Multi-attributes decision-making method for hesitant fuzzy linguistic term sets based on regret theory

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Abstract

In this paper, we study hesitant fuzzy linguistic multi-attributes decision-making (MADM) problems with incomplete weight information. We developed a new decision-making method considering the regret aversion of the decision makers. Firstly, we discussed three linguistic scale functions according to DMs' evaluation attitudes, and defined a novel score function to efficiently compare HFLTSs. Then, we defined a computational formula for the perceived utilities of alternatives under multi attributes to obtain DM's regret/rejoice values. Subsequently, on the premise that the attribute weights are completely unknown or partially unknown and different decision makers may have different psychological behaviors, optimism, neutrality and pessimism respectively, we set up a mathematical programming to determine the optimal attribute weights based on the maximum total comprehensive perceived utility in decision making, and compute the DM's weighted perceived utility values of the feasible alternatives and obtain the ranking order of them. Finally, an illustrative example is given to clarify the feasibility and practicality of the proposed method.

Keywords

HFLTS, Scale function, Score function, Regret theory.

1. Introduction

Multi-attributes decision making (MADM) is an important research branch in decision science, which aims to select the best alternative or rank alternatives based on the evaluation information under several attributes given by DMs [1][2]. The evaluation information are often given in the form of fuzzy sets, which was first proposed by Bellman et.al [3], owing to that the practical MADM problems are often with imperfect or imprecise information. After that, many extensions of fuzzy sets emerge as the times require, such as interval-valued fuzzy sets [4], type-2 fuzzy sets [5], intuitionistic fuzzy sets [6], interval-valued intuitionistic fuzzy sets [7], and hesitant fuzzy sets (HFS) [8], which were proposed to express imprecise information with DM's hesitation. Furthermore, Rodriguez et.al proposed a new tool named hesitant fuzzy linguistic term set (HFLTS) to handle more than one uncertainty in the form of linguistic information, which has received a number of scholars' attention. To accommodate this kind of uncertainty, discontinuous linguistic terms sets are required. Wang proposed the concept of EHFLTS [9]. Tremendous researches have been developed to compare HFLTSs [10][11][12], aggregate HFLTSs information [13][14][15], and construct hesitant fuzzy linguistic MADM methods, such as TOPSIS method [16], VIKOR method [17], the ELECTRE method [18][19] and etc.

Score function is a comparatively simple method to compare two or more HFLTSs, which takes into account the mean value and variance value [20]. We all know that the number of LTs in compared HFLTSs may be vary and the mean value may happen to be equal in HFLTSs, it may influence the score outcomes of HFLTSs such as $F(\{s_{-2}, s_0, s_2\}) > F(\{s_{-2}, s_2\})$, which obviously out the line with reality. Thus, a novel score function was proposed in this paper to avoid such mistake. In addition, as we know that the centered OWA operator [21] has the property of giving the most weight to the central value and least weight to the extreme values, which can efficiently manage some kinds of smoothing, such as seeking the averaging solution for some alternatives. Thus, in this paper, we extended the centered OWA operator to HFLTS environment to derive the balanced solution for some alternatives.

With regard to decision making method, most of the existing methods are based on the hypotheses of rational decision-making. However, in practical decision making, decision makers do not behave in a completely rational manner, but have a bounded rationality [22]. Kahneman and Tversky [23] made a lot of investigations and experiments on individual behavior research, and put forward the prospect theory (PT) based on the "bounded rationality". After that, many behavioral decision theories are proposed such as cumulative prospect theory (CPT) [24] and regret theory (RT) [25][26]. As one of the core methods, the core idea of regret theory is that decision makers not only focus on the outcomes obtained by the selected alternative, but also on the opportunity costs [25][26]. Furthermore, the regret theory has been applied widely in decision making problems [27][28][29]. Unfortunately, all of these efforts mostly not focus on the MCDM under hesitant fuzzy linguistic environment. Considering the three common types of psychological manner of decision makers, optimism, neutrality and pessimism respectively, we introduced a general decision model to calculate decision makers' perceived utility values compared with different reference alternatives.

Furthermore, considering that different sets of attribute weights will influence the ranking result of alternatives, we develop a novel model to determine the attribute weights based on the maximum total comprehensive perceived utility and minimum disparity between the attribute weights. In addition, we also integrate the subjective weight ideology into the proposed model by adding limitations when the attribute weights are partially unknown. However, most of the existing methods, which are based on the HFLTSs, didn't consider the regret behavior of decision makers when determining the attribute weights, which may directly influence the ranking of alternatives, and didn't consider decision makers' different psychological behavior, optimism, neutrality and pessimism respectively. Motivated by these weakness and gap in the existing research, in this paper we proposed a novel MADM decision making model under hesitant fuzzy linguistic environment.

The remainder of this paper is structured as follows. In section 2, we briefly reviewed the concepts of HFLTS and regret theory, and put forward a new score function for comparing the HFLTs. In section 3, the centered OWA operator was developed to hesitant fuzzy linguistic environment to obtain the averaging solution of alternatives, and we proposed a novel hesitant fuzzy linguistic MADM approaches to derive the attribute weights and obtain the ranking of the feasible alternatives based on the regret theory and the above methods. In section 4, a numerical example is given to illustrate the proposed method. This paper ends in section 5.

