Interval Information Risky Multiple Attribute Decision-Making Method Based On Regret Theory

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Abstract

Aiming at the problem of multi-attribute decision making with attribute values as interval numbers, this paper proposes a decision analysis method based on regret theory. Firstly, the risk decision-making evaluation matrix is obtained through expert questionnaires and statistical methods, and the influence of different physical dimensions on decision-making results is eliminated. Secondly, calculate the relative closeness of each scheme as the utility value of attribute value and construct the regret-rejoice matrix. Finally, the comprehensive regret-rejoice value of each scheme is calculated in turn, and then all schemes are sorted, and an example is given to verify the feasibility and effectiveness of the method.

Keywords

Risky Decision-Making, Interval Information, Regret Theory.

1. Introduction

In reality, some risk-based multi-attribute decision-making problems are often encountered. The main feature of the decision-making problem is to consider multiple natural states and predict the probability of occurrence of each state, and the attribute values of the scheme are different for different natural states. Risk-based multi-attribute decision making is widely used in new product development and investment project selection [1]. For example, LeEco Group has invested heavily in mobile phone manufacturing and smart car R&D. Due to the large scope of the company's layout, the risk of capital chain breakage was not taken into account, and ultimately the bankruptcy caused by the turnover of funds, which made many senior decision makers regret it. Therefore, scholars at home and abroad have carried out many researches on risk-based multi-attribute decision-making in recent years. The risky multi-attribute decision-making problem under interval information has also received much attention.

There are two main methods for solving risky multi-attribute decision-making problems. One class is a complete rational decision based on the theory of expected utility. The expected utility theory proposed by Ellsberg [7] is the main decision criterion of behavioral decision theory. The classic of this theory is that the form of utility function satisfies the nature of probability linearity. On the basis of subjective probability and expected utility theory, Savage [8] systematically gave seven axioms of subjective probability expectant utility theory.

The other is to consider that the behavior of decision makers is bounded rational, mainly with prospect theory and regret theory. Among the prospect theory, many experimental studies show that decision makers have reference dependence, loss avoidance, sensitivity decline, probability judgment distortion and other psychological characteristics [2]. At present, the research has been fruitful, such as: Zhang Xiao and Fan Zhiping [3] for the risky mixed multi-attribute decision-making problem that gives decision-makers the expectation information, according to the idea of prospect theory, calculated the size of the comprehensive foreground value Sort. Liu et al. [4] ranked the scheme by establishing the prospect value function and the interval probability weight function for the risky multi-attribute decision problem with the attribute value as the uncertain language phrase and the state probability as the interval number. At the same time, the theory of regret is to consider that the decision-making process of decision-makers will be affected by the psychological behavior of decision-making losses greater than expected, thus resulting in "repentance evasive" behavior and influence decision-making. This theory is proposed by Bell [5], Loomes [6], which provides a new idea for considering the decision-makers' psychological behaviors to deal with uncertain decision-making problems. For example, Zhang Xiao et al. [2] proposed a decision analysis method based on regret theory for risky multi-attribute decision making problem with interval number for both attribute value and state probability. Although both can reasonably solve the risky multi-attribute decision-making problem, the regret theory has certain characteristics and advantages. For example, the prospect theory needs to set the ideal reference point according to the expectations of the decision makers, with certain subjectivity. The regret theory avoids such defects. At the same time, the "regret value" and the "rejoice value" are included into the utility function for analysis which has better explanatory power [6].

There have been abundant research achievements on the problem of risky decision making, but the research results of risky decision-making based on regret theory are not rich, especially for risk-based multi-attribute decision-making problems with interval number. Based on this, this paper proposes an interval information risky multi-attribute decision-making method based on regret theory.

2. Regret theory

The core idea of the regret theory [6-7] is that in the decision-making process, the decision maker will compare the results of the options he considers with the possible results of other options. If they find that other programs get better results, they will regret it. On the contrary, feel rejoice.

