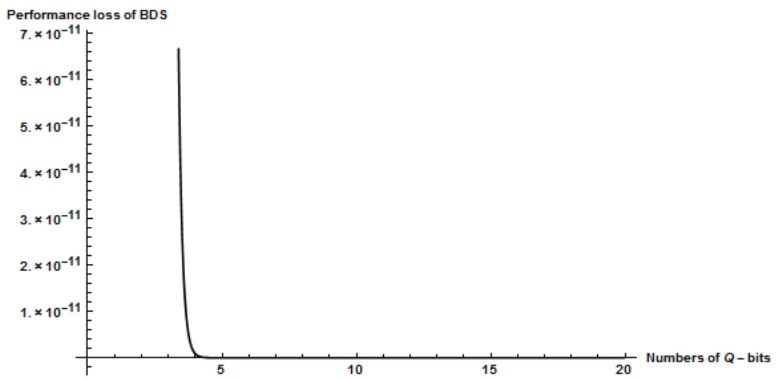

Fig.1 Performance loss of the bidirectional decoding strategy.

3. Full Interference Crossover

In the genetic algorithm, the crossover operation is the way in which the chromosomes in the population can exchange its information, and the intra-domain search performance of the algorithm is also reflected. Single point crossing, arithmetic crossing and uniform crossing [8] are common methods to achieve this. However, when the intersecting individuals are individuals of the same coding mode, the above operation loses the effect. At the same time, the quantum genetic algorithm stores the probability amplitude of each qubit, which represents the trend of a quantum bit taking 0 or 1, rather than a 0 or 1 binary form. If you still cross in the traditional way, it makes no sense. The resulting binary form of the quantum bit string may then dependent on the generated random number. Therefore, in the quantum genetic algorithm, in order to avoid the stagnation of the evolution process of the algorithm during operation, the local search performance of the algorithm is improved, and intra-domain communication is strengthened, and full interference crossover is used. The full interference crossover can make all the quantum bits after each generation of the population participate in the crossover operation, and use the known information as much as possible to try to construct a new individual for evaluation. The specific operation of the full interference cross is shown in Table 1.

Table 1 Full interference crossover

C1	C_{11}	C_{52}	C_{43}	C_{34}	C_{25}	C_{16}	C_{57}
C2	C_{21}	C_{12}	C_{53}	C_{44}	C_{35}	C_{26}	C_{17}
C3	C_{31}	C_{22}	C_{13}	C_{54}	C_{45}	C_{36}	C_{27}
C4	C_{41}	C_{32}	C_{23}	C_{14}	C_{55}	C_{46}	C_{37}
C5	C_{51}	C_{42}	C_{33}	C_{24}	C_{15}	C_{56}	C_{47}

4. Quantum Variation

The realization of quantum variation operation enables quantum genetic algorithm to obtain the ability to break through the domain search and expand the breadth of search. In terms of probability amplitude, the quantum variation makes the original probability amplitude corrected. This correction can be an exchange of 0, 1 probability amplitudes, or an arbitrary linear combination of the two. Quantum NOT gate is a means of realizing the exchange of probability and amplitude. It is a Pauli matrix [9]. Quantum non-gate is shown in equation 2:

$$\mathrm{Pauli-X}: \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

To achieve an arbitrary linear combination of probability amplitudes, the following matrix is required (3)

$$\mathrm{Mat} = \begin{bmatrix} \mu & \sqrt{1-\mu^2} \\ \sqrt{1-\mu^2} & -\mu \end{bmatrix}, \mu \in [0,1] \tag{3}$$

It is not difficult to find that the equation (3) does not change the probability amplitude after the Mat matrix transformation if and only if ${\alpha_i}^2 = \frac{1}{2}(1+\mu)$. The existence of quantum variability can make the algorithm maintain a certain diversity in the later stage when the probability amplitude tends to converge, thus avoiding the premature phenomenon that the algorithm may appear in the later stage of operation.

5. Algorithm Description

The procedure of IQGA is shown below. In the process of updating the probability amplitude, the rotation angle of the quantum gate is obtained by looking up the table in Table (2). In the table, f(x) represents the fitness value of the individual x, x_i represents the i-th bit of an individual, b_i represents the i-th bit of the optimal individual in the current population, and S=(α_i,β_i) indicates the direction of rotation of the rotation angle in polar coordinates. Corresponding rotation angle direction, θ_i is the rotation angle step length finally used by the individual i-th position. The formula for calculating the rotation angle step can be calculated by equation (4).

$$\theta_i = \delta\theta * S(\alpha_i, \beta_i) \quad (4)$$

Begin
Initialize Q(t)
Generate P(t) by observing Q(t)
Evaluate P(t) find the best P_best
While(G<MAXG) do
    G←G+1
    Generate P(t+1) by observing Q(t)
    Evaluate P(t+1) with bidirectional decoding and find current best P_current_best
    Full interference crossover
    Generate P(t+2) by observing Q(t)
    Evaluate P(t+2) with bidirectional decoding and find the best P_best
    Update Q(t) by quantum gates
    Quantum variation
End