2. Preliminary

2.1 Hesitant Fuzzy Linguistic Terms Sets

Definition 1 [30] Let $S=\{s_i|i=0,1,\ldots,2\tau\}$ be a linguistic term set (LTS). A HFLTS H_S is an ordered finite subset of the consecutive linguistic terms of S.

s_0 and $s_{2\tau}$ represent the lower and upper bounds of linguistic labels, respectively. τ is a positive integer, and S satisfies the following conditions:

If $i<j$, then $s_i<s_j$;

$neg(s_i)=s_{2\tau-i}$.

However, when a group of experts are authorized to evaluate an object using LS S, the group evaluation merging all possible linguistic terms may be no longer be denoted by consecutive linguistic terms. To accommodate this kind of uncertainty, discontinuous language terms sets are required. Wang proposed the concept of EHFLTS [9], which is shown as follows:

Definition 2 [9] Let S be a linguistic term set (LTS), then an ordered subset of linguistic term sets of S, that is

$$H_S(x)=\{s_i \mid s_i \in S\} \tag{1}$$

is called an extended hesitant fuzzy linguistic term set (EHFLTS).

Definition 3 [31] Let $S=\{s_i|i=0,1,...,2\tau\}$ be a LTS, H_S, H_S^1 and H_S^2 be three arbitrary EHFLTSs on S, $\lambda>0$, then:

1) $\lambda H_S = g^{-1}\left(\mathrm{U}_{\gamma\in g(H_S)}\left\{1-(1-\gamma)^{\lambda}\right\}\right)$;

2) $(H_S)^{\lambda} = g^{-1}\left(\mathrm{U}_{\gamma\in g(H_S)}\left\{(\gamma)^{\lambda}\right\}\right)$;

3) $\mathrm{neg}(H_S)=\mathrm{U}_{s_i\in H_S}\left\{s_{2\tau-i}\right\}$;

4) $H_S^1\oplus H_S^2 = g^{-1}\left(\mathrm{U}_{\gamma_1\in g(H_S^1),\ \gamma_2\in g(H_S^2)}\left\{\gamma_1+\gamma_2-\gamma_1\gamma_2\right\}\right)$;

5) $H_S^1\otimes H_S^2 = g^{-1}\left(\mathrm{U}_{\gamma_1\in g(H_S^1),\ \gamma_2\in g(H_S^2)}\left\{\gamma_1\gamma_2\right\}\right)$;

Where $g(\mathrm{H}_S)$ denotes the mapping of linguistic terms to numerical value. Furthermore, the mapping usually requires rigorous and fastidious processing since it can significantly affect the accuracy and reliability of the final decision results. Due to the attitudes of DMs may be different, some DMs may be strict such that the evaluation results appear to be lower integrally, some may be tolerant such that the evaluation results appear to be higher integrally, and some may be neutral. The corresponding attitudes of these DMs are pessimism, optimism and neutrality respectively. The most commonly used linguistic scale function is imprecise since the simple transformation of linguistic terms to crisp number (0-1) cannot objectively access decision makers' original evaluation preference.

The same subjective feelings may correspond to different preference values, which implies that the decision makers' evaluation attitudes are different. For example, when evaluating the rationality of location selection for a company, DMs may all deem that it is "medium". However, some DMs may be pessimistic, and the evaluation result "medium" is already relatively high in their mind, while some DMs who hold an attitude of optimism tend to deem that the evaluation result "medium" express a not very good meaning since they often give relative high evaluation in decision making. Therefore, in order to precisely reflect the DMs' subjective feelings, it is necessary to transform the linguistic terms into the numerical values according to DMs' subjective attitudes.

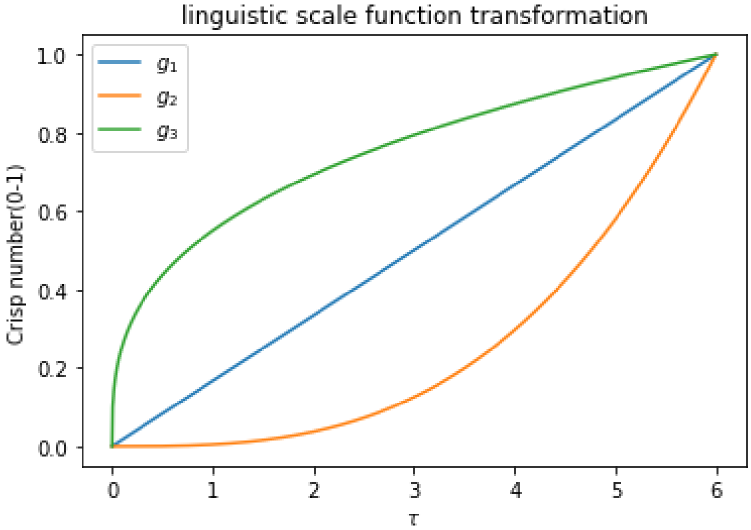
Thus, we choose the following three forms [32] as our scale functions, which correspond to neutral, optimistic and pessimistic attitudes.