Let x and y respectively indicate that the results can be obtained by selecting A and B, and the perceived utility of decision A is：

$$u(x,y)=v(x)+R\big(v(x)-v(y)\big) \tag{1}$$

$v(x)$ and $v(y)$ represent respectively the utility obtained by the decision maker from the results of scenarios A and B, and the function x is a monotonically increasing concave function. When $R\big(v(x)-v(y)\big)>0$ indicates as a rejoice value. When $R\big(v(x)-v(y)\big)<0$ indicates regret value.

3. Model for risky decision making with Interval information based on regret theory

Aiming at problem of risky multi-attribute decision making, let $A=\{A_1,A_2\cdots A_m\}$ represents a collection of m scenarios, where A_i represents the i alternative; suppose $C=\{C_1,C_2\cdots C_n\}$ represents a collection of n attribute, where C_j represents the j attribute, $C_1,C_2\cdots C_n$ are additively independent; w$=\{w_1,w_2\cdots w_n\}$ represents the weight vector of the attribute, Where w_j is the weight of C_j, and $\sum_{j=1}^{n} w_j=1$; $\theta=\{\theta_1,\theta_2\cdots\theta_h\}$ represents a collection of natural states, Where θ_t is the t state, p_t represents the probability of occurrence in state θ_t, and meets $0\le p_t\le 1$, $\sum_{t=1}^{h} p_t=1$; $E=\left[e_{ij}\right]_{m\times n}$ represents the risk decision matrix, Where e_{ij} represents the attribute evaluation value of item A_i for attribute C_j, $i=1,2\cdots,m$, $j=1,2\cdots,n$.

Above all, the calculation steps of method for decision making with risk of Interval information based on regret theory are as follows:

Step 1: Using the expert questionnaire survey and statistical methods to obtain the risk decision evaluation matrix, the evaluation value is recorded as e_{ij}.

Step 2: Make the evaluation information dimensionless. In order to eliminate the influence of different physical dimensions on the decision-making results, the evaluation information of each index is standardized by the method similar to literature [9]. The standardization process is shown in Table 1:

Table 1 Mixed information normalization processing

Type of information	Efficiency standardization	Cost-type normalization
Interval number $e_{ij}=\left[\underline{e}_{ij},\bar{e}_{ij}\right](j\in C_2)$	$Q_2=\sqrt{\sum_{i=1}^{m}\left(\left(\underline{e}_{ij}\right)^2+\left(\bar{e}_{ij}\right)^2\right)}$ $x_{ij}=\left[\underline{e}_{ij}/Q_2,\ \bar{e}_{ij}/Q_2\right]$	$Q_2^{'}=\sqrt{\sum_{i=1}^{m}\left(\left(1/\underline{e}_{ij}\right)^2+\left(1/\bar{e}_{ij}\right)^2\right)}$ $x_{ij}=\left[\left(1/\bar{e}_{ij}\right)/Q_2^{'},\left(1/\underline{e}_{ij}\right)/Q_2^{'}\right]$

Step 3: Calculate the relative closeness of each scheme as the utility value of attribute value. By comparing the case values under each attribute, two reference points are selected, which are positive and negative ideal point solutions of mixed information. Recorded as x^+ and x^- respectively, $x^+=\left(x_1^+,x_2^+,\cdots,x_n^+\right)$, $x^-=\left(x_1^-,x_2^-,\cdots,x_n^-\right)$, Table 2. The positive ideal solution is the most satisfactory solution for each index attribute, and the negative ideal solution is the most unsatisfactory solution. The theory of regret can be known that since other schemes are inferior to the positive ideal scheme, the decision makers will have regrets. Conversely, other schemes are better than negative ideals, and decision makers will be rejoice. $x-x^+$ represents the degree of regret between positive ideal values relative to other attribute values, $x-x^-$ represents the degree of rejoice between negative ideal values relative to other attribute values, The Euclidean distance formula is used to obtain the positive and negative ideal distance, and the degree of regret and the degree of rejoice are described. The scheme perceived utility value of this paper is expressed by the closeness of the positive and negative ideal distance ratio. Table 3. (Note: The following data is normalized data)