Table 2 Quantum Genetic Algorithm Rotation Angle.

b_i	x_i	f(x) > f(b)	δθ	$S = (\alpha_i, \beta_i)$			
				$\alpha_i\beta_i > 0$	$\alpha_i\beta_i < 0$	$\alpha_i = 0$	$\beta_i = 0$
0	0	True/False	0	-	-	-	-
1	1	True/False	0	-	-	-	-
1	0	True	0.05π	1	-1	0	-1/1
1	0	False	0.125π	-1	1	-1/1	0
0	1	True	0.05π	1	-1	0	-1/1
0	1	False	0.125π	-1	1	-1/1	0

6. Experiment Design and Results

6.1 Experiment Design

The experimental hardware platform run by this algorithm is: CPU: Intel(R) Core(TM) i7-3630QM 2.4GHz; memory: 8G; software platform: Microsoft visual studio 2010 version number 10.0.40219.1 SP1Rel.

We select the basic test functions of intelligent algorithms to test the function optimization ability of the algorithm. Algorithm reference Table (3) in parameter setting.

Table 3. Parameter Settings

Algorithm	Iterations	Qubit length	Pop Size	Crossover rate%	Mutation rate%
GA	500	20	500	0.1+0.4G/MAXG	0.1-0.099G/MAXG
QGA	500	20	500	-	-
IQGA	500	20	200	-	-

Test function 1: Shaffer's F6 Function.

$$f(x,y) = 0.5 - \frac{\sin^2\left(\sqrt{x^2+y^2}\right)-0.5}{\left(1+0.001(x^2+y^2)\right)^2}, x,y \in [-100,100] \quad (5)$$

Test function 2: Rosenbrock Function.

$$f(x,y) = 100(x^2-y)^2 + (1-x)^2, x,y \in [-2.048,2.048] \quad (6)$$

6.2 Result

Table 4. Summary Result of The Test Function Optimized by Algorithms

Test Func	Algorithm	Algorithm Performance Index（10 Times）					
		Optimal Solution	Worst Solution	Average Solution	Error Magnitude	Numbers of Convergence	Variance
F1	bGA	9.99960E-1	9.61138E-1	9.819978E-1	10^{-2}	1	1.89E-3
	QGA	9.99545E-1	9.88122E-1	9.921836E-1	10^{-2}	2	1.45E-4
	IQGA	1	9.90284E-1	9.979540E-1	10^{-3}	6	1.01E-4
F2	bGA	1.10000E-5	4.29900E-3	9.233000E-4	10^{-3}	7	1.56E-5
	QGA	1.65357E-5	7.88030E-4	3.383271E-4	10^{-4}	10	1.07E-6
	IQGA	4.60386E-6	4.35176E-5	2.045611E-5	10^{-5}	10	1.65E-10

It can be seen from Table (4) that in the optimization problem, the traditional genetic algorithm in the 10 times and 500th generation operations, the Shaffer function only converges to the global optimal solution once, and the Rosenbrock function converges to the global optimal solution 7 times. Under the same conditions, the quantum genetic algorithm of the native framework converges to the global optimal solution twice and 10 times respectively.

However, the improved quantum genetic algorithm can still converge to the global optimal solution 6 times and 10 times when the population is reduced by more than half. In terms of the magnitude of the error, the improved quantum genetic algorithm and its native framework as well as the traditional genetic algorithm did not open a particularly significant gap. In terms of variance, the improved quantum genetic algorithm is better on the test function 1, and the stability of the three algorithms is better. The improved algorithm is more stable on test function 2 than the previous two. In terms of the average value, the quantum genetic algorithm has little difference with the traditional genetic algorithm in the two test functions, and the improved quantum genetic algorithm performs better. In summary, the improved quantum genetic algorithm based on bidirectional decoding strategy is better at solving the optimization problem of complex functions [10].

7. Conclusion

In the basic framework of quantum genetic algorithm, this paper integrates the bidirectional decoding strategy, and implements quantum crossover and quantum variation using full interference crossover and quantum non-gate. In the process of running the algorithm, the binary form of the quantum bit observation is fully utilized, and the local search is performed to select an individual more suitable for evolution. On this basis, the operating efficiency of the algorithm is improved and the search space is reduced. The quantum crossover operation ensures that the algorithm can effectively cross and utilize the information contained in the qubit. The quantum variation operation ensures that the algorithm has a certain cross-region search capability, avoids the phenomenon that the algorithm converges in advance, and prevents the algorithm from falling into the misunderstanding of local optimization. Through the basic test function, the improved algorithm is verified by multiple times and multiple iterations. The quantum genetic algorithm applying the bidirectional decoding strategy exhibits strong global search ability and anti-premature ability, which also explains the algorithm is suitable for processing function optimization.

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