$$\begin{aligned}
&g_1:[0,2\tau]\to[0,1] && g_1(\mathrm{H}_S)=U_{s_i\in H_S}\left\{\frac{i}{2\tau}\right\}=\gamma\\
&g_1^{-1}:[0,1]\to[0,2\tau] && g_1^{-1}\left(g_1(\mathrm{H}_S)\right)=U_{s_i\in H_S}\left\{s_{2\tau\gamma}\right\}=\mathrm{H}_S
\end{aligned} \tag{2}$$

$$\begin{aligned}
&g_2:[0,2\tau]\to[0,1] && g_2(\mathrm{H}_S)=U_{s_i\in H_S}\left\{\left(\frac{i}{2\tau}\right)^{\tau}\right\}=\gamma\\
&g_2^{-1}:[0,1]\to[0,2\tau] && g_2^{-1}\left(g_1(\mathrm{H}_S)\right)=U_{s_i\in H_S}\left\{s_{2\tau\gamma^{\frac{1}{\tau}}}\right\}=\mathrm{H}_S
\end{aligned} \tag{3}$$

$$\begin{aligned}
&g_3:[0,2\tau]\to[0,1] && g_3(\mathrm{H}_S)=U_{s_i\in H_S}\left\{\left(\frac{i}{2\tau}\right)^{\frac{1}{\tau}}\right\}=\gamma\\
&g_3^{-1}:[0,1]\to[0,2\tau] && g_3^{-1}\left(g_1(\mathrm{H}_S)\right)=U_{s_i\in H_S}\left\{s_{2\tau\gamma^{\tau}}\right\}=\mathrm{H}_S
\end{aligned} \tag{4}$$

Given $\tau=3$, the feature of formulae (2), (3), (4) can be graphically shown in Fig. 1



2.2 Score functions for HFLTSs

Definition 5 [33] Let $h=\left\{h^{(1)},h^{(2)},...,h^{(n)}\right\}$ be a HFE, the following functions can be considered as the score functions for HFEs:

1. The arithmetic-mean function:

$$S_{AM}(\mathrm{h})=\frac{1}{n}\sum_{i=1}^{n}h^{(i)} \tag{5}$$

2. The fractional score function:

$$S_{F}(\mathrm{h})=\frac{\prod_{i=1}^{n}h^{(i)}}{\prod_{i=1}^{n}h^{(i)}+\prod_{i=1}^{n}(1\text{-}h^{(i)})} \tag{6}$$

Definition 6 [20] Let $S=\{s_t|t=0,1,...,2\tau\}$ be an LTS, and $H_S=\left\{s_{\delta_l}\,|\,l=1,2,\cdots,\#H_S\right\}$ be an HFLTS on S. A score function $F(H_S)$ is defined as follows:

$$F_W\left(H_S\right)=\overline{\delta}-\frac{\frac{1}{\#H_S}\sum_{l=1}^{\#H_S}(\delta_l-\overline{\delta})^2}{\mathrm{var}(S)} \tag{7}$$

where $\overline{\delta}=\frac{1}{\#H_S}\sum_{l=1}^{\#H_S}\delta_l$ and $\mathrm{var}(S)=\frac{\sum_{k=0}^{2\tau}(k-\tau)^2}{2\tau+1}=\frac{\tau(\tau+1)}{3}$.

For convenience of unified computing, we take the crisp number (0-1) which has been transformed into calculations.

***Example* 1** Let $S=\{s_t|t=0,1,...,6\}$ be an LTS, H_S^1 and H_S^2 be two different HFLTSs based on S. Suppose that $H_S^1=\{s_1,s_5\}$ and $H_S^2=\{s_1,s_3,s_5\}$, then the possibility degree of H_S^1 being not less than H_S^2 can be obtained by Eq. (5)-(6):

1) $S_{AM}(H_S^1)=0.5$, $S_{AM}(H_S^2)=0.5$;
2) $S_F(H_S^1)=0.5$, $S_F(H_S^1)=0.5$;
3) $F_W\left(H_S^1\right)=0.5-\frac{0.1109}{0.111}=-0.5$, $F_W\left(H_S^2\right)=0.5-\frac{0.074}{0.111}=-0.166$;

From example 1, we can find that the score function $F_W(H_S)$ has a preponderance over the one proposed by Farhadinia in that the score function $F_W(H_S)$ can not only express the mean value of the assessment value H_S but also denote the DMs' hesitant degree through the variation value, except there is one problem: H_S^2 deserves more hesitation than H_S^1 owing to an additional linguistic term s_3 included in H_S^2. We know that the higher mean and lower hesitation, the score function ought to be higher, but the fact is that we get the result $F_W(H_S^2) > F_W(H_S^1)$ based on Eq. (7). Obviously, the result doesn't conform to the facts. The reason for this is that the additional linguistic term s_3 in H_S^2 happen to be the mean value, which directly leads to the decrease of variance. So we will develop a more rational score function for comparing HFLTSs in the following:

Definition 7: Let $S = \{s_t | t = 0,1,...,2\tau\}$ be an LTS, and $H_S = \{s_{\delta_l} | l = 1,2,\cdots,\#H_S\}$ be an HFLTS on S. A novel score function $F(H_S)$ is defined as follows:

$$F(H_S) = \bar{\alpha} - \frac{\frac{1}{\#H_S}\sum_{i,j=1,i\neq j}^{\#H_S}|\gamma_i - \gamma_j|}{\frac{1}{\#S}\sum_{m,n=1,m\neq n}^{\#S}|\varsigma_m - \varsigma_n|} = \bar{\alpha} - \frac{\frac{1}{\#H_S}\sum_{i,j=1,i\neq j}^{\#H_S}|\gamma_i - \gamma_j|}{(\tau+1)/3} \tag{8}$$

where $\bar{\alpha} = \frac{1}{\#H_S}\sum_{l=1}^{\#H_S}\gamma_l$, $\varsigma = g(S)$ and $\gamma = g(H_S)$.

Example **2** (Continued with example 1) By Definition 7, suppose that $g(H_S) = U_{s_i \in H_S}\left\{\frac{i}{2\tau}\right\}$, we can get

$$F(H_S^1) = 0.5 - \frac{\frac{1}{2}\times 0.667}{\frac{4}{3}} = 0.249, \quad F(H_S^2) = 0.5 - \frac{\frac{1}{3}\times(0.333+0.667+0.333)}{\frac{4}{3}} = 0.167;$$

From the Example 2, we can see that the comparing results by Definition 7 can be distinguished more easily than Definition 5 and 6.