Table 2 Mixed information positive and negative ideal point

Type of information	Positive ideal point x^+	Negative ideal point x^-
Interval number $e_{ij}=\left[\underline{e}_{ij},\bar{e}_{ij}\right](j\in C_2)$	$\left[\max\limits_{1\le i\le m,1\le j\le n}\left(\bar{e}_{ij}\right),\max\limits_{1\le i\le m,1\le j\le n}\left(\bar{e}_{ij}\right)\right]$	$\left[\min\limits_{1\le i\le m,1\le j\le n}\left(\underline{e}_{ij}\right),\min\limits_{1\le i\le m,1\le j\le n}\left(\underline{e}_{ij}\right)\right]$

Table 3 Mixed information positive and negative ideal distance

Type of information	Positive ideal distance $d\left(x_{ij},x^+\right)$	Negative ideal distance $d\left(x_{ij},x^-\right)$
Interval number $e_{ij}=\left[\underline{e}_{ij},\bar{e}_{ij}\right](j\in C_2)$	$\sqrt{\frac{1}{2}\left[\left(\underline{e}_{ij}-\underline{e}_{ij}^{+}\right)^2+\left(\bar{e}_{ij}-\bar{e}_{ij}^{+}\right)^2\right]}$	$\sqrt{\frac{1}{2}\left[\left(\underline{e}_{ij}-\underline{e}_{ij}^{-}\right)^2+\left(\bar{e}_{ij}-\bar{e}_{ij}^{-}\right)^2\right]}$

The perceived utility value of mixed information scheme of the scheme i

$$C_{ij}=\frac{d\left(x_{ij},x^-\right)}{d\left(x_{ij},x^+\right)+d\left(x_{ij},x^-\right)} \tag{2}$$

Step 4: Formulas (3)-(5) Construct regret-rejoice matrix, and calculate the comprehensive regret-rejoice value of each scheme according to formulas (6)-(7). Because decision makers are risk-averse to rejoice and regret， regret-rejoice function $R(\Delta v)$ that compares the two schemes is a monotonically increasing concave function:

$$R(\Delta v)=1-\exp(-\gamma\Delta C) \tag{3}$$

$\gamma>0$ indicates the decision maker's regret avoidance coefficient, the greater the γ, the greater the degree of regret avoidance, ΔC indicates the difference in utility between the two schemes, Under natural state P_t and attribute C_j, the regret value of scheme A_i relative to scheme A_k is expressed as:

$$R_{ikj}^{t}=\begin{cases}1-\exp\left[-\gamma\left(C_{ij}^{t}-C_{kj}^{t}\right)\right] & C_{ij}^{t}<C_{kj}^{t}\\ 0 & C_{ij}^{t}\geq C_{kj}^{t}\end{cases} \tag{4}$$

The rejoice value of scheme A_i relative to scheme A_k is expressed as:

$$G_{ikj}^{t}=\begin{cases}0 & C_{ij}^{t}<C_{kj}^{t}\\ 1-\exp\left[-\gamma\left(C_{ij}^{t}-C_{kj}^{t}\right)\right] & C_{ij}^{t}\geq C_{kj}^{t}\end{cases} \tag{5}$$

Obviously, $R_{ikj}^{t}\leq 0$， $G_{ikj}^{t}\geq 0$， and $R_{iij}^{t}=\mathrm{G}_{iij}^{t}=0$.