2.3 Regret theory

Regret theory is one of the most popular methods to identify the best alternatives, which is proposed by Bell [30] and Loomes and Sugden [9]. According to regret theory, it is based on the intuition that the DMs care about not only the outcome they receive but also the opportunity outcomes if making other decisions. The decision maker will experience the feeling of rejoice and regret when the chosen outcome is greater or less than the opportunity outcome he didn't choose.

In the following, the regret-rejoice function $R(\Delta v)$ can be defined in the following [30]:

$$R(\Delta v) = 1 - e^{-\lambda\Delta x}, \tag{9}$$

Where λ represents the risk aversion degree of decision maker, and the greater the λ, the greater the risk aversion degree of decision maker. $\Delta v = v_1 - v_2$ denotes the difference between the utility values of the two alternatives. $R(\Delta v) = 0$ indicates that neither regret nor rejoice the decision maker felt when choosing alternative1; $R(\Delta v) > 0$ and $R(\Delta v) < 0$ represent rejoice and regret respectively when choosing alternative1 rather than alternative2. $R'(\Delta v) > 0$, and $R''(\Delta v) < 0$, $|R(-\Delta v)| > |R(\Delta v)|$ (if $\Delta v > 0$) can be derived indirectly, which implies that the intuition of the decision maker is more sensitive to gains than to losses.

Usually, the power utility function ($v(x) = x^{\alpha}$) [20] and exponential utility function ($v(x) = \frac{1-e^{-\beta x}}{\beta}$) [30] can be utilized to imitate the utility of the decision maker, where $v'(x) > 0$ and $v''(x) < 0$, and the risk aversion coefficient of the DM is represented as α and β, and satisfies $0 < \alpha, \beta < 1$. The smaller α and the greater β, the greater risk aversion degree. In this paper, for convenience, we choose $v(x) = x^{\alpha}$ as our utility function.

3. A decision making model for HFLTSs

3,1 HFLCOWA Operators

Yager [21] proposed a general class of OWA aggregation operators inspired by Gaussian distribution, called centered OWA operators. As we all know, this type of aggregation operators have the characteristics that the central value be in possession of the maximum weight and the extreme value be assigned the minimum weight, which can be more valid to derive the averaging solution of several alternatives. Moreover, the traditional centered OWA operator was extended as HFLCOWA operator to deal with hesitant fuzzy linguistic decision-making problems.

Definition 8 [21] A mapping F from

$$I^n \to I(where\ I \in [0,1])$$

is said to be a centered OWA aggregation operator of dimension n if:

$$F(a_1, a_2, a_3, ..., a_n) = \sum_{j=1}^{n} w_j b_j$$

where b_j is the j th largest element in the collection $U_{j=1,2,...,n} a_j$, $w = (w_1, w_2, \ldots, w_n)^T$ is the associated weighting vector with $w_j > 0$ $(j = 1, 2, \cdots, n)$, $\sum_{j=1}^{n} w_j = 1$, and satisfies the following conditions:

(1) Symmetric: $w_j = w_{n-j+1}$;

(2) Softly decaying: if $i \geq j \geq (n+1)/2$, then $w_i \leq w_j$ and if $i \leq j \leq (n+1)/2$, then $w_i \leq w_j$.

Definition 9 [21] The centering function $\varphi : [0,1] \to R$ satisfies the following three properties:

(1) $\varphi(x) > 0$;

(2) φ is symmetric about 0.5: $\varphi(0.5+z) = \varphi(0.5-z)$ for $\varphi \in [0, 0.5]$;

(3) φ is unimodal

(i) $\varphi(x) < \varphi(y)$ for $x < y \leq 0.5$;

(ii) $\varphi(x) < \varphi(y)$ for $x > y \geq 0.5$.

Definition 10 [34] Let φ be a centering function. The weighted vector $w = (w_1, w_2, \ldots, w_n)^T$ associated with centered OWA operator is defined as following:

$$w_j = f\left(\frac{j}{n}\right) - f\left(\frac{j-1}{n}\right) = K \int_{\frac{j-1}{n}}^{\frac{j}{n}} \varphi(y)\, dy \tag{10}$$

where $K = 1 \Big/ \int_0^1 \varphi(y)\, dy$.

Definition 11 Let $S = \{s_t \mid t = 0, 1, ..., 2\tau\}$ be a LTS and H_S^{i} $(i = 1, 2, ..., n)$ be a collection of HFLTSs on S. Then the hesitant fuzzy linguistic centered OWA (HFLCOWA) operator is defined as follows:

$$HFLCOWA(H_S^1, H_S^2, ..., H_S^n) = \mathop{\oplus}_{i=1}^{n} (K \int_{\frac{i-1}{n}}^{\frac{i}{n}} \varphi(y)\, dy H_S^{i}\,') \tag{11}$$

Where φ is a centering function and $K = 1 \Big/ \int_0^1 \varphi(y)\, dy$, $H_S^{i}\,'(i = 1, 2, ..., n)$ is a descending ordered collection of H_S^{i} $(i = 1, 2, ..., n)$, which means that $H_S^{i}\,' \geq H_S^{j}\,'$ if $i < j$.

Definition 12 Let $S = \{s_t \mid t = 0, 1, ..., 2\tau\}$ be a LTS and H_S^{i} $(i = 1, 2, ..., n)$ be a collection of HFLTSs on S, where $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of H_S^{i} $(i = 1, 2, ..., n)$. Then hesitant fuzzy linguistic weighted centered OWA (HFLWCOWA) operator is defined as follows:

$$HFLWCOWA(H_S^1, H_S^2, ..., H_S^n) = \mathop{\oplus}_{i=1}^{n} (\mathrm{n} \cdot w_i\,' K \int_{\frac{i-1}{n}}^{\frac{i}{n}} \varphi(y)\, dy H_S^{i}\,') \tag{12}$$

where φ is a centering function and $K = 1\Big/\int_0^1 \varphi(y)dy$, $H_S^{i}\,'(i=1,2,...,n)$ is an descending ordered collection of $H_S^{i}\,(i=1,2,...,n)$, and $w' = (w_1', w_2', ..., w_n')$ be the corresponding original weight of $H_S^{i}\,(i=1,2,...,n)$.

Theorem 1 Let the centering function $\varphi:[0,1]\to R$ satisfies the properties shown in Definition 9, $H_S^{i}\,(i=1,2,...,n)$ be a collection of HFLTSs, then the aggregated value obtained by using HFLWCOWA operator is also a HFLTS.

$$HFLWCOWA(H_S^1, H_S^2, ..., H_S^n) = g^{-1}\left(U_{\eta_1'\in g(H_S^1{}'),\eta_2'\in g(H_S^2{}'),...,\eta_n'\in g(H_S^n{}')}\left\{\left(1-\prod_{i=1}^{n}(1-\eta_i')^{\mathrm{n}\cdot w_i'K\int_{\frac{i-1}{n}}^{\frac{i}{n}}\varphi(y)dy}\right)\right\}\right) \tag{13}$$

***Example* 5** Let $S=\{s_t\,|\,t=0,1,...,6\}$ be a LTS. $H_S^1=\{s_3,s_4\}$, $H_S^2=\{s_4,s_5\}$ and $H_S^3=\{s_1,s_2\}$ be three HFLTSs on S, where $w=(0.3,0.5,0.2)$ be the associated weight vector. Moreover, suppose the centering function $\varphi(y)=4(y-y^2)$ (which is also used later in this paper) and the linguistic scale function $g(\mathrm{H}_S)=U_{s_i\in H_S}\left\{\frac{i}{2\tau}\right\}$. Such that, we can get the aggregation value of the three HFLTSs based on Eq. (8) and (13):

First of all, according to the score function, we can obtain the ranking of $H_S^{i}\ (i=1,2,3)$. Moreover, the HFLTSs $H_S^{i}\,'(i=1,2,...,n)$ can be derived.

$$F(H_S^1)=\frac{7}{12}-\frac{\frac{1}{2}\times 0.167}{\frac{4}{3}}=0.521;$$

$$F(H_S^2)=\frac{3}{4}-\frac{\frac{1}{2}\times 0.167}{\frac{4}{3}}=0.687;$$

$$F(H_S^3)=\frac{1}{4}-\frac{\frac{1}{2}\times 0.167}{\frac{4}{3}}=0.187;$$

Thus, $F(H_S^2)>F(H_S^1)>F(H_S^3)$ such that the collection of $H_S^{i}\,'(i=1,2,...,n)$ is $\left\{H_S^2, H_S^1, H_S^3\right\}$.

Moreover, according to the HFLWCOWA operator, we can obtain the aggregated results.

$$\begin{aligned}
&HFLWCOWA(H_S^1, H_S^2, ..., H_S^3)\\
&= \mathop{\oplus}_{i=1}^{n}(\mathrm{n}\cdot w_i'K\int_{\frac{i-1}{n}}^{\frac{i}{n}}\varphi(y)dyH_S^{i}\,') = \mathop{\oplus}_{i=1}^{n}\left(3\times w_i'\frac{3}{2}\cdot\int_{\frac{i-1}{n}}^{\frac{i}{n}}\varphi(y)dyH_S^{i}\,'\right)\\
&=\frac{9}{2}\left(0.5\times\frac{14}{81}\times H_S^1{}'\oplus 0.3\times\frac{26}{81}\times H_S^2{}'\oplus 0.2\times\frac{14}{81}\times H_S^3{}'\right)\\
&=\frac{9}{2}\left(\frac{7}{81}\times H_S^2\oplus\frac{13}{135}\times H_S^1\oplus\frac{14}{405}\times H_S^3\right)\\
&=g^{-1}\left(\{0.531,0.547,0.606,0.620,0.641,0.653,0.699,0.709\}\right)\\
&=\{s_3,s_4\}
\end{aligned}$$

3.2 The weighting model of attributes

In the MADM process, the attribute weights reflect the relative importance experts attach to different attributes, which directly affect the decision results. Considering the weight information is completely unknown or partly unknown, we must determine the attribute weights before any decision making. Based on the minimizing regret theory-in other words, maximum total comprehensive perceived utility, we can establish a single-objective optimization model.