On this basis, the regret matrix $R_j=\left[R_{ikj}\right]_{m\times n}$ and the rejoice matrix $G_j=\left[G_{ikj}\right]_{m\times n}$ of the pairwise comparison of the schemes for each attribute are respectively established. Where R_{ikj} and G_{ikj} respectively represent the regret value and the rejoice value for the attribute C_j scheme A_i. The calculation formula are

$$R_{ikj}=\sum_{t=1}^{h}p_t R_{ikj}^{t},\quad i,k\in 1,2\cdots m,\quad j\in 1,2\cdots n \tag{6}$$

$$G_{ikj}=\sum_{t=1}^{h}p_t G_{ikj}^{t},\quad i,k\in 1,2\cdots m,\quad j\in 1,2\cdots n \tag{7}$$

In this formula, $G_{ikj}\in[0,1]$, the probability p_t is calculated by using the PIGNISTIC probability transformation method and the risk probability f in the risk environment[11].

Let $\Theta=\{\theta_1,\theta_2,\cdots,\theta_l\}$ be the set of all possible states for solving decision problems, if there is a mapping $f:2^{\Theta}\to[0,1]$, so that $f(\phi)=0$, $\sum_{A\subseteq\Theta}f(A)=1$, Then f is called the basic belief assignment function defined on the set Θ.

$$P(\theta)=\sum_{\theta\in B}\frac{f(\theta)}{|B|} \tag{8}$$

In this formula, θ is the element in the set $\Theta=\{\theta_1,\theta_2,\cdots,\theta_l\}$, B is a subset of 2^{Θ} containing element θ. $P(\theta)$ represents the probability of occurrence in the natural state.

Step 5: Calculate comprehensive regret-rejoice value and sort all the schemes according to formulas (9)-(13).

Using a simple weighting principle, establish a comprehensive regret value matrix $\mathrm{R}=\left[R_{ik}\right]_{m\times n}$ and a comprehensive rejoice value matrix $G=\left[G_{ik}\right]_{m\times n}$ for the pairwise comparison of the schemes, where R_{ik} and G_{ik} respectively represent the comprehensive regret value and comprehensive rejoice value of scheme A_i relative to scheme A_k, the calculation formulas are

$$R_{ik}=\sum_{j=1}^{n}w_j R_{ikj},\quad i,k\in 1,2\cdots m \tag{9}$$

$$G_{ik}=\sum_{j=1}^{n}w_j G_{ikj},\quad i,k\in 1,2\cdots m \tag{10}$$

According to the matrices R and G, calculate respectively the overall regret value $R\left(A_i\right)$ and the overall rejoice value $G\left(A_i\right)$ of the scheme A_i relative to all other schemes :

$$R\left(A_i\right)=\sum_{k=1}^{m}R_{ik}\qquad i,k\in 1,2\cdots m \tag{11}$$

$$G\left(A_i\right)=\sum_{k=1}^{m}G_{ik}\qquad i,k\in 1,2\cdots m \tag{12}$$

In formula (11), $R(A_i)$ can be regarded as the psychological perception of decision makers that scheme A_i is inferior to all other schemes, $R(A_i)\le 0$, the larger $R(A_i)$ value, the smaller the disadvantage of scheme A_i relative to other schemes. In formula (12), $G(A_i)$ can be regarded as the psychological perception of decision makers that scheme A_i is better than all other schemes, $G(A_i)\ge 0$, the larger $G(A_i)$ value, the bigger the advantage of scheme A_i relative to other schemes. Further, according to the literature [10], calculate the sorting value of scheme A_i:

$$\Phi(A_i)=R(A_i)+G(A_i) \tag{13}$$

$\Phi(A_i)$ is considered as the relative overall psychological perception of decision makers on option A_i. The larger the value of $\Phi(A_i)$, the better the scheme A_i. So all schemes are sorted according to the value of $\Phi(A_i)$.