Firstly, we define the average solution for several alternatives under criteria c_j based on the HFLCOWA operator:

$$HFLCOWA(H_S^{1j}, H_S^{2j}, ..., H_S^{mj}) \\ = g^{-1}\left(U_{\eta_{1j}'\in g(H_S^{1j}'),\eta_{2j}'\in g(H_S^{2j}'),...,\eta_{mj}'\in g(H_S^{mj}')} \left\{ \left(1-\prod_{i=1}^{m}(1-\eta_{ij}')^{K\int_{\frac{i-1}{m}}^{\frac{i}{m}}\varphi(y)dy} \right) \right\} \right) \tag{14}$$

Definition 13 Let h_{ij} indicate the assessment value of i th alternative under j th attribute, and let $F(\mathrm{h}_{ij})$ be the score function value of h_{ij}. The balanced solution under j th attribute is denoted as h_j^*, and the score function value of the h_j^* is represented by $F(\mathrm{h}_j^*)$. Let $v(F(\mathrm{h}_{ij}))$ be the utility function. Then, the perceived utility function is defined as:

$$u_{ij} = v(F(\mathrm{h}_{ij})) + \mathrm{R}(\Delta v) \tag{15}$$

$\Delta v = v(F(\mathrm{h}_{ij})) - v(F(\mathrm{h}_j^*))$ denotes the difference in the utility value between h_{ij} and h_j^*. In the paper, we assume the power function $v(\mathrm{x}) = \mathrm{x}^\alpha$ as our utility function and $\mathrm{R(x)} = 1-\exp(-\gamma\Delta\,\mathrm{x})$ as our regret/rejoice function. When $\mathrm{R}(\Delta v) > 0$, $\mathrm{R}(\Delta v)$ denotes rejoice values; When $\mathrm{R}(\Delta v) < 0$, $\mathrm{R}(\Delta v)$ denotes regret values.

If we both know the attribute weight w_j and the assessment value h_{ij}, then the weighted normalized value of h_{ij} can be calculated using

$$F'(h_{ij}) = w_j \cdot F(h_{ij}) \tag{16}$$

Subsequently, the perceived utility value of h_{ij} can be rewritten as:

$$u_{ij}' = v(w_j \cdot F(\mathrm{h}_{ij})) + \mathrm{R}(v(w_j \cdot F(\mathrm{h}_{ij})) - v(w_j \cdot F(\mathrm{h}_j^*))) \tag{17}$$

Then the total perceived utility $\mathrm{P}(\cdot)$ of the decision matrix can be written as:

$$\mathrm{P}(\cdot) = \sum_{i=1}^{m}\sum_{j=1}^{n} u_{ij}' = \sum_{i=1}^{m}\sum_{j=1}^{n} (w_j)^\alpha (F(\mathrm{h}_{ij}))^\alpha + 1 - \exp\left\{-\gamma\cdot(w_j)^\alpha[(F(\mathrm{h}_{ij}))^\alpha - (F(\mathrm{h}_j^*))^\alpha]\right\} \tag{18}$$

Obviously, the bigger P corresponds to a higher degree of rejoice and a lower degree of regret for the decision making. Based on the basic idea, we designed a new method to determine the attribute weights, where we concentrated on integrating regret aversion in decision making. Thanks to the irrational character in reality decision making, the addition of risk aversion coefficient appears more credible. That is, a proper weight vector ensured that the total perceived utility is as large as possible, which contains not only the intrinsic values but also the rejoice/regret values. In addition, we hope that the weights of attributes should be equally important as possible so that it can avoid the occurrence of extreme weight. Thus, we can solve the following model to obtain the attribute weights:

$$\max\ P(\cdot) = \sum_{i=1}^{m}\sum_{j=1}^{n} (w_j)^\alpha (F(\mathrm{h}_{ij}))^\alpha + 1 - \exp\left\{-\gamma\cdot(w_j)^\alpha[(F(\mathrm{h}_{ij}))^\alpha - (F(\mathrm{h}_j^*))^\alpha]\right\} - \frac{1}{n-1}\sum_{i=1}^{n-1}(1-\frac{w_i}{w_{i+1}})^2 \\ s.t. \begin{cases} \sum_{j=1}^{n} w_j = 1 \\ 0 < w_j < 1\ (j=1,...,n) \\ w_j \in \Lambda\ (j=1,...,n) \end{cases} \tag{19}$$

Λ represents a weight set of partially known information, which generally appears in the following forms [35]: (i) weak ranking: $\{w_i > w_j\}$; (ii) strict ranking: $\{w_i - w_j \ge \alpha_i\}$; (iii) difference ranking: $\{w_i - w_j \ge w_k - w_l\}$; (iv) ranking with multiples: $\{w_i \ge \alpha_i w_j\}$; (v) interval ranking: $\{\alpha_i \le w_i \le \alpha_i + \varepsilon_i\}$.

We can see that the above model is established based on rejoice/regret values of each alternative to the balanced solution, actually, two other types of rejoice/regret values can be defined taking into account the distance from each alternative to the best and worst solution. The best and worst solutions are denoted as $h_j^+ = \max_{F(h_{kj})(\mathrm{k}=1,2,...,\mathrm{m})}\{h_{kj}\}$, and $h_j^- = \min_{F(h_{kj})(\mathrm{k}=1,2,...,\mathrm{m})}\{h_{kj}\}$. The different types of best alternatives derived by different models can reflect different preferences of the expert. The expert with different attitudes may choose different forms of comparative values to calculate rejoice/regret values of each alternative. If he/she is an optimist, he/she may choose the worst solution to compare; if he/she is a neutral, he/she may choose the balanced solution to compare; otherwise, if he/she is a pessimist, he/she may choose the best solution to compare.

3.3 MADM with HFLTSs

The MADM process aims to seek the optimal alternative from several practical alternatives. To better understand the procedure of solving MADM problem, we established a general framework based on the regret theory and the novel score function, which is shown as follows.

Step 1 Identify all the alternatives to be evaluated and the evaluated attributes. Construct an origin hesitant fuzzy linguistic decision matrix $H=(h_{ij})_{m\times n}$, and normalize H into the decision matrix H by Eq. (20). The set of incomplete weight information Λ is given in advance.

$$h_{ij}=\begin{cases} h_{ij} & for\ bebefit\ attribute\ c_j \\ neg(h_{ij}) & for\ cost\ attribute\ c_j \end{cases} \tag{20}$$

Step 2 Calculate the defuzzified values of h_{ij} by Eq. (2) and (8) according to DM's attitudes.

Step 3 Obtain the HFLTSs-positive ideal solution A^{+}, the HFLTSs-negative ideal solution A^{-} and the HFLTSs-balanced ideal solution $\bar{A}$ using Eq. (2-4) and (8).

Step 4 Construct and solve the mathematical programming (19) to efficiently use the DM's three attitudes from the risk aversion perspective. To do this, we can derive the optimal weight vector of the attributes $w^{*}=(w_1^{*},w_2^{*},...,w_n^{*})^T$.

Step 5 Compute the DM's optimal perceived utility values V_i^{*} for alternative A_i by Eq. (17). Further, obtain the ranking order of alternatives according to V_i^{*}, the greater the value, the better the alternative.

4 Illustrative example

In the following, we further verify the practicality and reliability of the proposed method by utilizing a practical example:

There is an investment company, which wants to seek an optimal technology company to invest. There is a panel with five possible alternatives: A_1 is a network technology company, A_2 is an information technology company, A_3 is an education technology company, A_4 is an environmental technology company, and A_5 is a biotechnology technology company. It is necessary to compare these alternatives so as to select the optional one, taking into account four attributes: (1) c_1: the risk analysis, (2) c_2: the growth analysis, (3) c_3: the social-political impact analysis, (4) c_4: the technical difficulty analysis. Suppose that the weight vector of the attributes is completely unknown. Moreover, assume that $S=\{s_0=Very\,Poor,\ s_1=Poor,\ s_2=Medium\,Poor,\ s_3=Fair,\ s_4=Medium\,Good,\ s_5=Good,\ s_6=Very\,Good\}$ and linguistic scale function $g=g_1$. The five possible alternatives $A_i\,(i=1,2,...,5)$ are evaluated under the attributes $c_j\,(j=1,2,...,4)$ using the HFLTSs by several decision makers, and the comprehensive decision matrix is listed in Table I.

Table 1. Decision matrix.

	c_1	c_2	c_3	c_4
A_1	$\{s_2,s_4\}$	$\{s_5\}$	$\{s_1,s_3\}$	$\{s_1,s_3\}$
A_2	$\{s_1,s_3\}$	$\{s_2,s_3\}$	$\{s_4,s_5\}$	$\{s_4,s_5\}$
A_3	$\{s_2,s_4\}$	$\{s_2,s_5\}$	$\{s_5\}$	$\{s_1\}$
A_4	$\{s_5\}$	$\{s_2,s_4\}$	$\{s_1,s_2\}$	$\{s_4,s_5\}$

A_5	$\{s_4, s_6\}$	$\{s_2, s_3\}$	$\{s_1, s_2\}$	$\{s_5, s_6\}$

Step 1. The decision matrix H is constructed from table I based on cost criteria c_1, c_4 and benefit criteria c_2, c_3 by Eq. (20).

$$H = \begin{bmatrix} \{s_2,s_4\} & \{s_5\} & \{s_1,s_3\} & \{s_3,s_5\} \\ \{s_3,s_5\} & \{s_2,s_3\} & \{s_4,s_5\} & \{s_1,s_2\} \\ \{s_2,s_4\} & \{s_2,s_5\} & \{s_5\} & \{s_5\} \\ \{s_1\} & \{s_2,s_4\} & \{s_1,s_2\} & \{s_1,s_2\} \\ \{s_0,s_2\} & \{s_2,s_3\} & \{s_1,s_2\} & \{s_0,s_1\} \end{bmatrix};$$

Step 2. The defuzzified values of H can be calculated by Eq. (2) and (8) as

$$F(H) = \begin{bmatrix} 0.375 & 0.833 & 0.208 & 0.542 \\ 0.542 & 0.354 & 0.688 & 0.188 \\ 0.375 & 0.396 & 0.833 & 0.833 \\ 0.167 & 0.375 & 0.188 & 0.188 \\ 0.042 & 0.354 & 0.188 & 0.021 \end{bmatrix};$$

Step 3. HFLTSs-PIS A^+, HFLTSs-NIS A^- and HFLTSs-BIS $\overline{A}$, as well as their defuzzified values are as follows:

$$A^+ = \left[\{s_3,s_5\},\{s_5\},\{s_5\},\{s_5\}\right], \qquad F(A^+) = [0.542, 0.833, 0.833, 0.833];$$

$$A^- = \left[\{s_0,s_2\},\{s_2,s_3\},\{s_1,s_2\},\{s_0,s_1\}\right], \qquad F(A^-) = [0.042, 0.354, 0.188, 0.021];$$

$$\overline{A} = \left[\{s_2,s_3\},\{s_3,s_4\},\{s_3,s_4\},\{s_2,s_3\}\right], \qquad F(\overline{A}) = [0.354, 0.521, 0.521, 0.354];$$

Step 4. According to the mathematical programming (19), we set $\alpha = 0.88$ and $\gamma = 0.3$ which was derived by experimental verifications in [24], and obtained the optimal weight vector of the attributes ($w^* = (w_1^*, w_2^*, ..., w_n^*)^T$) in three different situations using lingo. The optimal attribute weights based on the HFLTSs-BS $\overline{A}$ are as follows:

$$\overline{w}^* = (0.244, 0.258, 0.256, 0.242)^T;$$

Step 5. Compute the DM's optimal perceived utility values V_i^* for alternative A_i.