4. Analysis of examples

Considering the choice of a new product development project, a company wants to develop a new electronic product. There are five schemes (A_1, A_2, $\cdots$, A_5) that can be selected, and three attributes (C_1, C_2, C_3) are considered, where C_1 indicates development costs (unit: 10,000 Yuan), C_2 indicates product sales (unit: 10,000 units/year), C_3 indicates the rate of return (unit: %/year). Among the three attributes, C_1 is a cost type attribute, C_2 and C_3 are benefit type attributes, and attribute values of the three attributes are interval numbers. Here, it is assumed that the values falling within the interval obey the normal distribution, and the future market environment has three possible natural states (S_1, S_2, S_3), which represent good, medium, and poor, respectively, and their probability of occurrence is respectively $p_1=0.3$, $p_2=0.4$, $p_3=0.3$. The attribute weight vector provided by the decision maker is $w=(0.35,0.25,0.4)$, and the risk decision matrix is:

scheme	$p_1=0.3$			$p_2=0.4$			$p_3=0.3$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
A_1	[80,90]	[100,120]	[12,16]	[90,100]	[80,100]	[9,12]	[90,100]	[70,80]	[6,8]
A_2	[90,100]	[110,120]	[12,18]	[100,110]	[90,100]	[10,15]	[110,120]	[80,90]	[7,10]
A_3	[90,110]	[120,130]	[15,22]	[100,120]	[100,110]	[13,20]	[110,130]	[80,100]	[8,12]
A_4	[100,110]	[100,110]	[18,23]	[110,130]	[80,90]	[15,20]	[120,130]	[60,80]	[6,10]
A_5	[110,120]	[120,150]	[20,25]	[115,130]	[100,120]	[12,18]	[120,140]	[90,100]	[8,10]

In this paper, the natural state of the market p and the attribute weight w are the original data of the literature [11]. All attributes are normalized according to Table 1. Secondly, according to Table 2, the positive ideal solution x^+ and the negative ideal solution x^- of the interval information are obtained, and the positive ideal distance and the negative ideal distance of the interval information are obtained according to Table 3. Then calculate the utility value of attribute value of each scheme in different natural states according to formula (4), as shown below.

$$Z_1 = \begin{bmatrix} 0.7829 & 0.2554 & 0.2019 \\ 0.5311 & 0.3090 & 0.2890 \\ 0.4389 & 0.5000 & 0.5000 \\ 0.3022 & 0.1351 & 0.6325 \\ 0.1234 & 0.6422 & 0.7533 \end{bmatrix}, \quad p_1 = 0.3$$

$$Z_2 = \begin{bmatrix} 0.7878 & 0.3090 & 0.1807 \\ 0.5393 & 0.3828 & 0.3524 \\ 0.4446 & 0.6172 & 0.6258 \\ 0.2605 & 0.1667 & 0.7148 \\ 0.1933 & 0.6910 & 0.5350 \end{bmatrix}, \quad p_2 = 0.4$$

$$Z_3 = \begin{bmatrix} 0.6863 & 0.3828 & 0.2171 \\ 0.3993 & 0.6172 & 0.4336 \\ 0.3376 & 0.6910 & 0.6126 \\ 0.2294 & 0.3090 & 0.3874 \\ 0.1973 & 0.8333 & 0.5000 \end{bmatrix}, \quad p_3 = 0.3$$

Then, according to the formulas (5)-(9), the regret matrix R_j and the Xinxi matrix G_j for the comparison of the suppliers of the respective attributes are respectively established.

Regret-rejoice matrix when market status is good: ($R1\_C$ stands for regret matrix; $G1\_C$ stands for rejoice matrix，1 stands for attribute C_1)

```
R1_C =

         0         0         0         0         0
   -0.0785         0         0         0         0
   -0.1087   -0.0281         0         0         0
   -0.1551   -0.0711   -0.0419         0         0
   -0.2188   -0.1301   -0.0993   -0.0551         0

G1_C =

         0    0.0785    0.1087    0.1551    0.2188
         0         0    0.0281    0.0711    0.1301
         0         0         0    0.0419    0.0993
         0         0         0         0    0.0551
         0         0         0         0         0
```

Regret-rejoice matrix when market status is medium:

```
R1_C =

         0         0         0         0         0
   -0.0774         0         0         0         0
   -0.1084   -0.0288         0         0         0
   -0.1714   -0.0872   -0.0568         0         0
   -0.1952   -0.1093   -0.0783   -0.0203         0


G1_C =

         0    0.0774    0.1084    0.1714    0.1952
         0         0    0.0288    0.0872    0.1093
         0         0         0    0.0568    0.0783
         0         0         0         0    0.0203
         0         0         0         0         0
```

Regret-rejoice matrix when market status is poor:

```
R1_C =

         0         0         0         0         0
   -0.0899         0         0         0         0
   -0.1103   -0.0187         0         0         0
   -0.1469   -0.0523   -0.0330         0         0
   -0.1580   -0.0625   -0.0430   -0.0097         0


G1_C =

         0    0.0899    0.1103    0.1469    0.1580
         0         0    0.0187    0.0523    0.0625
         0         0         0    0.0330    0.0430
         0         0         0         0    0.0097
         0         0         0         0         0
```

Let $\gamma = 0.3$, then get the comprehensive regret matrix R and the comprehensive rejoice matrix G of the pairwise comparison according to the formulas (11)-(12), as shown below.

$$R = \begin{bmatrix} 0 & -0.0271 & -0.0644 & -0.0442 & -0.0727 \\ -0.0285 & 0 & -0.0388 & -0.0282 & -0.0472 \\ -0.0382 & -0.0089 & 0 & -0.0089 & -0.0172 \\ -0.0645 & -0.0448 & -0.0565 & 0 & -0.0443 \\ -0.0669 & -0.0355 & -0.0344 & -0.0185 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0.0285 & 0.0382 & 0.0645 & 0.0669 \\ 0.0271 & 0 & 0.0089 & 0.0448 & 0.0355 \\ 0.0644 & 0.0388 & 0 & 0.0565 & 0.0344 \\ 0.0442 & 0.0282 & 0.0089 & 0 & 0.0185 \\ 0.0727 & 0.0472 & 0.0172 & 0.0443 & 0 \end{bmatrix}$$

Finally, the ranking result of each logistics service provider is calculated according to formula (13)(14)(15) (Table 4).

Table 4 Comparison of the sorting results of the two methods when γ=0.3

γ	method	Sequencing results
γ=0.3	Literature [11] sorting results	$A_3 > A_5 > A_2 > A_1 > A_4$
	Sort results in this article	$A_3 > A_5 > A_1 > A_2 > A_4$

At the same time, the paper gives a comparison of the two methods under different γ values. The results are shown below (Table 5).

Table 5 Comparison of the results of the following two methods

γ	method	Sequencing results
γ=0.2	Literature [11] sorting results	$A_3 > A_5 > A_2 > A_1 > A_4$
	Sort results in this article	$A_3 > A_5 > A_1 > A_2 > A_4$
γ=0.3	Literature [11] sorting results	$A_3 > A_5 > A_2 > A_1 > A_4$
	Sort results in this article	$A_3 > A_5 > A_1 > A_2 > A_4$
γ=0.5	Literature [11] sorting results	$A_3 > A_5 > A_2 > A_1 > A_4$
	Sort results in this article	$A_3 > A_5 > A_1 > A_2 > A_4$

It can be seen from the above that the ranking result of the method is basically consistent with the results obtained in the literature [11], but the calculation of the method is simpler.

5. Conclusion

In the actual decision-making, the decision-makers' regret behavior and other psychological behaviors influence the decision-making process and results. Therefore, in the theoretical framework of risk decision-making, it is of theoretical and practical significance to study the influence of psychological behavior factors on decision-making results. This paper proposes a decision analysis method based on regret theory for risky multi-attribute decision making. Firstly, the risk decision-making evaluation matrix is obtained through expert questionnaire survey and statistical methods. Secondly, the relative closeness of each scheme is calculated as the utility value of the attribute value to construct the regret-rejoice matrix. Finally, the comprehensive regret-rejoice value of each scheme is calculated in turn. Sort all the scenarios. The method is clear in thought, involves fewer formulas, and extends the application scope of regret theory, which provides a new idea for solving the risky decision problem under interval information.

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