$$P = \begin{bmatrix} 0.140 & 0.316 & 0.045 & 0.144 \\ 0.209 & 0.119 & 0.221 & 0.040 \\ 0.140 & 0.137 & 0.271 & 0.222 \\ 0.049 & 0.128 & 0.037 & 0.040 \\ -0.014 & 0.119 & 0.037 & -0.018 \end{bmatrix};$$

$$V_1 = 0.645, V_2 = 0.589, V_3 = 0.770, V_4 = 0.254, V_5 = 0.124;$$

Thus, the most desirable alternative is A_3.

Table 2. Ranking results

DM's attitudes	Parameters	Results	Ranking
neutral	$\alpha = 0.88$, $\beta = 0.3$	$V_1 = 0.645, V_2 = 0.589, V_3 = 0.770, V_4 = 0.254, V_5 = 0.124$	$A_3 > A_1 > A_2 > A_4 > A_5$
	$\alpha = 0.66$, $\beta = 0.2$	$V_1 = 1.062, V_2 = 0.959, V_3 = 1.250, V_4 = 0.571, V_5 = 0.331$	$A_3 > A_1 > A_2 > A_4 > A_5$
	$\alpha = 0.44$, $\beta = 0.1$	$V_1 = 1.548, V_2 = 1.482, V_3 = 1.727, V_4 = 1.087, V_5 = 0.787$	$A_3 > A_1 > A_2 > A_4 > A_5$
optimistic	$\alpha = 0.88$, $\beta = 0.3$	$V_1 = 0.464, V_2 = 0.280, V_3 = 0.712, V_4 = 0.043, V_5 = 0.028$	$A_3 > A_1 > A_2 > A_4 > A_5$
	$\alpha = 0.66$, $\beta = 0.2$	$V_1 = 0.699, V_2 = 0.487, V_3 = 1.036, V_4 = 0.121, V_5 = 0.086$	$A_3 > A_1 > A_2 > A_4 > A_5$
	$\alpha = 0.44$, $\beta = 0.1$	$V_1 = 1.118, V_2 = 0.886, V_3 = 1.506, V_4 = 0.361, V_5 = 0.282$	$A_3 > A_1 > A_2 > A_4 > A_5$
pessimistic	$\alpha = 0.88$, $\beta = 0.3$	$V_1 = 0.873, V_2 = 0.857, V_3 = 0.984, V_4 = 0.653, V_5 = 0.295$	$A_3 > A_1 > A_2 > A_4 > A_5$

	$\alpha=0.66$, $\beta=0.2$	$V_1=1.301, V_2=1.273, V_3=1.397,$ $V_4=1.072, V_5=0.602$	$A_3 > A_1 > A_2 > A_4 > A_5$
	$\alpha=0.44$, $\beta=0.1$	$V_1=1.905, V_2=1.880, V_3=1.992,$ $V_4=1.699, V_5=1.154$	$A_3 > A_1 > A_2 > A_4 > A_5$

Table 3 Optimal alternative obtained by the proposed method

Alternatives	RT-based method		TOPSIS method		EDAS method	
	Comprehensive perceived utility value	The ranking order	The closeness coefficient(RC) to the ideal solutions	The ranking order	The appraisal score(AS)	The ranking order
A_1	0.645	2	0.6	2	0.785	2
A_2	0.589	3	0.525	3	0.613	3
A_3	0.770	1	0.725	1	0.994	1
A_4	0.254	4	0.25	4	0.079	4
A_5	0.124	5	0.175	5	0	5

To verify the effects of the parameters in RT, we took the relevant parameters for the PT method from the experimental data to do sensitivity analysis. The ranking information for all situations are summarized in Table II, which shows that the smaller parameter α and β are, the greater the comprehensive perceived utility values are. At the same time, when the parameters α and β remain unchanged and the DMs' evaluation attitudes changed, that is, the language scale function and the compared values of alternatives changed, the comprehensive perceived utility values are also different. The finding can allow decision makers to express their evaluations preference using several different linguistic scale functions, which can fully reflect DMs' attitudes and can help decision makers to identify different types of best alternatives by calculating the perceived utility of alternatives according to different compared alternatives. In addition, for a convenient comparison with the RT-based MADM method proposed in this paper, we calculated the results for the illustrative example using two other MADM methods: the TOPSIS method [16][36] and the DEAS method [37]. The relevant ranking results are presented in Table III. Table III shows that the optimal alternative obtained by the proposed method was consist with the results derived by the TOPSIS and the EDAS methods, which proved the validity of the proposed method.

5 Conclusion

It is not rational to assuming that the decision makers are fully rational in decision making process. In this paper, the hesitant fuzzy linguistic MADM method based on the regret theory has been investigated. According to the regret theory, decision makers may focus not only on the absolute values but also on the regret values of alternatives. Based on the DMs' different evaluation attitudes, several types of best alternatives have been defined in different uncertainty situations, and models have been established to identify the corresponding optimal attribute weights and best alternatives. The proposed methods, we have efficiently integrated regret aversion into the decision analysis process, which avoids decision bases or errors caused by judgments that only depend on the objective reference information. Through sensitivity analysis, we concluded that the DM's regret aversion affects the perceived utility values of alternatives, although the ranking order is sometimes the same as other MADM methods.

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